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Interfacial hydromagnetic waves with shear and inclined magnetic fields

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Abstract. The combined effect of non-parallel propagation, steady flow and inclined magnetic field on the propagation characteristics of hydromagnetic surface waves is examined. The fluid is assumed to be perfectly conducting, infinite in extent with an interface which supports body modes as well as surface modes. This model also supports fast, Alfvén and slow modes depending on the parametric values of the system. These modes have interesting applications in the solar corona and solar wind.

Keywords : Hydromagnetic Waves, Corona, Solar Wind

1. Introduction

Hydromagnetic Waves play an important role in the transfer of energy in the atmosphere of the Sun. The physical structure of the solar corona is very complicated. The interaction between the magnetic field, shear flows and the plasma plays a central role in the physical properties of this medium and is responsible for many phenomena occurring in the corona. The combined effect of non-parallel propagation, steady flow and inclined magnetic field on the propagation characteristics of hydromagnetic surface waves is examined. A dispersion relation is derived for a plasma which is infinite in extent, perfectly conducting and having an interface wherein the plasma parameters like density and pressure have discontinuities. This study is an extension to the earlier work of Joarder and Satya Narayanan (2000). The number of parameters characterizing the present model is large and hence one is forced to restrict the analysis to specific parametric values only.

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Depending on the values of the parameters, the model supports slow as well as fast magnetosonic waves. These modes have interesting application in the solar corona and solar wind.

2. Magnetic Interface with Flow

We assume the equilibrium such that

$$\frac{d}{dx}(p_0 + \frac{B^2}{2\mu}) = 0$$

where p_0, B are the gas pressure and magnetic field, respectively. Assuming small perturbations about the basic flow fields such as density, pressure, magnetic field and shear flow, as given below

$$\bar{\rho} = \rho_0(x) + \rho, \quad \bar{\mathbf{v}} = \mathbf{U}(x) + \mathbf{v}, \quad \bar{p} = p_0(x) + P, \quad \bar{\mathbf{B}} = \mathbf{B}(x) + \mathbf{b}$$

where $\mathbf{U} = (0, U_y, U_z)$, $\mathbf{B} = (0, B_y, B_z)$, we can simplify the basic equations of MHD to get a single differential equation for the velocity component v_x as

$$\hat{v_x}'' + (m_0^2 + k_Y^2)\hat{v_x} = 0, \qquad m_0^2 = \frac{\Omega^4 + k_z^2 c_f^2(\omega_T^2 - \Omega^2)}{c_f^2(\omega_T^2 - \Omega^2)}$$

where Ω, ω, k_x, k_y, c_f are the Doppler shifted frequency, frequency, wavenumbers and sound speed, respectively.

The solution to this equation is of the form

$$\hat{v_x} = Ae^{(m_0^2+k_y^2)^{1/2}x} - Be^{-(m_0^2+k_y^2)^{1/2}x} = 0.$$

Introducing suitable boundary condtions and simplifying, the dispersion relation can be written as

$$\rho_0(\omega_{A0}^2 - \Omega_0^2)(m_e^2 + k_y^2)^{1/2} + \rho_e(\omega_{Ae}^2 - \Omega_e^2)(m_0^2 + k_y^2)^{1/2} = 0$$

Using the trigonometric expressions for the wavenumber $\mathbf{k}$ and magnetic field $\mathbf{B}$ as follows:

$$\mathbf{k} = (\mathbf{0}, \mathbf{k}\mathbf{sin}\theta, \mathbf{k}\mathbf{cos}\theta), \qquad \mathbf{B} = (\mathbf{0}, \mathbf{B}\mathbf{sin}\gamma, \mathbf{B}\mathbf{cos}\gamma)$$

the final dispersion relation can be written as

$$\rho_0(k^2 c_{A0}^2 cos^2(\theta-\gamma) - \Omega_0^2)(m_e^2 + k_y^2)^{1/2} + \rho_e(k^2 c_{Ae}^2 cos^2(\theta-\gamma) - \Omega_e^2)(m_0^2 + k_y^2)^{1/2} = 0$$

$$m_{0,e}^2 = \frac{\Omega_{0,e}^4 + k^2 c_{f0,e}^2 cos^2\theta(k^2 c_{T0,e}^2 cos^2(\theta-\gamma) - \Omega_{0,e}^2)}{c_{f0,e}^2(k^2 c_{T0,e}^2 cos^2(\theta-\gamma) - \Omega_{0,e}^2)}.$$

The case of $\mathbf{U} = \mathbf{0}, \mathbf{k} = (\mathbf{0}, \mathbf{k}\mathbf{sin}\theta, \mathbf{k}\mathbf{cos}\theta)$ and $\mathbf{B} = (\mathbf{0}, \mathbf{B_{0,e}}\mathbf{cos}\gamma_{\mathbf{0,e}}, \mathbf{B_{0,e}}\mathbf{sin}\gamma_{\mathbf{0,e}})$ was discussed by Uberoi and Satya Narayanan (1986). For the case $\gamma = 0$, the above relation reduces to that derived by Joarder and Satya Narayanan (2000). When both $\theta = 0$ and $\gamma = 0$, the above reduces to the one derived by Nakariakov and Roberts (1995).

To begin with, let us consider the simple case of low β plasma. The pressure balance condition in this case is

$$p_0 + \frac{B_0^2}{2\mu} = p_e + \frac{B_e^2}{2\mu}$$

so that at the interface

$$\rho_0 c_{A0}^2 \approx \rho_e c_{Ae}^2.$$

Here 0 and e represent both sides of the interface. Define the following quantities :

$$\alpha = \frac{\rho_e}{\rho_0}, \quad \delta = \frac{U_e}{U_0} \quad \epsilon = \frac{U_0}{c_{A0}}, \quad x = \frac{\omega}{kc_{A0}}.$$

It can be shown that

$$\frac{\Omega_0}{kc_{A0}} = x - \epsilon \qquad \frac{\Omega_e}{kc_{A0}} = x - \delta\epsilon$$

$$(m_0^2 + k_y^2)^{1/2} = k(1 - (x-\epsilon)^2)^{1/2} \quad (m_e^2 + k_y^2)^{1/2} = k(1 - \alpha(x - \delta\epsilon)^2)^{1/2}.$$

The dispersion relation can be simplified as

$$\{cos^2(\theta-\gamma)-(x-\epsilon)^2\}\{1-\alpha(x-\epsilon\delta)^2\}^{1/2}+\{cos^2(\theta-\gamma)-\alpha(x-\epsilon delta)^2\}\{1-(x-\epsilon)^2\}^{1/2}=0.$$

The special case wherein $\mathbf{U}=\mathbf{0}$, i.e. $\epsilon=0$ was considered by Satya Narayanan (1997).

The relation for the case when $\mathbf{U}=\mathbf{0}$ and $\gamma=0$ reduce to

$$(1-\alpha x^2)^{1/2}(cos^2\theta-x^2)+(1-x^2)^{1/2}(cos^2\theta-\alpha x^2)=0$$

which can be simplified to yield

$$\alpha x^4-(1+\alpha)x^2+cos^2\theta(1+sin^2\theta)=0$$

so that

$$x^2=(1+\alpha)\pm[(1-\alpha)^2+4\alpha sin^4\theta]^{1/2}/2\alpha.$$

The above relation is the same as given in Jain and Roberts (1991).

The dispersion relation is to be solved for the relevant parameters of the model. Since the number of parameters is large, it is impossible to give a single criterion for the existence of surface waves. Moreover, depending on the value of the basic flow, negative energy instabilities may appear which was discussed in the earlier paper by Joarder and Satya Narayanan (2000). However, for the specific parametric values one can solve the dispersion relation for the existence of surface (interface) waves. This study is in progress and will be reported shortly. We also plan to apply this study for situations that may be found in coronal loops and also solar wind.

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