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Clustering of galaxies by inverse cascade in a turbulent medium

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SUMMARY
Non-linear interactions between small fluid elements in an energetically open non-linear system facilitate the formation of large coherent stable structures. This is known as self-organization. We interpret the clustering of galaxies on all scales as the result of self-organization processes occurring in a turbulent medium.

1 INTRODUCTION

The formation of the observed hierarchy of large-scale galactic structures in the Universe remains a challenging problem in conventional cosmology. The distribution of galaxies is no longer pictured as a random sprinkling. There are enormous superclusters as well as giant cellular voids, interspersed here and there with long lacy chains of galaxies. The painstaking survey by M. Geller and J. Huchra of thousands of galaxies in a relatively narrow strip of space shows that galaxies follow an intricate network of arcs and segments with huge rounded cells (Huchra *et al.* 1983). This distribution of galaxies into large-scale formations, far more ordered than previously thought, suggests the operation of well-defined physical processes.

It is to be noted that gravity by itself appears to be too weak to form such large structures from the initial density perturbations in time-scales comparable with the Hubble age. In other words, the conventional scenarios in which the density had subtle scale-invariant fluctuations which grew in amplitude and in scale with the expansion of the Universe to form large-scale structures seem untenable. Nevertheless, more detailed recent work indicates that it might be possible to form large structures through gravitational instability.

Many galaxies have independent motions (i.e. so-called peculiar velocities) at odds with the direction and speed of the overall Hubble flow, and it looks as if the galaxies are being drawn towards enormous concentrations of matter, i.e. high-density regions with masses $\simeq 10^{17}\, M_\odot$. For instance, the Local Group is moving at ~ 300 km s^{-1} towards the Virgo cluster in relation to Hubble flow. And, as shown by Burstein *et al.* (1986), galaxies between the local supercluster and the Hydra–Centaurus Supercluster share the same direction of motion, having peculiar velocities of several hundred kilometres per second with respect to a uniform Hubble flow. This reveals the presence of structures massive enough to draw several clusters towards them. Structures $\sim 10^{18}\, M_\odot$ are indicated (Tully 1986). More recently the Great Wall of galaxies (Huchra & Geller 1989) has been identified over scales ~ 100 Mpc. Also galaxies have been found at redshifts close to $z = 4$. This suggests the formation of galaxies, which are the smaller scale structures, at $z \gtrsim 5$. Then one is faced with the problem of how the smaller scale structures interacted, or what physical processes led to the formation of larger clusters and superclusters of galaxies from galactic-size smaller structures. In the conventional warm or hot dark matter scenarios (chiefly involving neutrinos with small rest masses as the dark matter), it appears that the largest scale structures formed first. Then one is faced with the problem of accounting for the presence of small-scale structures even at $z = 4$. The Zeldovich-type pancake fragmentation model also favours initial formation of large-scale structures. The cold dark matter scenario has had some measure of success in forming the smaller structures first. But it essentially invokes a particle (i.e. the axion), evidence for which seems to be becoming more and more meagre despite intensive searches.

The rejuvenation of cosmic turbulence in galaxy formation through energy cascading from larger to smaller scales has been discussed by Ozernoi & Chernin (1968) and Ozernoi & Chibisov (1971, 1972). The cascading takes place for scales for which the hydrodynamic interaction time is smaller than the cosmic expansion time. The sound speed in the cosmic medium prior to recombination is given by

$$\left(\frac{v_S}{c}\right) \simeq \frac{2}{3}\left(\frac{\rho_r}{\rho}\right)^{1/2}\left(1+\frac{4}{3}\frac{\rho_r}{\rho}\right)^{-1/2},$$

where ρ_r, ρ are radiation and matter densities, and for $\rho_r \gg \rho$, $v_S \sim (c/\sqrt{3})$. For an expanding fluid, with expansion rate given by a Friedmann equation, the minimum scale over which dissipation can occur is $l_{min} \sim (\nu t)^{1/2}$, which is obtained by equating time-scales for dissipation and expansion. The maximum length-scale is given by the maximum velocity v_{max} of the largest eddy, i.e. $l_{max} \sim v_{max} t$, and $v_{max} \ll v_S$. As a consequence of large Reynolds number and short hydrodynamic times, motions on comoving scales satisfying $l_{min} < l < l_{max}$ should rapidly reach the Kolmogorov spectrum characteristic of ordinary turbulence in an incompressible viscous fluid. From the point of view of the predictions of the theory, the fact that the turbulence spectrum is assumed to be Kolmogorov fixes the longest turbulence scale at the epoch of recombination.

The hypothesis of Ozernoi and coworkers is that the Kolmogorov spectrum $v(r) \propto r^{1/3}$, $r_{\min} < r < r_{\max}$, is set up on scales less than $r_{\max}$, for which (i.e. the largest scale) the dynamical time-scale equals the cosmic expansion scale. An important condition for the validity of the above relation is that motion on the largest scale (i.e. $v_{\max}$) should be subsonic (satisfied for sonic velocities $\simeq c/\sqrt{3}$). The fluid motion is then incompressible during the pre-recombination era which is about 20 per cent of the Hubble time. After recombination, the fluid motion becomes supersonic and therefore the Kolmogorov spectrum is not strictly valid. Generalization of inertial transfer theories of turbulence to an expanding universe has been done by Jones (1976). An equation for evolution of the energy spectrum of turbulence in an expanding universe was given by Tomita (1971) on the basis of a phenomenological model of energy transfer, and Jones showed by analogy with the non-expanding case that it is possible to have the Kolmogorov energy spectrum as a solution for this equation (i.e. the expanding case). The reasonableness of the inertial transfer theories in fact lies in their ability to reproduce the Kolmogorov spectrum.

Here we suggest a definite physical mechanism based on inverse cascading which quite naturally yields a bottom-up hierarchical structure. In an isolated turbulent medium, decay of large eddies into small eddies is a common occurrence. The converse, i.e. small eddies coalescing into large ones, can be accomplished in an energetically open system with very special properties. Generally, such a system must be dissipative and describable by a non-linear partial differential equation. In the absence of dissipation, the equation should have two or more quadratic or higher order conserved quantities called invariants. On the inclusion of dissipation, the invariants decay differently. The nature of the non-linear interaction between the fluid elements is such that the slow-decaying invariant (say B) cascades towards large spatial scales, and the fast-decaying invariant (say A) cascades towards small spatial scales. The initial random field of velocities can then be described through a variational principle in which the invariant A is minimized keeping the invariant B constant, i.e.

$$\delta A - \lambda \delta B = 0,$$

where δA, δB are small variations and λ is the Lagrange multiplier (Hasegawa 1985). Kraichnan (1967) found that, in two-dimensional hydrodynamic turbulence, the energy invariant cascades towards large spatial scales and the enstrophy invariant towards small spatial scales, where it suffers heavy dissipation. It is this property of selective decay which facilitates the formation of large structures. But can one use the results of 2D turbulence theory to explain phenomena in a real 3D system? What one needs is for the process of inverse cascade to occur in a 3D system and Levich & Tzvetkov (1985) have demonstrated such a possibility. This possibility has been used to explain solar granulation on all scales (Krishan 1989, 1991; Krishan & Mogilevskij 1990). Here we use these ideas to construct a model of cosmic clustering in its entirety, from clusters of galaxies to giant clusters of galaxies.

In our model, the elementary formations (or vortices) are identified with ordinary galaxies including dwarf galaxies. These would then form clusters by ordinary gravitational clustering as well as by turbulent cascading. We invoke inverse cascading only for the formation of superclusters and giant clusters like the Great Wall, beginning with clusters of galaxies. Velocities and scale sizes of the large-scale structures formed by inverse cascade are consistent with these two types of structures, i.e. superclusters and giant clusters. For instance, turbulent velocities of 300 km s^{-1} for the clusters give rise to structures on scales ~3 Mpc on a time-scale of 3×10^9 yr, and turbulent velocities of 10^4 km s^{-1} for superclusters give rise to structures on scales ~100 Mpc. There is a gap in the energy spectrum, situated between clusters of galaxies and superclusters. This is consistent with the observational absence of visible objects between galaxy clusters and superclusters (Einasto *et al.* 1988). The energy spectrum also shows a discontinuity at a scale where superclusters begin to develop into giant clusters, with a much steeper energy spectrum which may perhaps explain the rarity of the largest scale structures.

In this picture, large helicity fluctuations present in a turbulent medium play an essential role in the inverse cascading process. The helicity density γ, a measure of the knottedness of the vorticity field, is defined as $\gamma = \boldsymbol{v} \cdot \boldsymbol{\omega}$, $\boldsymbol{\omega} = \nabla \times \boldsymbol{v}$. It is found that the quantity I, defined as $I = \int d^3 r \langle \gamma(x) \gamma(x+r) \rangle$, is also an invariant of an ideal 3D hydrodynamic system in addition to the total energy. By assuming a quasi-normal distribution of helicities, the invariant I can be expressed as

$$I = c_I \int [E(k)]^2 \, dk, \tag{1}$$

where c_I is a constant and $E = \int E(k) \, dk$ is the total energy density. Following Kolmogorov, one finds the inertial range for the energy invariant to be

$$E(k) \propto k^{-5/3} \qquad \text{and} \qquad E \propto L^{2/3}, \tag{2}$$

and for the 'I' invariant to be

$$E(k) \propto k^{-1} \qquad \text{and} \qquad E \propto \log \frac{L(t)}{l}, \tag{3}$$

were $L(t)$ is the largest length-scale excited at time t (Levich & Tzvetkov 1985). By analogy with the 2D case (Hasegawa 1985) one expects that in 3D the invariant 'I' would cascade towards large spatial scales and the energy towards small spatial scales. The cascading of 'I' towards large spatial scales essentially enhances the correlation length of helicity fluctuations, without much increase in energy associated with it (equation 3). We propose that this regime of turbulence favours the formation of clusters of galaxies at the smallest scales. One can ask if there is an upper limit to the size of three-dimensional isotropic clusters of galaxies (the lower limit being determined by dissipation). We recall that a turbulent medium is restricted in the vertical direction by gravity. The largest dimension of fully 3D structures is given by the ratio $I/E^2 = L = L_z$, where L_z is the characteristic vertical scale. We identify this scale with that of clusters of galaxies of size ~0.3 Mpc. When the correlation length of helicity fluctuations reaches the limit L_z, it can only grow in the horizontal plane. Another consequence of the growth of the correlation length is that the velocity and vorticity become aligned, which reduces the non-linear term $(\boldsymbol{v} \cdot \nabla)\boldsymbol{v} = \nabla \times (\boldsymbol{v} \times \boldsymbol{\omega})$ of the Navier–Stokes equation and thus retards the flow of energy to small spatial scales. With the growth of correlation length only in the horizontal plane, the system becomes more and more anisotropic. In these circumstances, the vertical component of velocity v_z becomes independent of (x, y, z) and the horizontal components v_x and v_y independent of z, leading to $\omega_{x,y} = (\nabla \times v)x, y = 0$. The invariant I becomes

$$I = \int \langle (v_z \omega_z)^2 \rangle \, dx \, dy \, dz$$

$$\simeq L_z \langle v_z^2 \rangle k^2 v_k^2 k^{-2} \propto v_k^2 = kE(k) \propto L^{2/3}, \tag{4}$$

and from $I = \int I(k) \, dk$, one finds

$$I(k) \propto k^{-5/3}. \tag{5}$$

Here L is the largest length-scale in the horizontal plane. Thus the $I(k)$ spectrum coincides with the energy spectrum of 2D turbulence $E(k) \propto k^{-5/3}$, corresponding to the inverse cascade. One expects that an increasing fraction of energy is transferred to large spatial scales as the anisotropy in the system increases. We propose this part of the turbulence spectrum to be conducive to the formation of superclusters of galaxies which appear predominantly on large horizontal scales with energy spectrum given by equation (4). In the intermediate region, where a 3D system is developing anisotropy, the energy spectrum given by equation (3) can be identified with the gap between clusters of galaxies and superclusters. This may explain the observational absence of structures of sizes intermediate between clusters and superclusters. One can again ask if there is a limit to the size of superclusters. The growth of large structures in a highly anisotropic turbulence can be interrupted as a result of symmetry breaking caused by the Coriolis force. The length-scale L where the non-linear term of the Navier–Stokes equation becomes comparable to the Coriolis force can be determined from

$$(\boldsymbol{v} \cdot \nabla)\boldsymbol{v} = 2(\boldsymbol{v} \times \boldsymbol{\Omega}) - \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \boldsymbol{r})$$

or

$$L_c = v/\Omega, \tag{6}$$

where Ω is the angular velocity. Given sufficient energy, structures of size L_c must form. At these large spatial scales, the system simulates 2D behaviour and enstrophy conservation begins to play a role. One may consider scales $L \geq L_c$ as a source of vorticity injection into the system. The enstrophy then cascades towards small scales with a power-law spectrum given by

$$E(k) \propto k^{-3} \qquad \text{and} \qquad E \propto L^2. \tag{7}$$

Thus there is a break in the energy spectrum, as energy must cascade to larger spatial scales as $L^{2/3}$ and to small scales as L^2. The energy must therefore accumulate at $L \sim L_c$ and eventually pass on to the highest possible scales of the general circulation of the structure. It is tempting to identify this region of turbulence with the excitation of giant superclusters. This may be the complete story of the energy spectrum in the cosmic turbulent medium (Fig. 1).

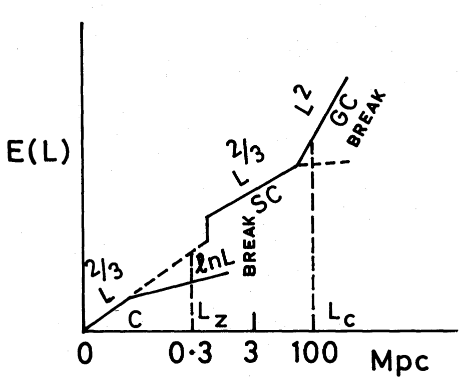

Figure 1. Turbulent energy spectrum. L_z = scale of the first break due to buoyancy, L_c = scale of the second break due to the Coriolis force, C = cluster, SC = super-cluster and GC = giant cluster.

2 ENERGETICS

According to the picture (Levich & Tzvetkov 1985) presented here, the energy of the larger structures has been inverse-cascaded from the smaller structures, in which case the energy density per unit mass $E(L)$ in the large scale should not exceed that in the small scale, $E_o(L)$. From the energy spectrum $E \propto L^{2/3}$ (equation 2), it follows that

$$E(L_{SC}) = \left[\frac{E_o(L_c)}{\tau_{SC}}\right]^{2/3} L_{SC}^{2/3} \lesssim E_o(L_c), \tag{8}$$

where τ_{SC} is the time for which energy injection must occur, which should be at least the lifetime of the large structure. If we take $L_{SC} \sim 3$ Mpc for a supercluster and $E_o(L_c) \simeq (300$ km s$^{-1})^2$ for the turbulent energy of a cluster, we obtain $\tau_{SC} \simeq 3 \times 10^9$ yr.

The giant clusters are formed in a turbulent medium which is stirred by the random motion of superclusters. The largest horizontal scale is limited by the Coriolis force. From equation (6), with L_{GC} = size of a giant cluster ~ 100 Mpc, and a random velocity of superclusters of 10 000 km s^{-1}; one finds the angular velocity $\Omega \sim 3 \times 10^{-18}$ rad s^{-1}. The energy content of giant clusters should not exceed that of superclusters. Using equation (8) for the energy spectrum in this region, one obtains

$$E_{GC} = \left[\frac{E(L_{SC})}{L_{SC}^2 \, \tau_{GC}}\right]^{2/3} L_{GC}^2 \leq E(L_{SC}), \tag{9}$$

where $[E(L_{SC})/L_{SC}^2 \tau_{GC}]$ is the enstrophy injection rate. For $L_{GC} \sim 100$ Mpc, $E(L_{SC}) \sim (10^4$ km s$^{-1})^2$, and $L_{SC} \sim 3$ Mpc, one finds $\tau_{GC} \simeq 10^{11}$ yr. It is clear that structures of the size of few hundred Mpc cannot be formed if the random velocity of a supercluster is less than 10 000 km s^{-1}. Furthermore, in the presence of Coriolis force, the pressure balance condition becomes

$$\frac{1}{\rho}|\nabla P| \simeq \frac{v^2}{L_{GC}} + F_c \simeq \frac{2v^2}{L_{GC}}, \tag{10}$$

in contrast to the case with no Coriolis force where

$$\frac{1}{\rho}|\nabla P| \simeq \frac{v^2}{L_{GC}}.$$

Thus one concludes that a larger energy density is required to form or maintain structures at scale L_{GC}. This may be the reason for their rarity. A very steep spectrum (equation 9) practically forbids further organization of turbulence into structures larger than L_{GC}.

The processes of inverse cascade have been shown to occur in an incompressible turbulent medium. The incompressibility fixes the epoch of formation of large structures before the recombination phase. Since the recombination takes about 20 per cent of the Hubble time, the structures formed then should be detectable at present. The eddy turnover time for the largest structure of say ~ 100 Mpc, with an associated fluid velocity of 10 000 km s^{-1}, is $\sim 10^{10}$ yr, which is less than the Hubble time. This is also true at smaller scales. Within the framework of the present theory, the smaller structures are more or less isotropic whereas the larger ones are anisotropic and become nearly two-dimensional at the largest scale. One may also note that the turbulent velocity scales as $v \propto L$ at the largest scales, instead of $L^{1/3}$ as for a Kolmogorov spectrum (Fig. 1). Peebles (1971a,b) has obtained a non-Kolmogorov spectrum by including the effects of expansion, while Silk (1973, 1974) has shown that the effects of expansion caused little deviation in turbulence from the incompressible case.

3 TURBULENCE GENERATION, INFLATION AND COSMIC MICROWAVE BACKGROUND

In the above picture we did not take into account the earliest epoch of the Universe which, according to current thinking, is described by an exponential expansion of the scale factor as $R = R_0 \exp(Ht)$. It has been argued that such an exponential expansion would considerably dilute any initial vorticity and magnetic fields. For instance, with flux conservation assumed, an initial magnetic field would be diluted by a factor of $\exp(2Ht) \sim e^{60}$ for most inflation models. However, this need not necessarily be so. There are very recent models where B decreasing as $1/r^2$ is averted by breaking the conformal invariance of electromagnetism by considering non-minimal gravitational couplings of photon field to gravity such as RA^2, $RF^{\mu\nu}$, $F^{\mu\nu}$, etc. These terms in fact result in the *production* of large-scale magnetic fields during inflation [Turner & Widrow (1988); also de Sabbata & Sivaram (1988) for post-inflation generation of magnetic field through vorticity in general relativity]. Since magnetic fields can be coupled to differential rotation of ions and electrons in the plasma prior to recombination, this would also generate vorticity in addition to magnetic field. Moreover, MHD turbulence aids inverse cascading. So inflationary expansion in the earliest phase need not be a drawback to this picture, which gives a scale-invariant description independent of inflation. Generation of the magnetic field during inflation would also indirectly generate vorticity. Moreover, the process of direct cascade occurring in the first branch of the spectrum (Fig. 1), i.e. where clusters decay into galaxies in the 3D turbulent medium before the anisotropy is set up, would be a source of vorticity production. In addition, it has been shown (Hasegawa 1985) that, during the spectral transfer characterized by $E(k) \propto k^{-3}$, the largest scales are vorticity injection scales. It is in the intermediate region that the organized structures are formed, being continuously fed at the two ends.

Since, in our picture, the large-scale structures are essentially flow patterns set up by the transfer of energy from small scales to large scales, there need not necessarily be attendant temperature fluctuations which might distort the cosmic microwave background radiation. An example is the solar supergranulation which is essentially observed as a velocity field without attendant brightness or temperature flutuations, although the smaller scale granules are associated with temperature fluctuations.

In conclusion we must emphasize that, although the role of turbulence in galaxy formation has been considered by many authors, this is the first time that the inverse cascading process has been applied to try to account for the formation of the whole hierarchy of structures from giant clusters to clusters. The result is consistent with the increasing evidence that galaxies formed first and the larger structures later, i.e. inverse cascading naturally giving rise to bottom-up hierarchical structures.

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