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Effect of Lyman radiation transfer on the formation of the Hα line in supergiant chromospheres

S.V. Mallik, D.C.V. Mallik, and D. Mohan Rao

Indian Institute of Astrophysics, Bangalore 560034, India

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Abstract. We present radiative transfer calculations of the Hα line in spherically expanding chromospheres of late-type supergiants where the assumption of radiative detailed balance in Lyman transitions has been relaxed. The chromospheres have been modelled as regions with outward-positive velocity and temperature gradients. Relaxation of the assumption of radiative detailed balance resulting in the escape of both Lyman continuum and line photons affects the ionization structure of hydrogen as well as the distribution of level populations in the line-forming region. The shape and strength of the new Hα profiles are different though the overall changes are not drastic for the densities and temperatures considered here. The implications of the new results for the chromospheres of a sample of G and K supergiants are discussed.

Key words: radiation transfer: Lyman continuum – radiation transfer: Hα line – stars: chromospheres – stars: late-type supergiants

1. Introduction

The Hα line is now recognized as an important chromospheric diagnostic in cool giants and supergiants. A number of studies have treated the radiative transfer problem of Hα in a fair amount of detail in the chromospheres of these stars in order to interpret the observed characteristics of the line (Boesgaard & Hagen 1979; Cram & Mullan 1979, 1985; Dupree et al. 1984a, b; Mallik 1982, 1986). In most of these the excitation of the Hα line is studied on the assumption of radiative detailed balance in Lyman lines and the continuum. This results in the simplification of the statistical equilibrium equations and therefore in computational convenience.

The assumption of radiative detailed balance in the Lyman continuum and lines is certainly valid for the lower chromospheres of cool stars where the densities are high and the temperatures sufficiently low. However, chromospheres of late type supergiants extend to several stellar radii and the temperature rise and density drop across them are quite significant. Hydrogen is partially ionized at temperatures of 7000 K and above and the optical depth in the Lyman continuum is reduced as a result. At this point in the outer atmosphere, the optical depth is neither large enough to assume radiative detailed balance nor small enough to ignore transfer effects. Also, in the low-density extended chromospheres, the thermalisation depth of the Lyman line photons may often exceed the total line-center optical depth so that the condition of radiative balance in line transitions is not satisfied. Hα is a strong absorption feature in the chromospheres of late G and K supergiants and comparison with our model calculations indicates that for reasonable extents and in the presence of chromospheric temperature gradients, the optical depth unity in the line is reached close to the surface or at the surface itself. Therefore, a more correct description of radiative transfer in this line is obtained when the assumption of radiative detailed balance in Lyman transitions is relaxed.

In an earlier paper (Mallik & Mallik 1988) we reported on the radiative transfer calculations of the Hα line in typical supergiant chromospheres with temperature and expansion velocity increasing outward and the total density dropping in accordance with steady state flow. Radiative detailed balance was assumed to hold for both Lyman lines and the continuum. The results showed significant differences with earlier isothermal calculations in similar chromospheres (Mallik 1986). In the present paper we report on new calculations where we have solved for the transfer of Lyman continuum radiation in the chromospheres of cool supergiants and also allowed for the effects of escape of the Lyman line radiation. We have assumed a temperature distribution similar to what has been used by Dupree et al. (1984a, b) and also adopted by us in our previous paper (Fig. 1). In the absence of radiative detailed balance, the first two levels of hydrogen are no longer collisionally coupled and the departure coefficients b_1 and b_2 have quite a different behaviour. The escape of the Lyman line and continuum radiation tends to overpopulate the ground state while the escape of Lyα photons specifically tends to underpopulate the second level. The net effect is to alter the population distribution in the first two levels of hydrogen which in turn changes the optical depth distribution of the Hα line. The computed Hα profiles have larger widths as well as larger equivalent widths. The effects are more drastic at lower densities and for higher extensions of the chromosphere.

In Sect. 2, we describe the theoretical framework of the problem and the method we use to obtain simultaneous solution of the transfer equation and the equations of statistical equilibrium. In the concluding section we describe the results and discuss their implications.

Send offprint requests to: S.V. Mallik

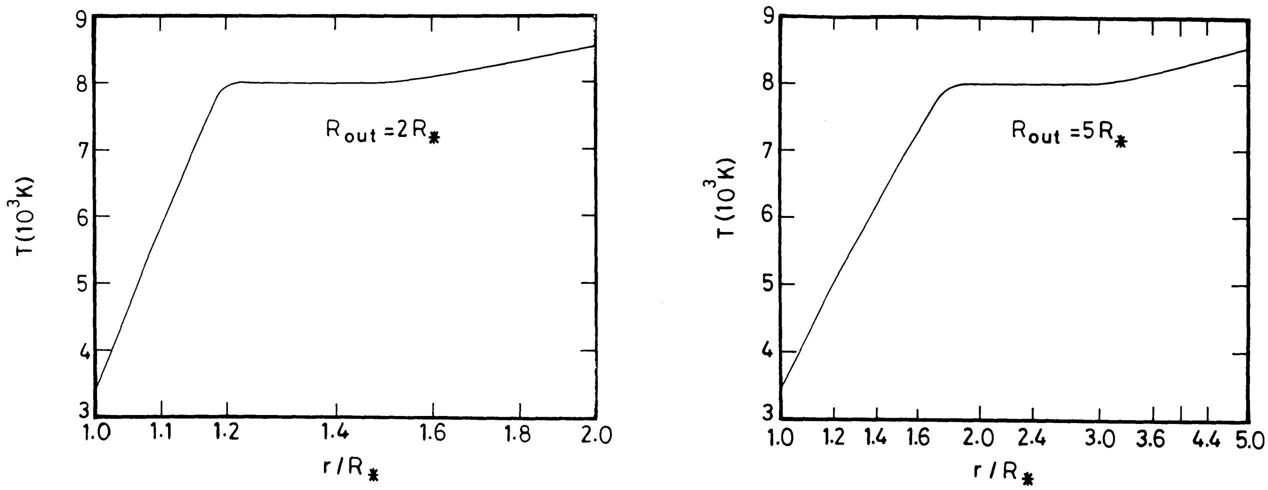


Fig. 1. The temperature structure for two different extensions $R_{\rm out}/R_* = 2$ and 5, adopted from Dupree et al. (1984a, b)

2. Transfer of Lyman continuum and line radiation

To treat the transfer of Lyman radiation we consider the simple model of a hydrogen atom with two discrete levels and the continuum. Our chromosphere is a pure hydrogen chrompsphere. We neglect collisional ionization and its reverse process the three-body recombination, because for the temperatures and densities of relevance to our problem these processes are orders of magnitude less efficient than the corresponding radiative processes. On the other hand, densities are large enough to ensure that H I 2^2S level depopulates faster by collisions to the 2^2P level than by two-photon decay to the 1^2S level. Since we are ultimately concerned with the computation of the Hα line the only way species other than H can affect our analysis is by providing extra sources of continuum opacity as well as changing the electron density distribution. The Hα line being a very strong absorption feature the ratio of continuum to line opacity is very small (10^{-8} or less) and the inclusion of other sources of opacity including H$^-$ produces negligible change in this value. The bulk of our chromosphere is at $T \geqq 6000$ K and it is well known from the work of Vernazza et al. (1973) that for all practical purposes $n_{\rm e} \approx n_{\rm p}$ at these temperatures.

Further, the transfer of radiation in the Lymanα line in these atmospheres is very poorly understood. Hartmann & Avrett (1984) obtained approximate solutions of the line transfer equations in the chromosphere of α Ori and found that the net radiative rates corresponding to these solutions differed little from those given by the simple escape probability formula of Rybicki (1983). To keep our analysis simple and the computations within manageable proportions we have adopted the same approach. We write for the net radiative bracket in Lymanα

$$n_2 A_{21} \varrho_{21} = n_2 (A_{21} + B_{21} \int_L \phi_\nu^L J_\nu^L \, {\rm d}\nu) - n_1 B_{12} \int_L \phi_\nu^L J_\nu^L \, {\rm d}\nu$$

and use the escape probability formula to approximate ϱ_{21} as τ_{21}^{-1}, where τ_{21} is the integrated optical depth at the line center of Lyα. With these assumptions and modifications in mind we obtain for levels 1 and 2 the statistical equilibrium equations

$$n_1 R_{1{\rm c}} + n_1 C_{12} = n_2 (C_{21} + A_{21} \varrho_{21}) + n_{\rm e} n_{\rm p} \alpha_1 \,,$$

$$n_2 R_{2{\rm c}} + n_2 (C_{21} + A_{21} \varrho_{21}) = n_1 C_{12} + n_{\rm e} n_{\rm p} \alpha_2 \,,$$

where C's are the collisional rates and

$$R_{1{\rm c}} = \int_{\nu_0}^{\infty} 4\pi/h\nu \, J_\nu \alpha_\nu (n=1) \, {\rm d}\nu \quad \text{and}$$

$$R_{2{\rm c}} = \int_{\nu_2}^{\infty} 4\pi/h\nu \, J_\nu \alpha_\nu (n=2) \, {\rm d}\nu \,.$$

ν_0 and ν_2 are the Lyman and Balmer threshold frequencies respectively, α_ν's are the photoionization cross-sections, and α_1 and α_2 are the recombination coefficients. The conservation equation

$$n_{\rm H} = n_1 + n_2 + n_{\rm p}$$

completes the set. Explicit expressions for n_1, n_2 and $n_{\rm p}$ are derived from these.

A measure of the impact of Lyα escape on the population distributions is provided by the importance of the term $A_{21}\varrho_{21}$ relative to the collision term C_{21}. For the total densities and temperatures we have used, the optical depths at the line center of Lyα vary between 10^6 and 10^8 and $A_{21}\varrho_{21}$ is as large as or larger than C_{21}. Moreover, since we are generally dealing with $\tau_{21} \geqq 10^6$, the escape probability formula is reasonably accurate as has been shown by Frisch (1983).

The radiation field in the Balmer continuum is specified by a radiation temperature $T_{\rm B}$ and a dilution factor W i.e. $J_\nu({\rm Balmer}) = W B_\nu(T_{\rm B})$. The Lyman continuum radiation field is obtained from an explicit solution of the transfer equation where the information from the statistical equilibrium equations is introduced directly. The transfer equation for Lyman continuum in spherical symmetry is

$$\mu \frac{\partial I_\nu}{\partial \tau_0} - \frac{1-\mu^2}{r} \frac{1}{\chi_0} \frac{\partial I_\nu}{\partial \mu} = \phi_\nu (I_\nu - S_\nu) \,,$$

where τ_0 is the optical depth at the Lyman threshold, χ_0 is the absorption cross-section and $\phi_\nu = (\nu_0/\nu)^3$ is the profile function. S_ν is the source function. Assuming only ground state recombinations lead to photons of frequencies larger than ν_0, we can show that

$$S_\nu = \frac{1}{b_1} B_\nu \,,$$

where B_ν is the Planck function. b_1 obtained from the statistical equilibrium equations turns out to be of the form

$$\frac{1}{b_1}=\frac{J_L+\eta}{1+\varepsilon_l},$$

where

$$J_L=\frac{\int_{\nu_0}^{\infty}\frac{\alpha_\nu}{h\nu}J_\nu\,\mathrm{d}\nu}{\int_{\nu_0}^{\infty}\frac{\alpha_\nu}{h\nu}B_\nu\,\mathrm{d}\nu},$$

$$\eta=\frac{1}{4\pi\int_{\nu_0}^{\infty}\frac{\alpha_\nu}{h\nu}B_\nu\,\mathrm{d}\nu}\,\frac{C_{12}R_{2c}}{(C_{21}+A_{21}\varrho_{21}+R_{2c})},$$

and

$$\varepsilon_l=\frac{1}{4\pi\int_{\nu_0}^{\infty}\frac{\alpha_\nu}{h\nu}B_\nu\,\mathrm{d}\nu}\,\frac{n_p^*}{n_1^*}\frac{(C_{21}+A_{21}\varrho_{21})R_{c2}}{(C_{21}+A_{21}\varrho_{21}+R_{2c})}.$$

The source function for the Lyman continuum can then be cast exactly in the form of the line source function of a two-level atom with continuum:

$$S_\nu=\frac{1}{b_1}B_\nu=\gamma_\nu\int_{\nu_0}^{\infty}\frac{\phi(\nu)}{\nu}J_\nu\,\mathrm{d}\nu+\varepsilon B_\nu,$$

where

$$\gamma_\nu=\frac{B_\nu}{(1+\varepsilon_l)}\,\frac{1}{\int_{\nu_0}^{\infty}\frac{\phi(\nu)}{\nu}B_\nu\,\mathrm{d}\nu},$$

and

$$\varepsilon=\frac{\eta}{1+\varepsilon_l}.$$

Thus

$$S_\nu=\gamma_\nu\int_{\nu_0}^{\infty}\Phi(\nu)J_\nu\,\mathrm{d}\nu+\varepsilon B_\nu,$$

where $\Phi(\nu)=\nu^{-1}\phi(\nu)$.

The source function has a scattering term where the intensities at all frequencies in the continuum are coupled and a thermal term. As pointed out by Dietz & House (1965), the source function above bears a strong resemblance to the line source function for a two-level atom although it differs from the latter in two important aspects. First, the profile function has a ν^{-1} dependence and hence strongly peaks at $\nu=\nu_0$. Secondly, the factor γ_ν in front of the scattering integral also has a strong frequency dependence through the Planck function. The character of the transfer problem here is thus quite different from a line transfer calculation.

We modified the line transfer code of Peraiah (1981) (see Mallik 1986; Mallik & Mallik 1988 for details) to solve the transfer equation, the conservation equation and the statistical equilibrium equations simultaneously. As a test run we tried a plane parallel monochromatic transfer case with $\nu=\nu_0$, keeping only the scattering term in the source function. No radiation was assumed incident at the top of the atmosphere and at the inner surface a Planckian radiation field characterised by a temperature of 7000 K was applied. The program was tested for positivity of fluxes and flux conservation. The solution was also tested against the analytic solution given in Mihalas (1978) for monochromatic transfer in the Lyman continuum.

For the transfer in supergiant chromospheres we chose a grid of twenty frequency points spanning the interval between the Lyman threshold and the He I threshold. For the Balmer radiation temperature T_B we have chosen a value of 4000 K. The line and the continuum forming region is divided into a number of shells starting from R_*, the stellar radius, to R_{out}. The initial guesses at the level populations and the ionization of hydrogen were made on the assumption of radiative detailed balance in Lyman continuum and the Lyman α line. Using these population distributions, the optical depth distribution in the Lyman continuum was determined. The line-center optical depth in Lyman α is $10474\,\tau_0$. We assumed that no radiation was incident from outside at $r=R_{out}$ while at $r=R_*$ a Planckian radiation field characterised by a temperature $T_R=4000$ K was incident. With these inputs the transfer equation and the non-LTE rate equations were simultaneously solved. An iterative procedure was followed where the J_ν in Lyman continuum and the resulting R_{1c} from the current cycle were used in the next to obtain improved estimates of n_1, n_2, n_p and the optical depth distribution. Convergence was obtained in 5 to 6 iterations.

The converged solution was then used to solve for the Hα source function and in the computation of the final Hα line profile in the observer's frame following the procedure outlined in Mallik (1986). We have assumed the velocity increasing outward up to V_{out} at R_{out}. This particular choice of velocity law has been prompted by the observations of blueshifted cores of Hα in these chromospheres. Earlier work by Mallik (1979) and Boesgaard & Hagen (1979) have shown that the asymmetries in the shapes could only be reproduced by a positive velocity gradient. For simplicity we assume a linear gradient, i.e., $V(r)\propto r$. In order to conserve mass flux in a steady flow, we adopt $n(r)\propto r^{-3}$. We have computed profiles over a range of densities $n_H(R_*)=10^9-10^{10}\,\mathrm{cm}^{-3}$, for velocities V_{out} ranging up to $30\,\mathrm{km\,s}^{-1}$ and for two extensions of the chromosphere $R_{out}/R_*=2$ and 5.

3. Results

The main aim of the computations undertaken here was to study the effect of the transfer of the Lyman continuum and Lyman α line radiation on the formation of the Hα line in late type supergiant chromospheres. The earlier computations, where radiative detailed balance in all the Lyman transitions was assumed to hold, provided a reference situation against which the new results could be studied. In general, the principal non-LTE effect revealed in the computations is a drastic reduction in the level of ionization. For the temperatures and densities considered here the LTE ionization would be very high. When the LTE assumption is replaced by the assumption of radiative detailed balance in the Lyman continuum and the lines, as in Mallik & Mallik (1988), the ionization drops down by a few orders of magnitude. The level 1 is overpopulated and since the levels 1 and 2 are still collisionally coupled, as n_1 goes up, so does n_2. The increase in n_2 is quite substantial. When the radiative detailed balance condition is relaxed, first in the Lyman continuum, there is a further drop in the ionization and the levels 1 and 2 are overpopulated even more although they are no longer collisionally coupled. For $R_{out}/R_*=2$ and $n_H(R_*)=10^9\,\mathrm{cm}^{-3}$, b_1 goes up

Table 1. Hydrogen ionization parameters

r/R_*	T(K)	n_1/n_p			
		LTE	RDB [a]	Lyc [b]	Present [c]
(a) $n_H(R_*) = 10^9\ cm^{-3}$, $R_{out}/R_* = 5.0$					
1.6	7125	0.71 (−3)	0.135	0.281	0.607
2.9	8000	0.99 (−5)	0.006	0.007	0.819
3.5	8125	0.40 (−5)	0.005	0.006	1.241
4.2	8300	0.16 (−5)	0.004	0.005	2.040
5.0	8500	0.56 (−6)	0.003	0.030	11.529
(b) $n_H(R_*) = 10^9\ cm^{-3}$, $R_{out}/R_* = 2.0$					
1.2	6850	4.93 (−3)	0.385	0.416	0.911
1.5	8000	0.07 (−3)	0.010	0.022	0.291
1.6	8125	0.038 (−3)	0.009	0.013	0.341
1.8	8275	0.038 (−3)	0.008	0.008	0.431
2.0	8500	0.008 (−3)	0.008	0.053	2.023

[a] On the assumption of radiative detailed balance in Lyman transitions.
[b] Only Lyc transfer allowed.
[c] Lyc transfer and Lyα escape.

from 50 at $r = 1.16\,R_*$ to 5000 at the surface, while b_2 varies between 100 and 2000 over the same region. The effects are marginal at the base and more drastic near the surface. Next, as the escape of the Lyman α radiation is also taken into account and the full set of statistical equilibrium equations is solved for n_1, n_2, and n_p, we find a further drop in the level of ionization. The inclusion of Lyman α escape has changed things enormously. The level 2 is drained with further enhancement of n_1. Moreover, since in these cool star chromospheres the major part of hydrogen ionization occurs through photoionization out of the level 2 due to the underlying stellar photospheric radiation, this process is now rendered inefficient resulting in a further reduction in the level of ionization. For the same model quoted above b_1 now varies between 100 at $r = 1.16\,R_*$ and $8\ 10^4$ at the surface while b_2 varies from 50 to 100. To illustrate these effects quantitatively, we present in Tables 1a and 1b the variation of n_1/n_p with height in the atmosphere for two of our models. The various columns correspond to the different approximations made in computing the ionization. The distributions of n_2 are displayed in Figs. 2–4. In general n_2 is higher than in the radiative detailed balance case almost all through the chromosphere, except at the surface where the depopulation due to Lyman α escape is very high indeed. At lower densities all these effects are enhanced as they are also for higher extensions of the chromosphere, if the density were held constant.

The choices of the initial density $n_H(R_*)$ and the extensions R_{out}/R_*, that we have made, were prompted by the need to reproduce the observed characteristics of the chromospheric Hα absorption in G and K supergiants. A sample of these stars was observed by Mallik (1982, 1986) using the Coude Echelle Spectrograph at the 1-m Zeiss telescope of Vainu Bappu Observatory in Kavalur, India. Our previous computations indicated that the observed line strengths implied chromospheric optical depths in Hα in the range 50–1000 and densities in the range 10^9–$10^{11}\ cm^{-3}$. In the present calculations we find enhanced n_2 for the same initial densities. Hence the observed characteristics are reproduced in models with lower initial densities. The Hα profiles

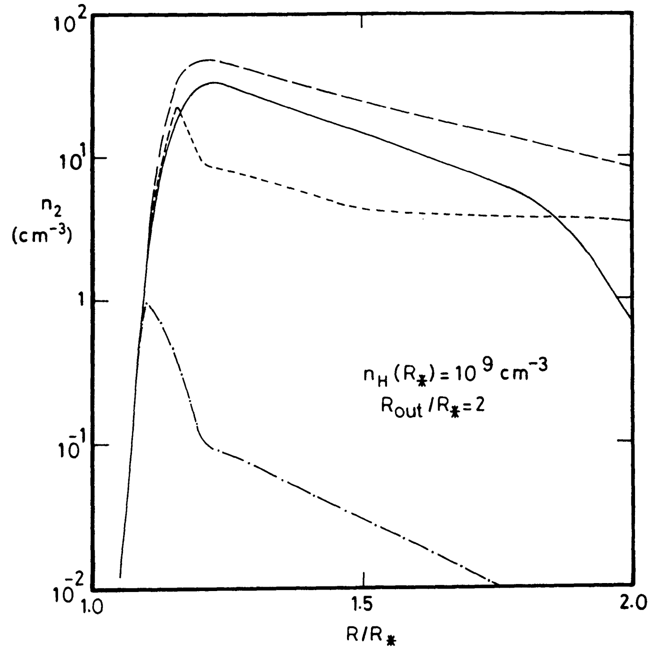


Fig. 2. The distribution of n_2 as a function of the radial coordinate for $n_H(R_*) = 10^9\ cm^{-3}$ and $R_{out}/R_* = 2$. The various cases: *dot-dash line*, LTE; *short dashes*, radiative detailed balance; *long dashes*, Lyc transfer: *continuous*, Lyc and Lyα transfer

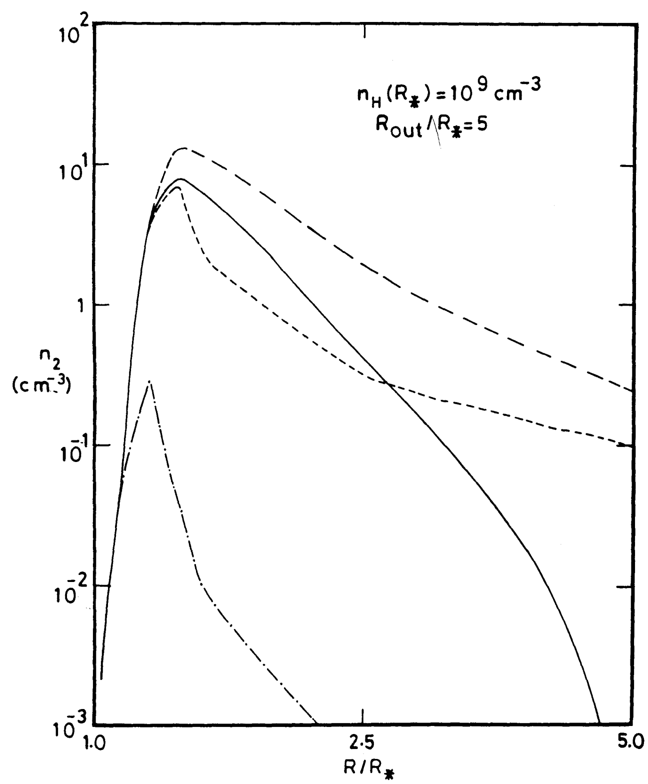


Fig. 3. Same as Fig. 2 for $n_H(R_*) = 10^9\ cm^{-3}$ and $R_{out}/R_* = 5$

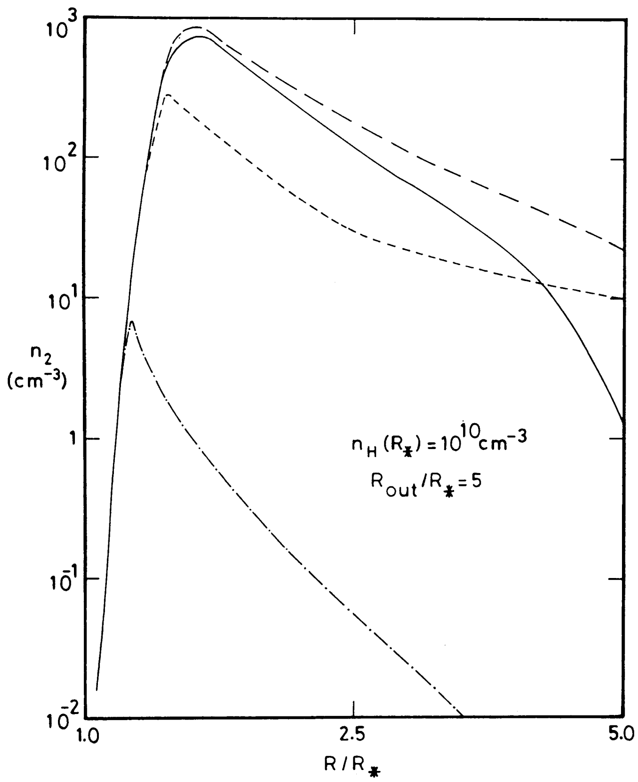
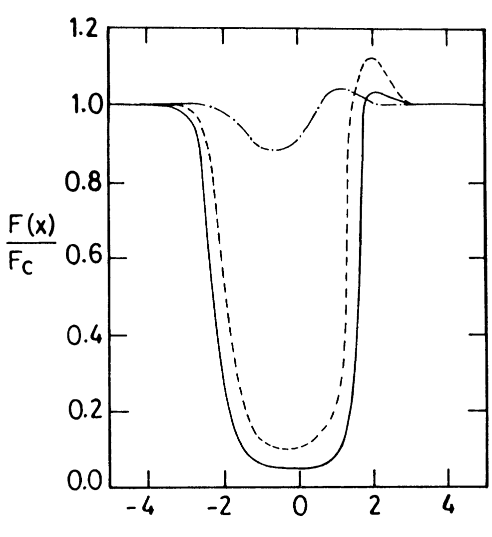


Fig. 4. Same as Fig. 2 for $n_H(R_*) = 10^{10}\,cm^{-3}$ and $R_{out}/R_* = 5$

are displayed in Figs. 5–8. In each case the LTE profile as well as the one obtained on the assumption of radiative detailed balance in Lyman transitions are shown for comparison. The equivalent widths as well as the full widths at half maximum are higher in the present case, although the increase is by no means very drastic. At $n_H(R_*) = 10^{10}\,cm^{-3}$ for both $R_{out}/R_* = 2$ and 5 the effects are reduced. It appears that a factor of 2 to 5 reduction in $n_H(R_*)$ over the previously inferred values would suffice to match the observed Hα strengths. The computed mass loss rates should also be decreased by the same factor. Hence they should now lie in the range 10^{-7} to $2\ 10^{-9}\,M_\odot\,yr^{-1}$.

We had noted in our previous work that microturbulent velocities of the order 10–15 $km\,s^{-1}$ failed to reproduce the observed widths satisfactorily. While we find a marginal increase in FWHM in our present computations, this is hardly adequate to make up for the difference in widths. Within the constraints of the present models, therefore, only an increased microturbulent velocity can explain the large widths observed. The computed profiles are always deeper than the observed ones. This effect was also present in the earlier models with assumed radiative detailed balance in Lyman transitions. The relaxation of this assumption in the present case did not enhance the agreement in this respect to any tangible extent. The problem of matching the width and the depth of the Hα profile is present even when consistent multilevel computations of the Hα line transfer are carried out. Kunasz & Morrison (1982) modelled the Hα line in the wind of α Cyg by performing transfer calculations very similar to ours but based on a multilevel configuration. They concluded that the shallowness of the observed profiles could not be synthesized by fiddling around with the various model parameters. From the results of the detailed computations of the Hα line in the solar chromosphere by Vernazza et al. (1981), it is obvious that although the multilevel computations raised the depth of the core somewhat, it fell short of matching it exactly with the observed Hα profile. From these published results we infer that the principal results derived here would still remain valid when replaced by consistent multilevel computations.

Finally, although the new Hα profiles do not show drastic changes with respect to the earlier ones, the chromosphere that produces the line is quite different. The reduced level of ionization implies lower electron densities and reduced cooling. We cannot comment on the exact effects of these any further, since our temperature structure has been assumed and not computed. The change in the ionization structure of the chromosphere due to the transfer of Lyman radiation would perhaps affect collision-dominated lines more strongly. It would be worth undertaking a study of the lines that are more sensitive to the local values of the electron density and n_1/n_e. It is clear that in a complete chromospheric model the radiative detailed balance assumption in Lyman transitions has to be properly relaxed to enable a



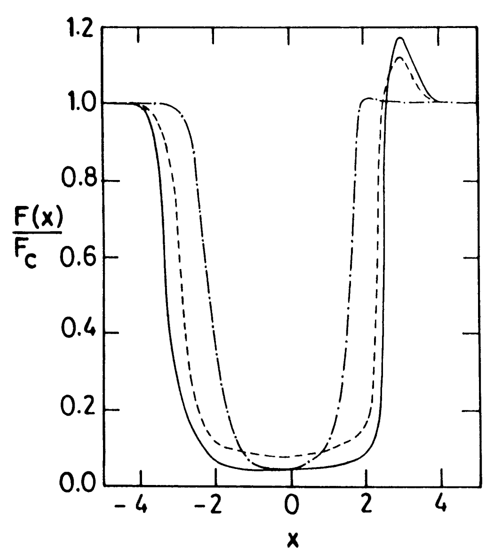


Fig. 5 (*left*). Emergent flux profile of Hα for $n_H(R_*) = 10^9\,cm^{-3}$, $R_{out}/R_* = 2$, $V_{max} = 10$. Here $x = (v - v_0)/\Delta v_D$ where Δv_D is the Doppler width at R_{out}. *Dot-dash line*, LTE; *short dashes*, radiative detailed balance; *continuous*, present paper

Fig. 6 (*right*). Same as Fig. 5 for $n_H(R_*) = 10^{10}\,cm^{-3}$, $R_{out}(R_*) = 2$, $V^m{}_{ax} = 10$

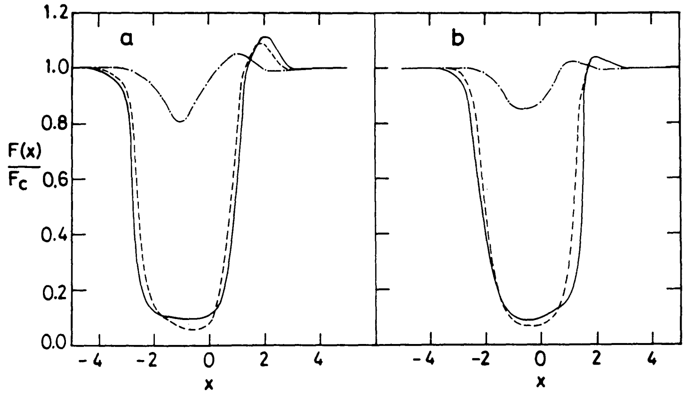


Fig. 7a and b. Same as Fig. 5 for $n_H(R_*) = 10^9\,cm^{-3}$, $R_{out}/R_* = 5$, $V_{max} = 20$ and 10

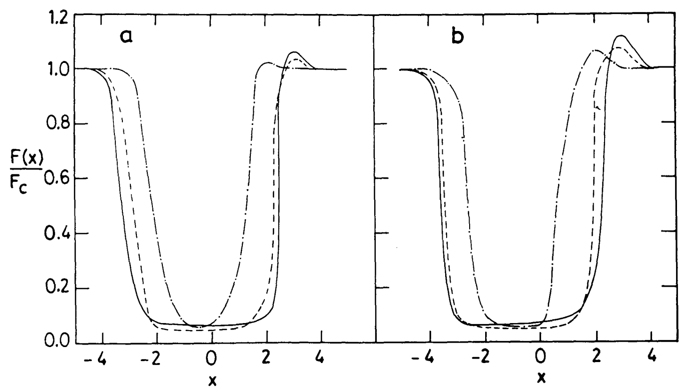


Fig. 8a and b. Same as Fig. 5 for $n_H(R_*) = 10^{10}\,cm^{-3}$, $R_{out}/R_* = 5$. $V_{max} = 10$ and 20

realistic calculation of the cooling rate and for proper determination of chromospheric line profiles.

In our program we have been able to compute the emergent Lyman continuum fluxes from the chromospheres of late G/early K supergiants. The possibilities of actually observing the far uv spectrum of these stars in selected regions of the sky may not be too remote. The results of these computations may then be rigorously tested.

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