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Research Note

Implications of the crustal moment of inertia for neutron-star equations of state

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Abstract. The dynamical models for neutron star glitches and postglitch behaviour, and the information they yield on the crustal moment of inertia, are applied to a catalogue of neutron star models. The implications for neutron star equations of state are discussed. The postglitch timing data from the Vela pulsar are interpreted to rule out one of the softer equations of state for neutron matter with an admixture of hyperons, if the neutron star mass is 1.4 solar mass.

Key words: neutron star: equation of state – neutron star: glitches

1. Introduction

Models for neutron star glitches and postglitch behaviour yield information on the fraction of moment of inertia residing in the superfluid in the star's crust. This superfluid has interesting dynamical properties because the quantized vortices whose distribution determines its rotational dynamics are pinned by the crust lattice. Although the bulk of the neutron star matter is believed to be superfluid, it is only the pinned crust superfluid that is linked to the dynamical relaxation of the star through observed changes in the spin-down rate according to these models. Detailed models to the eight postglitch data sets from the Vela pulsar indicate that the crust superfluid comprises about $3.4\,10^{-2}$ of the star's moment of inertia (Alpar et al. 1992). This is a lower bound to the fractional moment of inertia of the entire crust, lattice plus superfluid, at densities below $\rho = 2\,10^{14}\,\mathrm{g\,cm^{-3}}$. Here we discuss the dynamical models and the information they yield on the crustal moment of inertia, and apply it to a catalogue of neutron star models. In general, lower bounds on the fractional moment of inertia in the crust will tend to disfavour the softest equations of state for a given mass, and select lower mass models for a given equation of state.

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2. Equation of state and crustal moment of inertia

The key input that determines neutron star models is the equation of state (EOS), namely, $P = P(\rho)$, where P and ρ are the pressure and density of degenerate matter at high densities. The density expected in neutron stars spans a rather wide range: from about $7.8\,\mathrm{g\,cm^{-3}}$ near the surface to more than $10\rho_0$ in the interior, where $\rho_0 = 2.8\,10^{14}\,\mathrm{g\,cm^{-3}}$, the equilibrium nuclear matter density. The composition of matter at $\rho > \rho_0$ is expected to be mostly neutron matter in beta equilibrium. However, significant admixtures of other elementary particles, such as pions and hyperons, remains a distinct possibility. Despite two decades of theoretical investigations, there still exists a lack of consensus on the exact composition and behaviour of the EOS of high density matter (Lattimer et al. 1990). The nearly two dozen EOS models available in the literature comprise a rather broad set, and at best represent sophisticated parameterizations. The usual practice is to perform such parameterizations so as to obtain desirable properties for nuclear matter at the saturation density ρ_0. However, this in itself does not necessarily provide the trend of the EOS for $\rho \geq 4\rho_0$ (Prakash & Ainsworth 1987; Horowitz & Serot 1987; Stock 1989; Baym 1991), which is the density regime of greatest importance for neutron star structure.

The EOS used in this survey were taken from the following references: (A) Pandharipande (1971a), neutron matter (B) Pandharipande (1971b), hyperonic matter, (C) Moszkowski (1974), hyperonic matter, (D) Bethe–Johnson (1974), model V (neutron matter), (E) Bethe–Johnson (1974), model V (hyperonic matter), (F) Canuto & Chitre (1974), solid neutrons, (G) Walecka (1974) neutron matter, (H) Brown & Weise (1976), neutron matter with pion condensation, (I–L) Glendenning (1985), hyperonic matter, models I–IV, (M–P) Prakash et al. (1988), models PAL 1, PAL 2, PAL 3 and PAL 4 and (Q–S) Wiringa et al. (1989), models AV 14+UV II, UV 14+UV II and UV 14+TN I. Models (M–S) refer to neutron matter in beta equilibrium. The above EOS models constitute a representative set to describe high density neutron star matter, and span the range of very soft EOS to very stiff ones. Model (G) is a very stiff EOS whereas models (B) (F), (H) are very soft EOS, with the others intermediately placed.

The moment of inertia of a neutron star rotating with a uniform angular velocity Ω (as seen by a distant observer), that is "slow" in comparison to that for which equatorial mass shedding occurs, is given by (Hartle & Thorne 1968):

$$I=\frac{8\pi}{3}\int_0^R \frac{r^4(\rho+P/c^2)}{(1-2Gm/rc^2)}\frac{\bar{\omega}(r)}{\Omega}\,e^{-\nu(r)}\,dr, \qquad (1)$$

where R is the radius of the star and $\bar{\omega}(r)$ is the angular velocity of the fluid element relative to the local inertial frame. For a given EOS, $P=P(\rho)$, the mass, density, pressure and potential profiles: $m(r), \rho(r), P(r)$ and $\nu(r)$ respectively are obtained by solving numerically the Tolman–Oppenheimer–Volkoff equation for stellar structure (Misner et al. 1970).

For each of the EOS models, we have calculated the following structure parameters: the gravitational mass, the radius (R), the superfluid pinned region of the inner crust, defined as the radial extent corresponding to the density interval ($2\,10^{14}$–$2\,10^{13}$) g cm^{-3} and $\alpha\equiv I_p/I$, the ratio of the moment of inertia of the pinned superfluid crust with respect to the total moment of inertia of the star. We find that although the magnitude of the inner crust thickness increases with an increase in the rotation rate of the star (as expected), the ratio α does not exhibit any appreciable variation with respect to the rotation rate, which was varied from $\Omega=\Omega_s$ (the secular rotational instability point) down to period = 89.296 ms (to correspond to the Vela pulsar). The rotating configurations with $\Omega=\Omega_s$ are relevant to rapidly rotating neutron stars such as millisecond pulsars (Datta 1988), while the conclusions summarized below are expected to be general, relevant for the Vela pulsar and slower pulsars as well as for more rapidly rotating pulsars.

3. Results and discussion

Our calculations indicate the following general behavior. For a given EOS (and a fixed rate of rotation), the inner crust thickness decreases as the mass of the star increases. Consequently, as the mass increases, the ratio α decreases. Table 1 gives the calculated values of (a) the maximum gravitational mass of stable neutron stars, (b) the maximum mass that is consistent with $\alpha\geq 3.4\,10^{-2}$ and (c) the value of α for the $1.4M_\odot$ neutron star, for each EOS model. As mentioned earlier, the lower limit of α estimated from the postglitch data of the Vela pulsar is $3.4\,10^{-2}$ (Alpar et al. 1992). This rules out higher gravitational masses for all the EOS (except models (H–N), for which all allowed stable masses are consistent with the Vela pulsar constraint).

Table 1

Equation of state	$M_{max}/M_\odot$	$M_{max}/M_\odot$ consistent with $\alpha\geq 3.4\,10^{-2}$	α for neutron star of $1.4M_\odot$
A	1.658	1.572	$5.518\,10^{-2}$
B	1.415	1.214	$1.394\,10^{-2}$
C	1.751	1.481	$4.060\,10^{-2}$
D	1.758	1.742	$7.103\,10^{-2}$
E	1.653	1.645	$6.797\,10^{-2}$
F	1.367	1.242	—
G	2.236	2.230	$1.221\,10^{-1}$
H	1.038	—	—
I	1.812	—	$3.161\,10^{-1}$
J	1.787	—	$3.691\,10^{-1}$
K	1.973	—	$3.725\,10^{-1}$
L	1.978	—	$3.200\,10^{-1}$
M	1.510	—	$1.360\,10^{-1}$
N	1.770	—	$1.938\,10^{-1}$
O	1.431	1.400	$4.916\,10^{-2}$
P	1.705	1.700	$9.203\,10^{-2}$
Q	2.127	1.532	$4.181\,10^{-2}$
R	2.182	1.786	$6.329\,10^{-2}$
S	1.839	1.671	$5.902\,10^{-2}$

All measured neutron star masses are consistent with a value of $1.4M_\odot$ (Van Paradijs 1991). Assuming that the mass of the Vela pulsar is also $1.4M_\odot$, we ask whether the $1.4M_\odot$ neutron star model for the various EOS considered here can support a crust that corresponds to $\alpha=3.4\,10^{-2}$. Table 1 shows that for most of the EOS models, a $1.4M_\odot$ neutron star is able to support a crust with $\alpha=3.4\,10^{-2}$. A notable exception to this is the EOS model (B), for which the calculated α for a $1.4M_\odot$ neutron star is less than half the limit estimated from observations of the Vela pulsar.

The assumption that the Vela pulsar mass is $1.4M_\odot$ and the requirement that α be at least $3.4\,10^{-2}$ in magnitude, rule out the EOS model (B). This EOS is one of the softer EOS, corresponding to neutron matter along with admixtures of protons and hyperons such as Λ, $\Sigma^{\pm,0}$ and Δ^0. Other soft neutron matter EOS, like the models (M–P) due to Prakash et al. (1988) are not constrained by these criteria. The presence of pion condensate in high density matter also leads to a soft EOS (Brown & Weise 1976), model (H). The maximum mass of a neutron star with a pion condensate (and in beta equilibrium) is lower than $1.4M_\odot$ (see Lattimer et al. 1990). Another soft neutron matter EOS, due to Canuto & Chitre (1974), model (F), gives neutron star maximum less than $1.4M_\odot$. For this reason we have not applied the $1.4M_\odot$ assumption to these two EOS models in the present survey, but have listed the maximum masses consistent with $\alpha \geq 3.4\,10^{-2}$.

The EOS model (B) is based on the use of a non-relativistic approach: a variational many-body method and a potential description for the interactions. The presence of hyperons implies lesser value for the fermi momentum of the neutrons, the dominant constituents of high density matter. This makes the hyperonic EOS relatively soft, and is reflected in the value for the maximum gravitational mass, which is comparatively low. The other hyperonic EOS considered here, namely models (C), (E) and (I–L) satisfy the constraint on α. Models (C) and (E) also use a potential description for the interactions and a non-relativistic many-body theory. Models (I–L) are based on relativistic mean field theory. Unlike nucleon–nucleon interactions, hyperon–nucleon and hyperon–hyperon interactions at nuclear densities are not well determined from experiments, and are even more uncertain at higher densities. So, there exists a considerable spread among the various hyperonic EOS models.

It may be mentioned here that observational data provide a very precise estimate for the mass of the binary pulsar PSR 1913+16, namely, $(1.442 \pm 0.003) M_{\odot}$ (Taylor & Weisberg 1989). This mass constraint would already discard the EOS models (B), (F), (H) and (O) as unrealistic, quite independently of the crustal moment of inertia criterion that we have considered in this paper. If we use the PSR 1913+16 mass constraint, the maximum gravitational mass for the Vela pulsar implied by the limit on α is in the range $(1.48–2.23) M_{\odot}$ (see Table 1).

To summarize, we have applied the information on the neutron star crustal moment of inertia, implied by the dynamical models for pulsar glitches and postglitch behaviour, to constrain the equation of state of high density matter. Using the data from the Vela pulsar, we find that the hyperonic EOS model (B), due to Pandharipande (1971b), is disfavoured. A study applying a weaker, model independent bound to postglitch data allows even this EOS at neutron star mass equal to $1.4 M_{\odot}$ (Link et al. 1992). A reexamination of the role of hyperons in the EOS of high density matter clearly deserves further investigation. The method of using postglitch timing data has proved restrictive for only one EOS, with an assumed mass value. This method may prove to be a more effective criterion to distinguish neutron star EOS, if future postglitch data yield larger values of the fraction of moment of inertia residing in the neutron star crust superfluid.

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