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A model of solar granulation through inverse cascade

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SUMMARY

Non-linear interactions between small fluid elements in an energetically open system facilitate the formation of large coherent stable structures. This is known as self-organization. We interpret solar granulation on all scales to be the result of self-organization processes occurring in the turbulent medium of the solar atmosphere. This mechanism provides explanations for the intrinsic weakness of mesogranulation and the rare appearance of giant cells in addition to the sizes and lifetimes of these structures. The entire energy spectrum from the smallest granules to the largest giant cells brings out the prevalence of Kolmogorov's $K^{-5/3}$ law.

1 INTRODUCTION

In an isolated turbulent medium, decay of large eddies into small eddies is a common occurrence. The converse, i.e. small eddies coalescing into large ones can be accomplished in an energetically open system with very special properties. Generally, such a system must be dissipative and describable by a non-linear partial differential equation. In the absence of dissipation, the equation should have two or more quadratic or higher-order conserved quantities called invariants. On the inclusion of dissipation the invariants decay differentially. The nature of the non-linear interaction between the fluid elements is such that the slow decaying invariant (say B) cascades towards large spatial scale, and the fast decaying invariant (say A) cascades towards small spatial scales. The initial random field of velocities and magnetic fields can then be described through a variational principle in which the invariant A is minimized keeping the invariant B constant, i.e.

$$\delta A - \lambda \delta B = 0, \tag{1}$$

where δA and δB are small variations and, λ is the Lagrange multiplier Hasegawa (1985). Kraichnan (1967) found that in two-dimensional hydrodynamic turbulence, the energy invariant cascades towards large spatial scales and the enstrophy invariant towards small spatial scales where it suffers heavy dissipation. It is this property of selective decay that facilitates the formation of large structures. But can one use the results of 2-D turbulence theory to explain phenomena in a real atmosphere which is a 3-D system? What one needs is the process of inverse cascade to occur in a 3-D system. Several attempts have been made to explore the formation of large-scale structures in three-dimensional flows. Sulem *et al.* (1989) and Frisch, She & Sulem (1987) have performed 3-D simulations of the Navier–Stokes equations to show that an anisotropic stirring force can give rise to large helical structures. The inhibition of energy transfer to small scales due to helicity cascade has been shown to occur by Andre & Leisieur (1977). The generalization of the helicity conservation law and the vorticity theorem which would be applicable for a large class of flows has been suggested by Gaffet (1985). The generation of large cloud complexes in the Earth's atmosphere invoking the role of helicity fluctuations has been discussed by Levich & Tzvetnov (1985). We will use the ideas developed in this work to construct a model of solar granulation in its entirety from granules to giant cells.

2 MODELLING SOLAR GRANULATION

Radiation and convection are the two main energy transport processes in the solar interior. The convective transport becomes operative where the temperature and density gradients are such that a fluid element, when displaced from its equilibrium position, continues to move away from it. This stratification through unstable convection produces turbulence in the medium. Fluid eddies of varying sizes then carry energy as they propagate and dissipate. The cellular velocity patterns observed on the solar surface are believed to be manifestations of convective phenomenon occurring in the subphotospheric layers. The cellular velocity fields are seen prominently on two scales: the granulation and the supergranulation, though mesogranulation and giant cells are also suspected to be present. The formation of granules with an average size of 1000 km and a lifetime of a few minutes is understood either from the mixing length (Schwarzschild 1975) or from the linear instability (Bogart, Gierasch & MacAuslan 1980) description of the convection in the hydrogen ionization zone of the subphotospheric medium. The supergranules with an average size of 30 000 km and a lifetime of 20 hr do not have an unambiguous association with a subphotospheric region. The attempts have been to seek an explanation for the energy concentration at the supergranular scale and to identify the region. Simon & Leighton (1964) suggested helium ionization as responsible for accumulation of energy at supergranular scales. Convective modes with dominant growth rates at the two scales have been favoured by Bogart, Gierasch & MacAuslan (1980) and Antia, Chitre & Narasimha (1984).

The quality observations obtained at Pic-du-Midi indicate the existence of a continuum of sizes instead of one dominant scale in the granulation. The fractal dimension studies reveal that small eddies are more regular than the large ones (Roudier & Muller 1986). Further, a plot of kinetic energy $E(k)$ contained in the scales between K and $K+dK$, versus the wave number, K shows two slopes of ~ -0.70 and -1.70 with a break at $K^{-1} \simeq 2000$ km (Malherbe *et al.* 1987). The slope of (-1.70) is very close to the Kolmogorov's law, $K^{(-5/3)}$, for homogeneous isotropic turbulence, where large eddies cascade to small eddies, setting up an inertial range. Similar conclusions confirming the turbulent nature of granulation have been derived through the SOUP observations by Title *et al.* (1986). Prompted by these observations it was suggested that perhaps the energy injection into the solar atmosphere occurs at the supergranular scale and the continuum of granules is formed by the direct cascading of energy from large scales to small scales (Zahn 1987). Assigning the convective energy transport to supergranules also alleviates the problem of getting too large a measure of vertical velocities and temperature fluctuations if one restricts the energy transport only to granules (Nordlung 1982, 1985). The large-scale motions reported by Ribes, Mein & Mangeney (1985) could also be sustained by supergranules or may be a consequence of the instability of the convection zone against large-scale disturbances. Mesogranulation refers to the scales lying between the granules and supergranules as reported by November *et al.* (1981). Here we try to determine if the excitation of random small-scale motions can lead to the large organized structures which are observed in the form of granules, mesogranules, supergranules and giant cells. In this picture large helicity fluctuations present in a turbulent medium play an essential role in the inverse cascading process. The helicity density γ, a measure of the knottedness of the vorticity field is defined as $\gamma = \mathbf{V}\cdot(\nabla \times \mathbf{V})$. It is found that the quantity I, defined as

$$I = \int \langle \gamma(x)\, \gamma(x+\varepsilon) \rangle \, d^3\varepsilon, \tag{2}$$

is also an invariant in an ideal 3-D hydrodynamic system in addition to the total energy. Assuming a quasi-normal distribution of helicities, the invariant I can be expressed as

$$I = C \int [E(k)]^2 \, dk, \tag{3}$$

where C is a constant and $E = \int E(k)\, dK$ is the total energy density. Using Kolmogorovic arguments one finds the inertial range for energy invariant to be

$$E(k) \propto k^{-5/3}$$

and

$$E \propto L^{2/3}, \tag{4}$$

and for I invariant to be

$$E(k) \propto K^{-1}$$

and

$$E \propto \log \frac{L(t)}{l}, \tag{5}$$

where $L(t)$ is the largest length scale excited at time t (Levich & Tzvetkov 1985). In analogy to the 2-D case where the inertial range corresponding to energy invariant is $E(k) \propto k^{-5/3}$ and that corresponding to enstrophy invariant (Hasegawa 1985) is $E(k) \propto K^{-3}$, and where the energy is found to cascade to large spatial scales and enstrophy to small spatial scales, one expects that in 3-D the invariant I would cascade towards large spatial scales and the energy towards small spatial scales. The cascading of I towards large spatial scales essentially enhances the correlation length of helicity fluctuations, without much increase in energy associated with it (equation 5). We propose that this regime of turbulence favours the formation of solar granulation at the smallest scales. More and more evidence is also piling up in favour of the abundance of smaller granules. One can ask if there is an upper limit to the size of granules, the lower limit of course is determined by dissipation. We recall that all atmospheres are restricted in the vertical direction due to gravity. The largest dimension of fully 3-D structures is given by the ratio $I/E^2 = L = L_z$ where L_z is the characteristic vertical scale. We identify this scale with the size of the region with superadiabatic temperature gradient, since this is the region that provides energy in the vertical velocity field which then drives the horizontal flow. This limit (Nelson & Musman 1978) of 1000 km puts the 3-D granules right at the top of the convection zone. When the correlation length of helicity fluctuations reaches the limit L_z, it can only grow in the horizontal plane. Another consequence of the growth of correlation length is that the velocity and vorticity get aligned which reduces the non-linear term

$$(\mathbf{V}\cdot\nabla)\,\mathbf{V} = (\mathbf{V} \times \boldsymbol{\omega}) + \tfrac{1}{2}\nabla(V^2)$$

of the Navier–Stokes equation and thus retards the flow of energy to small spatial scales. With the growth of correlation length only in the horizontal direction, the system becomes more and more anisotropic. Under these circumstances, the vertical component of velocity V_z becomes independent of x, y and z, and the horizontal components V_x and V_y independent of z, leading to $\omega_{x,y} = (\nabla \times V)x,\ y = 0$. The invariant I becomes

$$I = \int \langle (V_z \omega_z)^2 \rangle \, dx\, dy\, dz$$

$$\simeq L_z \langle V_z^2 \rangle K^2 V_k^2 K^{-2} \propto V_k^2 = KE(k) \propto L^{2/3} \tag{6}$$

and from

$$I = \int I(k)\, dk,$$

one finds

$$I(k) \propto K^{-5/3}. \tag{7}$$

Here L is the largest length-scale in the horizontal plane. Thus the $I(k)$ spectrum coincides with the energy spectrum of 2-D turbulence $E(K)\propto K^{-5/3}$ corresponding to the inverse cascade. One expects that an increasing fraction of energy is transferred to large spatial scales as the anisotropy in the system increases. We propose this part of the turbulence spectrum to be conducive to the formation of supergranules which appear predominantly on large horizontal scales with energy spectrum given by equation (6). The intermediate region where a 3-D system is developing anisotropy, the energy spectrum being given by equation (5), can be identified with mesogranulation or the gap between granulation and supergranulation. Again one can ask if there is a limit to the size of supergranules. The growth of large structures in a highly anisotropic turbulence can again be interrupted as a result of symmetry breaking caused by the Coriolis force. The length-scale L_c where the non-linear term of the Navier–Stokes equation becomes comparable to the coriolis force, can be determined from

$$(\mathbf{V}\cdot\nabla)\,\mathbf{V}=2(\mathbf{V}\times\mathbf{\Omega})-\mathbf{\Omega}\times(\mathbf{\Omega}\times\boldsymbol{\varepsilon}) \tag{8}$$

or

$$L_c\simeq\frac{V}{\Omega}, \tag{9}$$

where $\mathbf{\Omega}$ is the angular velocity. Given sufficient energy structures of size L_c must form. At these large spatial scales the system simulates 2-D behaviour and enstrophy conservation begins to play its role. One may consider scales $L\geqslant L_c$ as a source of vorticity injection into the system. The enstrophy then cascades towards small scales with a power-law spectrum given by

$$E(k)\propto k^{-3} \quad \text{and} \quad E\propto L^2. \tag{10}$$

Thus there is a break in the energy spectrum as energy must cascade to larger spatial scales as $L^{2/3}$ and to small scales as L^2. Therefore the energy must accumulate at $L\sim L_c$ and

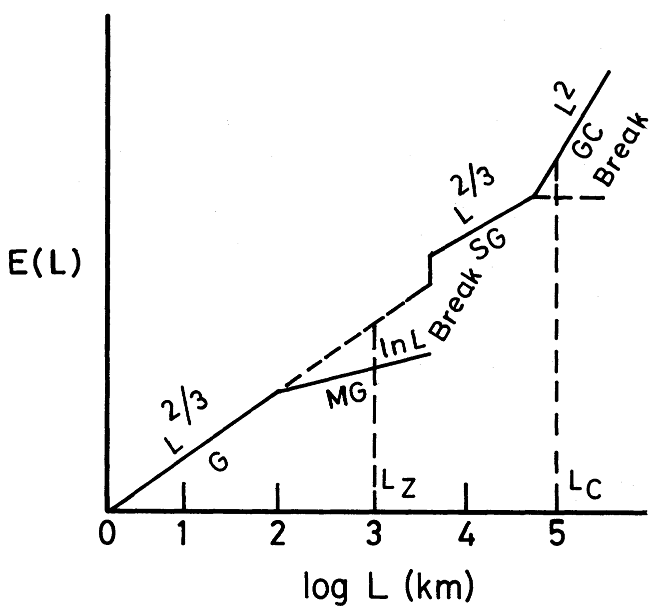

Figure 1. Turbulent energy spectrum. L_z – scale of the first break due to anisotropy; L_C – scale of the second break due to the Coriolis force; G – granule; MG – mesogranule; SG – supergranule; and GC – giant cell.

eventually pass on to the highest possible scales of the general circulation of the atmosphere. It is tempting to identify this region of turbulence with the excitation of giant cells. This is perhaps the complete story of the energy spectrum (Fig. 1) in the turbulent medium of the solar atmosphere.

3 ENERGETICS

According to the description (Levich & Tzvetkov 1985) presented here, the energy in the larger structures has been inverse cascaded from the smaller structures, and so the energy density per unit gram, $E(L)$, in the large-scale L should not exceed that in the small scale (1). From the energy spectrum $E\propto L^{2/3}$, it follows that

$$E(L)=\left(\frac{E_0(l)}{\tau}\right)^{2/3}L^{2/3}\leq E_0(l), \tag{11}$$

where τ is the time for which energy injection must occur. If we take τ to be the lifetime of the larger structure then for $\tau\sim 20$ hr for supergranules and $E_0(l)\simeq(0.5)^2$ km^2 s^{-2} for energy density gm^{-1} in the granules, one gets

$$L\sim L_{SG}\sim 36\,000\ \text{km}, \tag{12}$$

which is the typical size of a supergranule. The size of a giant cell can be determined from equation (9) for $\Omega=(2\pi/27)$ d^{-1} and assuming $V\sim 0.3$ km s^{-1} for supergranular velocity (since they provide the stirring force for turbulence that organizes itself into giant cells) one gets

$$L_c=L_{GC}\sim 1.17\times 10^5\ \text{km}. \tag{13}$$

Again the energy content of giant cells should not exceed that of supergranules. Using equation (10) for the energy spectrum in this region, one gets

$$E_{GC}=[E(L_{SG})/L_{SG}^2\,\tau]^{2/3}L_{GC}^2\lesssim E(L_{SG}), \tag{14}$$

where

$$\left[\frac{E(L_{SG})}{L_{SG}^2\,\tau}\right]$$

is the enstrophy injection rate.

For $L_{GC}\sim 10^5$ km, $\tau=30$ d = lifetime of giant cell, $L_{SG}\sim 30\,000$ km, and $E_{SG}\sim(0.3)^2$ km^2 s^{-2}. One finds that equation (14) can be barely satisfied. Furthermore, in the presence of coriolis force, the pressure balance condition becomes

$$\frac{1}{\rho}|\nabla P|\simeq\frac{V^2}{L_C}+F_C\simeq 2V^2/L_C, \tag{15}$$

in contrast to the case with no coriolis force where $(1/\rho)|\nabla P|\simeq V^2/L_C$. Thus one concludes that larger energy density is required to maintain structures at scale L_C. This may be the reason for their rare observability. The appearance of structures at L_C must be accompanied by a corresponding increase in the convective flux and therefore probably of total solar flux. Total solar luminosity changes of 1 per cent have been observed. If we attribute all of this 1 per cent to increase in the convective flux, equation (15) can be satisfied and structures of size L_C can get excited. The differential rotation of the Sun favours the formation of larger structures at the polar regions in comparison to the equatorial regions. This is further substantiated by the fact that the dominantly open magnetic fields in polar regions do not inhibit the flow of convective flux. Thus one may look for probable correlation between polar phenomena and solar luminosity enhancements with the appearance of giant cells. A very steep spectrum, equation (10), practically forbids further organization of turbulence into structures larger than L_c.

4 CONCLUSION

It does seem possible to account for many of the observed features of solar granulation by invoking the role of helicity fluctuations in the inverse cascade of energy. The deduced energy spectrum. Malherbe *et al.* 1987) with a Kolmogorov law of ($-5/3$) and a flatter branch with a slope of (-0.7) is in good agreement with the predictions of the theory in the region of granules (with $-5/3$ slope) and a flatter region of mesogranulation (with a slope of -1). The rest of the spectrum can be tested as and when quality observations become available for supergranular and giant cell regions.

The energy spectrum in the extreme case of anisotropy or almost two-dimensional turbulence is expected to be K^{-3} or K^{-4} depending upon whether the vorticity discontinuities that develop in the flow are packed together or well separated, as discussed by Gilbert (1989). The presence of vorticity in the photospheric motions has been reported by Brandt *et al.* (1988), though more measurements of this quantity would really help resolve the turbulent nature of solar granulation. The organization of small-scale magnetic fields and the simultaneous organization of velocity and magnetic fields (Krishan 1990) are some of the areas that should be explored in future.

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