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Research Note

On the Maxwellian alternative to the galactic dark matter problem

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Abstract. Recently a new type of gravitational coupling due to a gravo-inductive force field generated by mass currents was introduced to account for constant rotation velocities at large distances from galactic cores. It is pointed out that the exact analogy with electromagnetism assumed in postulating this force would for reason of consistency imply a vanishingly small coupling rather than the anomalously large coupling required.

The gravo-inductive radiation is compared with gravitational radiation for its role in the damping of the motion of bound systems.

Key words: galactic dynamics – non-Newtonian gravitation

In a recent paper Fahr has proposed (Fahr 1990), yet another alternative explanation of the flat rotation curves of spiral galaxies different from the dark matter hypothesis. By analogy with electromagnetism which has two distinct field components E and B, he has postulated a gravo-inductive force B_g (the analogue of B) distinct from the "electric" force E_g which is the familiar Newtonian force. He further assumes that masses couple gravitationally with different strengths y_1 and y_2 to the fields E_g and B_g.

In close analogy, a Lorentz-force type of relation is written for the equation of motion of a mass m, i.e.:

$$m\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{v}=y_1 m\boldsymbol{E}_g+y_2 m\frac{1}{c_g}(v\times\boldsymbol{B}_g). \quad (1)$$

The field $\boldsymbol{B}_g$ like magnetism is velocity dependent (i.e. proportional to $\boldsymbol{v}$). Being the "radiation" part of the field it would fall off with distance only as $1/r$ rather than $1/r^2$, so that balance with the centripetal force mv^2/r, would imply a constant v (i.e. independent of r) as distances larger than the outer boundary of the galactic core where this force is expected to dominate. In order that the B force dominate and balance the centripetal force at large distances its coupling to mass would have to be about 10^6 times larger than to the E_g part of the field, i.e. $y_2/y_1\approx 10^6$. y_1^2 is identified with the Newtonian gravitational constant G.

Furthermore the analogy implies that the velocity of propagation of the gravito-inductive field, i.e. c_g in Eq. (1) is to be identified with light velocity c in free space. This is also required for the Lorentz invariance of the equations. Now in electromagnetism the velocity of light is related to the permittivity and permeability, i.e. dielectric coefficients, respectively through

$$c=\frac{1}{\sqrt{\varepsilon_0\mu_0}}. \quad (2)$$

Now for the "electric" part of the field, the analogy of Newton's law with Coulomb's force law for electric charges, would imply

$$\varepsilon_g=1/G, \quad (3)$$

where ε_g is the gravitational analogue of dielectric constant. So, if the speed of the gravo-inductive perturbation is also to be c, e.g. Eq. (2) would suggest that the corresponding gravo-inductive permeability is

$$\mu_g=G/c^2=7.4\ 10^{-29}. \quad (4)$$

This is the factor which multiply B_g in Eq. (1) and wherever the gravo-inductive force is present. This is such a small coupling that its effects would be very difficult to observe. Of course the idea of a gravo-inductive force is not new, and has been considered by several authors. There are many arguments in the literature to justify the existence of such a force with just this value of μ_g. A magnetic type gravitational force with μ_g as given by Eq. (4) was suggested for instance by Forward (1963) and Salisbury et al. (1974). Moreover, it is well known that equations of motion in general relativity already contain gravi-magnetic forces. The geodesic equation can take a form exactly similar to the Lorentz force with:

$$\boldsymbol{E}_g=\nabla \boldsymbol{V}, \qquad \boldsymbol{B}_g=v\times\nabla\times A,$$

with

$$A_i=ch_{0i}, \qquad V=c^2h_{00}.$$

However unlike in electromagnetism, the Lorentz type equations of motion are *not independent* of the field equations. The electric and magnetic parts of the field are components of the same linearised metric tensor and therefore must have the same coupling constant. This would again imply a μ_g given by Eq. (4). This is the factor we would expect to have rather than the postulated 10^6 enhancement.

Fahr (1990) has also made connection with other anomalous couplings like the fifth force which would couple to baryon number or isospin. In fact one can easily see that such composi-

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tion dependent forces would give a coupling 10^{-3} times weaker than the Newtonian force.

For instance for the baryonic charge of a mass

$$M=(A-Z)m_n+Zm_p$$

we can write

$$A\,m_p=M+(A-Z)\,(m_p-m_n), \quad (5)$$

m_p, m_n are the proton and neutron masses, Z the atomic number, $A-Z$ the number of neutrons.

For the baryon dipole vector moment we can write

$$D_{bi}=\Sigma m r_i+(A-Z)\,(m_p-m_n)\,r_i,$$

and so for a system of mass M, velocity v and size L, we would have a typical dipole radiation power given by

$$P_{dip}\approx\alpha[(m_n-m_p)/m_p]^2\,M^2\,v^4/L^2c^3. \quad (6)$$

The factor in brackets has a value $\approx 10^{-6}$. So such couplings cannot generate the large coupling sought. The quadrupolar gravitational radiation for the system would have a power:

$$P_{quadr}\approx M^2v^6/L^2c^5. \quad (7)$$

Thus in considering the damping effects of gravo-inductive dipole radiation, it would be useful to realize that

$$P_{dip}/P_{quad}\approx 10^{-6}(c/v)^2.$$

So only for low velocities $v<10^{-3}c$, would the dipole radiation be larger than the usual gravitational quadrupolar radiation.

For the Earth–Sun system where $v\approx 10^{-4}c$, this would apply; the damping time for the system with gravitational radiaiton is $\approx 10^{19}$ yr. The damping time can be estimated as

$$t\simeq\frac{GMm}{RP_{dip}}=\frac{\text{orbital energy}}{p_{dip}} \quad \text{or} \quad t\simeq\frac{\text{orbital energy}}{P_{quad}}.$$

In conclusion gravo-inductive forces cannot account for the flat rotation curves.

References

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