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Clustering of galaxies by the α-effect

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ABSTRACT
The development of large-scale flows from the stresses generated by small-scale flows in a Navier–Stokes system is studied in order to model the large-scale structure of the Universe. This process is an analogue of the α-effect, by which a large-scale magnetic field can be generated beginning with small-scale fields. It is shown that, under similar conditions of anisotropy and non-zero mean helicity, large-scale flows can be generated. The clustering of galaxies on all possible scales is then suggested to be a result of the hydrodynamic hierarchy.

Key words: hydrodynamics – magnetic fields – turbulence – galaxies: clustering – large-scale structure of Universe.

1 INTRODUCTION

The conventional understanding of the clustering of galaxies rests on the proposed existence of several types of dark matter, the nature of which is one of the many issues that these theories have to settle (Peebles 1971; Kolb & Turner 1990). The one-way turbulence theory, by which small structures form from large ones, was forsaken some time ago due to the large hydrodynamical time-scales and the 'bottom-up' dictates of the observations (von Weizsacker 1951; Ozernoi & Chernin 1968; Ozernoi & Chibisov 1971, 1972). Kurskov & Ozernoi (1974a, 1976) have considered the possibility of producing the 'bottom-up' hierarchy through the gravitational growth of density fluctuations. Recent developments in fluid turbulence theory point to the possibility of making large structures from small ones (Levich & Tzvetkov 1985 and references therein; Hasegawa 1985; Sulem et al. 1989). The first applications of the inverse cascade have been to planetary atmospheres (Hasegawa 1985), particularly the Earth's atmospheric turbulence (Levich & Tzvetkov 1985). Attempts to apply inverse cascade to model solar granulation on all scales have proved to be encouraging, as the predicted theoretical energy spectrum is similar to that inferred from observations of motions on the solar surface (Krishan 1991). The hierarchical structure of the Universe offers another attractive situation where inverse cascade processes might be operative (Verschuur 1991). An initial attempt in this direction was made recently (Krishan & Sivaram 1991) in which, while stressing the role of helicity fluctuations, the entire energy spectrum encompassing scales from a few kpc to a few Mpc was derived using Kolmogorovic arguments. It may be relevant here to respond to the conventional view that the decay of vorticity with the expansion of the Universe could be an impediment to the development of non-linear cascading processes essentially based on the existence of vorticity and helicity. One must remember that the decay of vorticity is concluded from a linear perturbation analysis of the fluid equations (Kurskov & Ozernoi 1974b; Kolb & Turner 1990). Now it is well known that the vorticity-generating mechanisms are embedded in the non-linear terms of the Navier–Stokes equations describing a three-dimensional system, and it is precisely for this reason that the enstrophy (square of vorticity) is not an invariant of a three-dimensional system. Furthermore, as discussed in Landau & Lifshitz (1959), a turbulent fluid exhibits two kinds of flow: the rotational flow and the potential flow confined to separate regions of the medium, and the exchange between them always occurs in one direction, i.e. fluid can always enter the region of rotational flow from the region of potential flow but never leaves it. This has been sufficiently confirmed by experiments. It is then argued that a potential flow is more akin to a laminar flow, and it is the rotational flow which brings out the cascading properties of a turbulent flow. In the light of these comments, we consider vorticity generation as a separate problem meriting discussion in its own right and proceed, assuming that there are parts of the Universe that possess sufficient vorticity to warrant the discussion of hydrodynamical processes dominated by rotational flow.

The phenomenon of fluid turbulence is approached mainly by two routes: (i) through the study of the statistically stationary states using Kolmogorovic arguments, and (ii) through the solutions of the Navier–Stokes equation in the hope that its stationary solutions will confirm the predictions of the former. Following this line of thought here, we study clustering of galaxies through the solutions of the Navier–Stokes equations and put forward two conjectures: (i) that the Universe has distinct regions of rotational and potential flow and it is in the rotational flow regions that the phenomenon of clustering takes place, whereas the voids may correspond to regions dominated by potential flow, and (ii) that the two phenomena of the large-scale structure of the Universe and the generation of large-scale magnetic fields are close relatives, if not identical twins. In the next section, we study the excitation of a large-scale flow from a small-scale flow, following Sulem et al. (1989) but incorporating the expansion of the fluid. In this paper, only linear excitation is considered, as the non-linear treatment including expansion becomes complex and may need to be tackled by numerical methods.

2 EXCITATION OF A LARGE-SCALE FLOW

Analogous to the generation of large-scale magnetic fields by the α-effect, a large-scale flow can also develop from a small-scale flow in a purely hydrodynamic system, essentially under the same conditions of anisotropy and/or parity violation. Following Sulem et al. (1989), we write the Navier–Stokes equations for the small-scale as well as for the large-scale flow including their interaction through the Reynolds stresses. We also restrict ourselves to incompressible flows. The formation of large-scale flows from small-scale compressible flows for a non-expanding fluid has been considered by Moiseev et al. (1983), and we plan to include compressibility effects in an expanding fluid in the near future. The excitation of spatial scales, much larger than those that one starts with, justifies the procedure of scale separation. The Navier–Stokes equations for a small-scale incompressible flow $\boldsymbol{U}$ in the presence of a large-scale flow $\boldsymbol{W}$ in a coordinate system comoving with the mean expansion can be written as (Kurskov & Ozernoi 1974c)

$$\frac{\partial U_i^*}{\partial t^*} + W_j^* \partial_j U_i^* = \nu^* \nabla^2 U_i^* + f_i^*, \tag{1}$$

$$\partial_j U_j^* = 0, \tag{2}$$

where

$$\boldsymbol{U} = \frac{g}{2}\boldsymbol{U}^*, \qquad \mathrm{d}t = \frac{2a}{g}\,\mathrm{d}t^*, \qquad \nu = \frac{ag}{2}\nu^*,$$

$$f_i = (g^2/4a)\, f_i^*, \qquad g = (\rho_t a^4)_{\rm eq}(\rho_t a^4)^{-1},$$

and ν is the kinematic viscosity. f_i is the anisotropic force periodic in space and time with space–time average $\langle f_i \rangle = 0$, which produces the small-scale flow $\boldsymbol{U}$. The scale factor $a(t)$ is related to the redshift z as $a(t) = (1+z)^{-1}$, and ρ_t is the sum of the matter and radiation energy densities.

The Reynolds number is given by $R = \ell_0 V_0/\nu$ for the small-scale flow $\boldsymbol{U}$ characterized by the spatial scale ℓ_0 and a typical velocity V_0.

The Navier–Stokes equation for the large-scale flow $\boldsymbol{W}$ can be written as

$$\frac{\partial}{\partial t^*} W_i^* + \partial_j (W_i^* W_j^* + \alpha_{ij}) = \nu^* \nabla^2 W_i^*,$$
$$\partial_j W_j^* = 0, \tag{3}$$

$$\alpha_{ij} = \langle U_i^* U_j^* \rangle, \tag{4}$$

and the angular brackets represent the space–time or the ensemble average. In order to calculate α_{ij}, we solve equation (1) under the assumptions that (i) the large-scale flow $\boldsymbol{W}$ is independent of space–time, (ii) the Reynolds stress term $\partial_j(U_i^* U_j^*)$ is small compared to the other terms. With these assumptions, equation (i) describes a small-scale forced flow $\boldsymbol{U}^*$ carried by the large-scale flow $\boldsymbol{W}^*$. In order to simulate a helical and parity-violating flow, the forcing term f_i can be chosen to be

$$f_x^* = f_0 \cos\left(\frac{y}{\ell_0} + \frac{\nu t^*}{\ell_0^2}\right) \exp\left[-\int\left(\frac{\nu^* - \nu}{\ell_0^2}\right) \mathrm{d}t^*\right],$$
$$f_y^* = f_0 \cos\left(\frac{x}{\ell_0} - \frac{\nu t^*}{\ell_0^2}\right) \exp\left[-\int\left(\frac{\nu^* - \nu}{\ell_0^2}\right) \mathrm{d}t^*\right], \tag{5}$$
$$f_z^* = \beta(f_x^* + f_y^*),$$

where β is a measure of the anisotropy and $\langle f_x^* \rangle = \langle f_y^* \rangle = 0$. From equation (1), we find

$$U_x^* = \frac{V_0 \nu}{\ell_0^2} \cos\left(\frac{y}{\ell_0} + \frac{\nu t^*}{\ell_0^2} - y_0\right) \exp\left[-\int\left(\frac{\nu^* - \nu}{\ell_0^2}\right) \mathrm{d}t^*\right],$$
$$\sin^2 y_0 = \left(1 + W_y^* \frac{\ell_0}{\nu}\right)^2 \left[1 + \left(1 + W_y^* \frac{\ell_0}{\nu}\right)^2\right]^{-1}, \tag{6}$$

$$u_y^* = \frac{V_0 \nu}{\ell_0^2} \cos\left(\frac{x}{\ell_0} - \frac{\nu t^*}{\ell_0^2} - x_0\right) \exp\left[-\int\left(\frac{\nu^* - \nu}{\ell_0^2}\right) \mathrm{d}t^*\right],$$
$$\sin^2 x_0 = \left(1 - W_x^* \frac{\ell_0}{\nu}\right)^2 \left[1 + \left(1 - \frac{W_x^* \ell_0}{\nu}\right)^2\right]^{-1}, \tag{7}$$

$$U_z^* = \beta(U_x^* + U_y^*), \qquad f_0 = \frac{\nu V_0}{\ell_0^2}. \tag{8}$$

By substituting for the Us in equation (4), one determines the components of the stress tensor α_{ij} as

$$\alpha_{xx} = \langle U_x^{*2} \rangle = \frac{V_0^2}{1 + [1 + (\ell_0/\nu)\, W_y^*]^2} \langle\,\rangle_{y,t^*},$$

$$\alpha_{yy} = \langle U_y^{*2} \rangle = \frac{V_0^2}{1 + [1 - (\ell_0/\nu)\, W_x^*]^2} \langle\,\rangle_{x,t^*},$$

$$\alpha_{xy} = \alpha_{yx} = 0; \qquad \alpha_{xz} = \alpha_{zx} = \beta \alpha_{xx}, \tag{9}$$
$$\alpha_{zz} = \beta^2(\alpha_{xx} + \alpha_{yy}); \qquad \alpha_{yz} = \beta \alpha_{yy},$$

$$\langle\,\rangle_{y,t^*} \equiv \left\langle \exp\left[-2\int\left(\frac{\nu^* - \nu}{\ell_0^2}\right) \mathrm{d}t^*\right] \cos^2\left(\frac{y}{\ell_0} + \frac{\nu t^*}{\ell_0^2} - y_0\right)\right\rangle$$

$$\langle\,\rangle_{x,t^*} \equiv \left\langle \exp\left[-2\int\left(\frac{\nu^* - \nu}{\ell_0^2}\right) \mathrm{d}t^*\right] \cos^2\left(\frac{x}{\ell_0} - \frac{\nu t^*}{\ell_0^2} - x_0\right)\right\rangle.$$

The linear characteristics of the large-scale flow can be studied by retaining terms linear in W_x^* and W_y^*. The linearized large-scale flow is governed by the equations

$$\frac{\partial W_x^*}{\partial t^*} - \delta \frac{\partial W_y^*}{\partial z} = \nu^* \frac{\partial^2}{\partial z^2} W_x^* \tag{10}$$

and

$$\frac{\partial W_y^*}{\partial t^*} + \delta \frac{\partial W_x^*}{\partial z} = \nu^* \frac{\partial^2}{\partial z^2} W_y^*, \tag{11}$$

which admit a solution of the form

$$W_x^* + \mathrm{i}\, W_y^* \propto \mathrm{e}^{\mathrm{i}Z/L}\, \mathrm{e}^{\Gamma(t^*)t^*}, \tag{12}$$

where

$$\Gamma(t^*) = \frac{\delta}{L} - \frac{\ell_0 V_0}{RL^2} - \frac{16}{9}\frac{\ell_0 V_0}{RL^2}\frac{T}{t^*} z_{\rm eq}^2 \ln(t^*/T),$$

$$\delta = \frac{\beta R V_0}{2}\{\exp[(1-B)\ln(2\pi)]/2\pi(1-B)\},$$

$$B = \frac{32}{9}\frac{TV_0}{\ell_0 R} z_{\rm eq}^2, \qquad z_{\rm eq} = 1.77 \times 10^4 \Omega h^2,$$

$$T = 1.1 \times 10^{11} (\Omega h^2)^{-2}.$$

L is the spatial scale of the large-scale flow, and h is Hubble's constant normalized to 75 km s^{-1} Mpc^{-1}. The time variation of the scale factor $a(t)$ is given by the Friedmann equation, as discussed by Kurskov & Ozernoi (1974c). In the above analysis we have used the large $Z (Z \gg Z_{\rm eq})$ form for the factor g. It is clear that growing modes are excited for $\Gamma > 0$. Thus the linear analysis points towards the excitation of large-scale flow of dimension L at the expense of the small-scale flow of dimension ℓ_0. The growth rate in the non-comoving frame is given by

$$\Gamma_{\rm non} = \Gamma(t^*) - H$$

and

$$t^* = \frac{4\sqrt{2}}{3} z_{\rm eq}\sqrt{Tt} \quad \text{for} \quad z \gg z_{\rm eq},$$

where H is Hubble's constant. We choose typical values of the parameters as

$$R = R_8 \times 10^8, \qquad V_0 = V_6 \times 10^6 \text{ cm s}^{-1},$$

$$\ell_0 = \ell_{18} \times 10^{18} \text{ cm}, \qquad t^* = t_{20} \times 10^{20} \text{ s},$$

$$\Omega h^2 = 10^{-1}, \qquad \beta = 1.$$

We find

$$1 - B = \frac{1.225\, V_6}{\ell_{18} R_8},$$

$$t^* = 1.1 \times 10^{10} \sqrt{t}\, \text{s},$$

$$\Gamma(t^*) = \frac{R_8 V_6}{\alpha \ell_{18}} (0.5 \times 10^{-4}) \frac{\left\{\frac{1}{2}\exp\left[\frac{1.225\, V_6}{\ell_{18} R_8}\ln(2\pi)\right]\right\}}{(2\pi \times 1.225\, V_6/\ell_{18} R_8)}$$

$$- \frac{10^{-20} V_6}{\alpha^2 R_8 \ell_{18}} - \frac{1.4 \times 10^{-20} V_6}{t_{20}^* R_8 \ell_{18} \alpha^2} \ln\left[\frac{t_{20}^*}{1.1} \times 10^9 (\Omega h^2)^2\right] \text{s}^{-1}$$

and

$$\alpha = L/\ell_0.$$

A plot of $\Gamma(t^*)$ versus t_{20}^* for different values of α is shown in Fig. 1. It is seen that the e-folding time $(\Gamma)^{-1}$ of the scales as large as 10^{27} cm (333 Mpc) is $\sim 10^{14}$ s, which is much less than the Hubble time. Furthermore, the e-folding time is a very weak function of time. Fig. 2 exhibits the variation of the growth rate versus t_{20}^* for $\ell_{18} = 10^4$ and a smaller value of the Reynolds number $R_8 = 10^{-3}$. A plot of the growth rate versus α for two values of the small scale ℓ_{18} and the Reynolds number R_8 is shown in Fig. 3. Again the e-folding times of the scales of a few hundred Mpc are much smaller than the Hubble time. In addition, a larger value of the Reynolds number R_8 admits a larger range of the flow scales.

3 CONCLUSION

In order to determine the velocity W at the large-scale L, a non-linear treatment of the Navier–Stokes equations

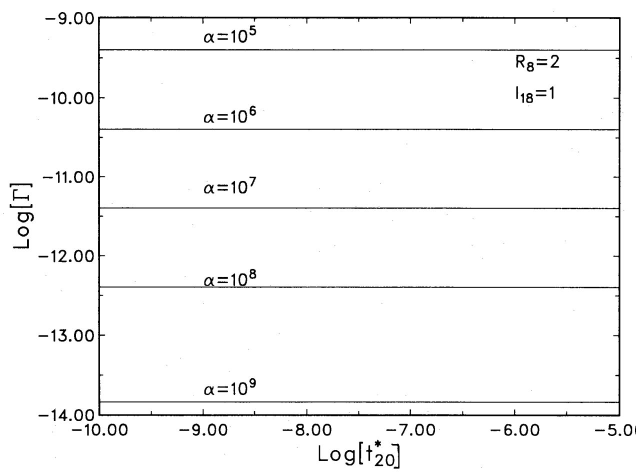

Figure 1. The growth rate Γ is plotted versus time t^* in the comoving frame with $\ell_0 = 10^{18}$ cm and $R = 2 \times 10^8$, for different values of the parameter $\alpha = L/\ell_0$.

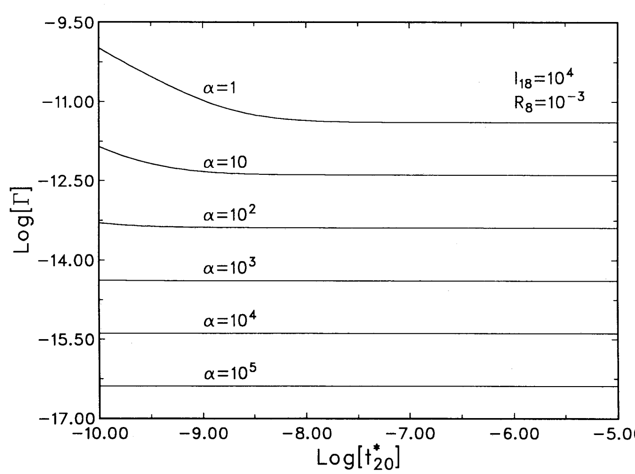

Figure 2. The growth rate Γ is plotted versus time t^* in the comoving frame for $\ell_0 = 10^{22}$ cm and $R = 10^5$, for different values of the parameter $\alpha = L/\ell_0$.

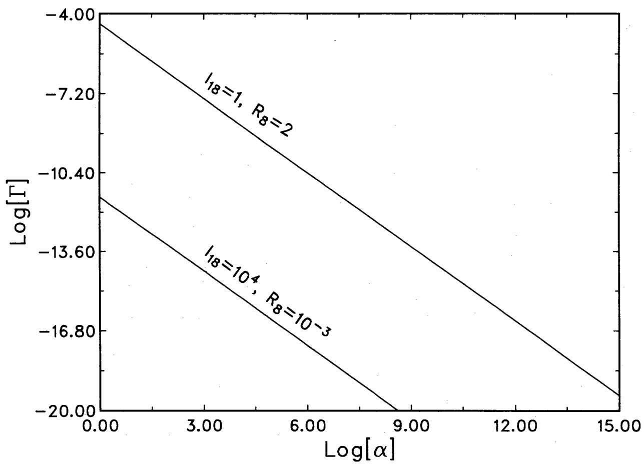

Figure 3. The growth rate Γ is plotted versus the scale factor α for (i) $\ell_0 = 10^{18}$ cm and $R = 2 \times 10^8$, and (ii) $\ell_0 = 10^{22}$ cm and $R = 10^5$.

describing the formation of large-scale flows is needed. This has been done by Sulem et al. (1989) for a stationary fluid. Incorporation of the expansion of the fluid is a non-trivial exercise, and we are working at it. However, on the basis of this linear analysis and earlier work on the statistical studies of the turbulence (Krishan & Sivaram 1991), we put forward the following suggestions.

(1) The Universe has distinct regions of rotational and potential flows. The phenomenon of clustering takes place in the rotational flow region through the inverse cascade of energy.

(2) In the irrotational or potential-flow-dominated regions, organized structures cannot form. The fluid elements are in a random state of small-scale motions which may appear as a continuum, and this is what we may be interpreting as voids. Thus, contrary to the popular belief, voids are not devoid of matter but of helicity. In fact, they may contain motions which may be more compressible than those associated with the clustering phenomenon.

(3) The seemingly disparate phenomena of (i) non-equilibrium motions on stellar surfaces, (ii) the generation of large-scale magnetic fields, and (iii) the large-scale structure of the Universe have their origin in the inverse cascade of energy leading to self-organization in an otherwise turbulent medium.

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