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Fluctuations in Moon's period and its relationship with the perigees of the Moon and the Sun

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Abstract. Just as the regression of the nodes of the lunar orbit is connected with the periodicity in the eclipse phenomenon, the advance of perigee is connected with the periodicity in the fluctuations of Moon's synodic period. From the data for 495 synodic months it is found that the gross fluctuation pattern repeats itself after about 112 synodic months, which is close to the period of one complete revolution of the Moon's perigee. Within each gross fluctuation pattern there are eight smaller cycles of about 14 synodic months. The causes of these features are suggested in terms of the angles between the new moon and the perigees of the Moon and the Sun. The dependence of the amplitude of variation with respect to various angular separations can possibly be understood by the fact that the tidal effect on the Moon varies with its relative distance from the Sun.

1. Introduction

If the Earth and the Moon are regarded as point masses and no other body is present then all the six elements of the Moon's orbit will remain the same forever. However the Earth and the Moon are not point masses and the system is not an isolated one. It is influenced by the gravitational attraction of other bodies in the solar system. Sun being the most massive and sufficiently close causes appreciable perturbation on the orbital elements compared to other bodies. On account of these perturbations orbital elements experience both periodic and secular changes (Abhyankar 1999; Danby 1962; Moulton 1970).

Advance of the perigee and regression of the nodes are two important secular perturbations on the lunar orbit due to the Sun. It is well known that the latter is related to the periodicity in occurrences of eclipses. The relationship of the former with the periodicity of the fluctuations in Moon's period is generally not mentioned in astronomy books. It is the aim of this paper to highlight this phenomenon and indicate how the angular separation between the new moon and the perigees of the Moon and the Sun alter the average value of the synodic period of the Moon.

2. Advance of perigee and its relationship with periodicity of fluctuation in moon's period

According to Kepler's third law of planetary motion the square of the orbitl period 'P' is proportional to the cube of the semi-major axis 'a' i.e. P^2 / a^3 = constant. We have noted above that 'a' changes periodically due to perturbations hence the period 'P' of the Moon's orbit will also undergo periodic changes. It is well known that the synodic period (time taken by the moon to return to the same phase as seen from the Earth i.e. to say from one new moon to the next) fluctuates between 29 and 30 days, the average being 29.530588 days.

To study various features of the fluctuations in the synodic period and its relationship with the advance of perigee, we have taken times of 495 new moons from Jan. 1976 to Dec. 2015 (Meeus 1995). The difference between the actual synodic period and its mean value, called the amplitude of fluctuation, ΔP, is shown in Fig. 1 as a function of the synodic month number.

The following characteristics of variation in the period may be noted. Starting from a value of ΔP close to zero we notice that ΔP goes on increasing on the positive side and attains a maximum value. It then starts decreasing and reaches a peak on the negative side of ΔP, then reversing the trend it reaches a value of ΔP close to zero. This cycle is completed in about 14 synodic months. Eight such cycles form another gross fluctuation cycle of about 112 months, which is close to the period of one complete revolution of Moon's perigee. We shall discuss these features qualitatively.

In Table 1 we give the value of ΔP for each of the eight peaks on either side of the mean for the four gross cycles along with various angular separations of interest at that time of the new moon. It may be noted that the peaks of a new gross cycle have slightly different amplitudes which increases and decreases with the increase and decrease in the value of the angular separation of the new moon and the Moon's perigee. This indicates a strong correlation between the two. It is clear that the gross fluctuation cycle depends upon the phase difference between the perigee of the Moon and Perigee of the Sun, while the smaller cycle depends on the phase difference between the new moon and the perigee of the Moon's orbit. In one synodic period the perigee of the Moon moves ahead of the new moon by about 26°. Hence it will require about 14 synodic months for the new moon to come the same position with respect to its perigee. This explains the 14 synodic months cycle of fluctuation.

It is found that for the gross cycle the amplitude for each of the eight peaks depends on the angular separation of the new moon with the Moon's perigee, with the Sun's perigee and with the separation of Moon's and Sun's perigees.

We have seen above that 14 synodic months is the time interval for the recurrence of lunar perigee (or apogee) at new moon. Further it may be noted that 8 x 14 synodic months $\approx$ 9 solar years. Therefore this is the time interval for the recurrence of both the lunar and solar perigees

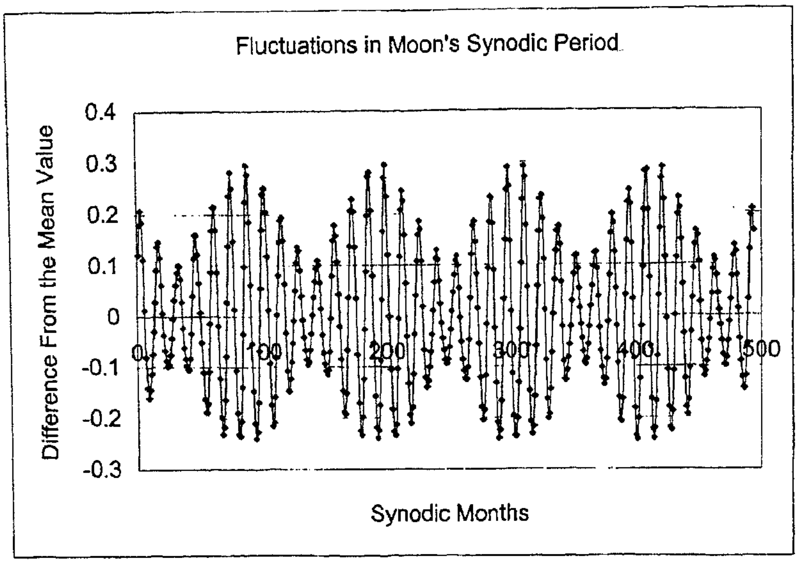
Table 1. Maximum deviations of the synodic period from the mean value.

On the positives side of the mean On the negative side of the mean.

Peak	Cycles 1/2/3/4					Peak	Cycles 1/2/3/4				
No.	Date	ΔP	EMP	ESP	SMP	No.	Date	ΔP	EMP	ESP	SMP
1	Jul 5	.101	199	209	10	1	Feb 26	–.104	48	84	36
	Jul 25	.110	210	228	18		Mar 18	–.115	60	104	44
	Aug 14	.117	221	148	26		Apr 7	–.125	71	123	52
	Sep 3	.124	233	267	34		Mar 29	–.136	57	114	57
2	Aug 22	.161	200	255	56	2	Apr 15	–.188	49	131	82
	Sep 11	.178	211	275	64		May 5	–.192	60	150	90
	Oct 1	.185	223	295	72		Apr 26	–.203	46	142	95
	Sep 29	.198	208	286	77		May 16	–.207	57	161	103
3	Oct 9	.215	201	303	102	3	May 4	–.231	25	149	125
	Oct 29	.229	213	323	110		May 24	–.233	35	168	133
	Oct 20	.232	199	314	115		May 15	–.240	21	159	138
	Nov 9	.245	211	334	123		Jun 3	–.242	32	178	146
4	Nov 26	.283	204	352	148	4	Jun 22	–.235	24	195	171
	Dec 17	.281	216	12	156		Jun 12	–.239	10	186	176
	Dec 7	.290	202	3	161		Jun 2	–.235	21	205	184
	Dec 27	.284	214	23	169		Jun 22	–.240	7	196	189
5	Dec 15	.296	180	11	190	5	Jul 10	–.240	0	213	213
	Jan 4	.296	193	31	199		Jul 29	–.234	11	232	222
	Dec 25	.292	178	22	204		Jul 20	–.231	356	223	227
	Jan 15	.289	190	42	212		Aug 10	–.224	8	242	235
6	Feb 1	.251	183	60	236	6	Jul 28	–.215	335	231	256
	Jan 22	.246	169	50	241		Aug 17	–.210	346	250	264
	Feb 12	.235	181	70	250		Aug 8	–.201	332	241	270
	Feb 3	.230	166	61	255		Aug 29	–.194	343	261	278
7	Mar 21	.194	185	107	282	7	Sep 14	–.147	336	278	302
	Mar 12	.186	170	98	288		Sep 5	–.140	322	269	307
	Apr 1	.175	182	118	296		Sep 26	–.127	333	289	316
	Mar 22	.165	168	108	301		Sep 16	–.119	319	279	321
8	Apr 9	.137	160	125	325	8	Dec 1	–.095	5	356	352
	Apr 29	.130	171	145	334		Dec 2	–.094	17	17	360
	Mar 19	.118	182	164	342		Jan 10	–.096	29	37	8
	May 10	.114	168	155	347		Jan 30	–.099	41	57	16

ΔP : Actual synodic period - Value of mean synodic period
EMP : Longitude of moon - Longitude of moon's perigee
ESP : Longitude of sun - Longitude of sun's perigee
SMP : Longitude of sun's perigee - Longitude of moon's perigee

(or lunar apogees and solar perigee) at new moon, which is the period of the gross cycle. These recurrences explain the features in Fig. 1.



3. Discussion

The results highlight the effects of the eccentricity of the Moon's orbit around the Earth (0.05) and the eccentricity of the Earth-Moon orbit around the Sun (0.02) on the fluctuations in the period of the Moon.

Increase in period means increase in the semi-major axis and hence increase the energy. Maximum energy occurs when the Sun is at perigee and the Moon is at apogee. Conversely the maximum decrease in energy with respect to the average occurs when the Sun is at apogee and the Moon is at perigee. The amplitude of various peaks on the positive and the negative sides of the mean can be understood by this underlying physics.

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