MAXIMUM ACCELERATION AND MAGNETIC FIELD IN THE EARLY UNIVERSE

(Letter to the Editor)

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(Received 18 June, 1990)

Abstract. The maximum acceleration at the Planck epoch is shown to be related to the maximum magnetic field and curvature as well as temperature in that era. Spin-torsion effects at that epoch also lead to same value.

Recently there has been a lot of interest in the notion of maximum acceleration following the early work of Caianiello (1981, 1984) who showed that it arises when quantum considerations, i.e., position-momentum uncertainty relations are incorporated into the geometry of a particle in eight-dimensional phase space. Subsequently, other authors (Gasperini, 1987; Gasperini and Scarpetta, 1989) also obtained the same maximum acceleration from other considerations, i.e., the existence of an upper limit to the acceleration of material bodies given by

$$a_{\max} \simeq mc^3/\hbar\,, \tag{1}$$

seems to be recurrent feature in all those works. Thus although the classical theory does not require any limit on particle acceleration (but only on velocity), quantum considerations do seem to impose a maximum acceleration (Toller, 1988; Wood *et al.*, 1989). The question of maximum acceleration in the early universe has also been considered by some authors recently (Voráček, 1989). Again in a recent work (de Sabbata, 1988) we had dealt wth magnetic fields in the early universe, obtaining a maximum primeval magnetic field at the Planck epoch of $\approx 10^{58}$ G. Subsequently, flux conservation would have made the field behave as

$$Bt = BT^{-2} = \text{constant}\,, \tag{2}$$

with expansion time t and temperature T. Such a magnetic field would also have accelerated charged particles in the early universe. For a particle with charge e, the relativistic Larmor frequency is given by

$$\omega_L = (ecB/\hbar)^{1/2}\,, \tag{3}$$

implying a magnetic energy $\hbar\omega_L \approx (e\hbar cB)^{1/2}$. It is known that this would imply a critical

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Astrophysics and Space Science **176**: 145–148, 1991.

magnetic field when $(e\hbar cB)^{1/2}$ equals the rest mass energy mc^2 of the particles – i.e., when the gyroradius r/G becomes smaller than the Compton length. Thus quantum considerations impose a critical magnetic field strength of

$$B_c = m^2c^3/e\hbar\,. \tag{4}$$

At the Planck epoch when $m \simeq m_{\mathrm{Pl}} = (\hbar c/G)^{1/2}$, this implies a $B_{\max}$ of $c^4/eG \approx 10^{58}$ G, the same value found earlier (de Sabbata and Sivaram, 1988). Now Equation (3) would imply a circular or helical acceleration in the magnetic field of the charged particle with $a \simeq \omega_{\mathrm{L}}^2 r_G$, or

$$a \simeq (ecB/\hbar)r_G\,. \tag{5}$$

If we substitute $B_{\max} \approx c^4/Ge$ and the corresponding $r_G \approx (\hbar G/c^3)^{1/2}$ at the Planck epoch in Equation (5), gives the maximum acceleration experienced by the charged particles in the early universe as

$$a_{\max} \simeq c^{7/2}/(\hbar G)^{1/2} \simeq m_{\mathrm{Pl}}c^3/\hbar = 5 \times 10^{53}\ \mathrm{cm\ s^{-2}}\,. \tag{6}$$

Note that in Equation (6) for the maximum acceleration of a charged particle it is not involved the electric charge. Instead there is the Planck constant which is connected with spin.

Conversely using this value for $a_{\max}$ in Equation (5) and noting that the value of $B_{\max}$ as obtained earlier (de Sabbata, 1988) from flux conservation and torsion is also 10^{58} G, we can fix the value of the fundamental electric charge e. Also in de Sabbata and Sivaram (1990) we had obtained an expression for the classical electron radius (and thereby the electric charge) by considering a finite energy model of the electron supported by spin-torsion showing that spin is more fundamental and the charge can be derived from it.

In general, substituting Equation (4) for the critical field and Equation (3) for ω_{L} with the corresponding $r_G \sim \hbar/mc$ in Equation (5), gives for the limiting acceleration for particles of mass m the expression

$$a_{\max} \approx mc^3/\hbar\,, \tag{7}$$

which agrees with that obtained by other authors and which for $m = m_{\mathrm{Pl}}$ becomes that given by Equation (6). Alternatively an $a_{\max}$ as given by Equation (6) would imply a maximum magnetic field of $B_{\max} \approx 10^{58}$ G at the Planck epoch in agreement with our earliest work (de Sabbata and Sivaram, 1988), showing that the spin h is more fundamental. Moreover, as implied by the Unruh–Davis effect, an accelerated body would find itself immersed in a heat bath with a black-body temperature given by

$$T = \hbar a/K_B c\,. \tag{8}$$

Thus an $a_{\max}$ at the Planck epoch of $c^{7/2}/(\hbar G)^{1/2}$ as given by Equation (6) would imply a maximum temperature of

$$T_{\max} \simeq (1/k_B)\,(\hbar c^5/G)^{1/2} \simeq 10^{32}\ \mathrm{K} \tag{9}$$

at the Planck epoch from Equation (8). This again is consistent with the earlier work. Also the Hawking effect would connect temperature with the curvature of space. For a space of constant curvature given by Λ {with $R_{\mu\nu} = \Lambda g_{\mu\nu}$ and $R_{\mu\nu\rho\sigma} = (\Lambda/3)(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\nu}g_{\rho\sigma})$}, the Hawking temperature is

$$T_{\rm H} = \hbar c \Lambda^{1/2}/K_B \,. \tag{10}$$

As quantum effect on geometry of space-time would imply a maximum curvature of $\Lambda_{\rm max} \sim c^3/\hbar G \approx 10^{66}\ {\rm cm}^{-2}$ (see Equation (12)) for ($\hbar \Rightarrow 0$, in the classical case, it can be infinite, i.e., unbounded), Equation (10) would imply that

$$T_{\rm max} \approx (1/K_B)(\hbar c^5/G)^{1/2} \simeq 10^{32}\ {\rm K}\,, \tag{11}$$

i.e., the same as Equation (9)! In turn from Equation (7) this would imply a maximum acceleration of $a_{\rm max} \approx c^{7/2}/(\hbar G)^{1/2}$ at the Planck epoch. Thus the equivalence of the Hawking and Unruh temperatures would imply the connection between acceleration and curvature as

$$a = c^2(\Lambda)^{1/2} \,. \tag{12}$$

As quantum gravity effects would impose a limiting curvature $\Lambda_{\rm max} \approx c^3/\hbar G$, this would imply a limiting acceleration of $a_{\rm max} = c^{7/2}/(\hbar G)^{1/2}$ while the equivalent of Hawking temperature and Hagedorn temperature ($T = mc^2/K_B$) would imply $a_{\rm max} = mc^3/\hbar$ in agreement with the above results. When $\hbar \Rightarrow 0$, i.e., at classical regime, all these quantities are unbounded.

We can still note that spin-torsion effects at the Planck epoch would give the same value of $a_{\rm max}$. With torsion dominating at the earliest epoch (de Sabbata, 1990), the acceleration for the Robertson–Walker universe is given by

$$\ddot{R} = -(3G^2s^2)/2c^4R^5 \,, \tag{13}$$

as $s = \hbar$ (the spin), Equation (11) is maximized for $R = R_{\rm min}$ which is just the Planck length. So with $R = (\hbar G/c^3)^{1/2}$, $s = h$, this gives $a_{\rm max} \simeq c^{7/2}/(\hbar G)^{1/2}$, the same as found earlier! In all these expressions it is the spin which enters the formula. It is so to be noted that in Equation (13), the sign of $\ddot{R}$ is negative. This implies that with torsion (as discussed earlier by de Sabbata and Sivaram, 1990) we have the required condition for an inflationary expansion with negative $\ddot{R}$.

As Equations (5), (8), and (12) which relate the acceleration to the magnetic field, temperature, and curvature, respectively, show, the maximum acceleration in the early universe would be epoch-dependent.

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