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A radiative transfer calculation using R_{III} in an expanding spherically symmetric stellar atmosphere

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ABSTRACT

With the availability of very high-resolution stellar spectra, even small differences between line profiles resulting from complete redistribution and partial frequency redistribution may be important in the quantitative analysis of stellar spectra. A differentially expanding and spherically symmetric stellar envelope is considered in order to study the effects of the angle-averaged partial frequency redistribution function R_{III} on spectral line formation. The line transfer equation is solved in the rest frame of the star, assuming a two-level atomic model. The differences between the emergent mean intensity profiles resulting from complete redistribution and the angle-averaged R_{III} are found to be much smaller in expanding spherically symmetric stellar atmospheres than has been reported previously for plane-parallel atmospheres.

Key words: line: profiles – radiative transfer – stars: atmospheres.

1 INTRODUCTION

The limiting case of complete redistribution (CRD) is usually taken to be an adequate representation of the partial frequency redistribution (PRD) function R_{III} in radiative transfer problems to study spectral line formation in stellar atmospheres. The works of Finn (1967) and Vardavas (1976a,b) have shown that, for plane-parallel (static and moving) stellar atmospheres, the differences between the emergent intensities due to R_{III} and CRD are below 20 per cent. With the advent of very high-resolution stellar spectroscopy (resolving power $\sim 10^5$–2×10^5), however, highly accurate line profiles (e.g. for bright giants and supergiants: see Sanner 1976) are available, and therefore even small differences between CRD and PRD profiles could be important in a quantitative analysis of stellar spectra [for example, in stellar winds and mass-loss phenomena in giants and supergiants (Hempe 1984)]. Also, the combined effect of sphericity and macroscopic velocity on these differences cannot be estimated a priori.

To the author's knowledge, there has so far been no paper in the literature showing the effects of R_{III} on spectral line formation in spherically symmetric and differentially expanding stellar atmospheres. In this paper, therefore, I consider a parametrized spherically symmetric and differentially expanding stellar envelope to study the effects of the angle-averaged R_{III} on spectral line formation. I solve the line transfer equation in the rest frame of the star, assuming a two-level atomic model. As a part of this work, I also examine the behaviour of the emergent mean intensity profiles (J_x), the source function (S) and the emergent flux profiles (F_x) as a function of the atmospheric extent (b/a = outer/inner stellar radius), the maximum velocity of expansion (V_b) and the thermalization parameter (ε). The results are in agreement with the earlier results of Kunasz & Hummer (1974) and Mihalas (1978). The emergent mean intensity profiles arising from $R_{\mathrm{III}}(x', x)$ and CRD are compared. Here, x' and x are the frequency displacements from the line centre in Doppler units of the incident and scattered photons, respectively.

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2 BASIC EQUATIONS AND COMPUTATIONAL PROCEDURE

If x' and x are the frequency displacements from the line centre (in units of standard Doppler width) of the incident and scattered photons, respectively, seen in the *rest frame*, then the corresponding frequency displacements, seen in the *fluid frame* at radius r, are

$$X' = x' \pm V(r)\mu, \qquad (1)$$

$$X = x \pm V(r)\mu, \qquad (2)$$

where $\mu(\in[0, 1])$ is the cosine of the angle between the radial vector and the direction of propagation of the radiation, $\pm$ stands for the oppositely directed beams of radiation, and $V(r)$ gives the macroscopic velocity at radius r.

The line transfer equation for spherical geometry in the rest frame for a two-level atomic model has the following form (Peraiah 1978):

$$\pm\mu\frac{\partial I(x,\pm\mu,r)}{\partial r}\pm\frac{1-\mu^2}{r}\frac{\partial I(x,\pm\mu,r)}{\partial\mu}$$
$$=K_l[\beta+\phi(x,\pm\mu,r)][S(x,\pm\mu,r)-I(x,\pm\mu,r)], \quad (3)$$

where $\pm$ stands for the oppositely directed beams of radiation, and $I(x, \pm\mu, r)$ represents the specific intensity of the ray making an angle $\cos^{-1}\mu$ ($\mu\in[0,1]$) with the radial vector at the radial point r. Also, $\phi(x, \pm\mu, r)$ represents the profile function, given as

$$\phi(x,\pm\mu,r)=\phi(X,r)=\int_{-\infty}^{+\infty}R_{\mathrm{III}}(X',X)\,\mathrm{d}X', \quad (4)$$

where the angle-averaged partial frequency redistribution function $R_{\mathrm{III}}(X', X)$ has the following form (Mihalas 1978):

$$R_{\mathrm{III}}(X',X)=\pi^{-5/2}\int_0^\infty\left[\tan^{-1}\left(\frac{X'+u}{a}\right)-\tan^{-1}\left(\frac{X'-u}{a}\right)\right]$$
$$\times\left[\tan^{-1}\left(\frac{X+u}{a}\right)-\tan^{-1}\left(\frac{X-u}{a}\right)\right]\mathrm{d}u, \quad (5)$$

a being the damping parameter, set equal to 10^{-3} in this calculation. $S(x, \pm\mu, r)$ is the total (line plus continuum) source function, which is written as

$$S(x,\pm\mu,r)=\frac{\phi(X,r)S_l(x,\pm\mu,r)+\beta S_c(r)}{\phi(X,r)+\beta}, \quad (6)$$

where $\beta=K_c/K_l$ is the continuum-to-line opacity ratio, and S_c is the continuum source function, set equal to 1 in this computation. $S(x, \pm\mu, r)$ represents the line source function, which is given by the following expression:

$$S_l(x,\pm\mu,r)=\frac{1}{2}\frac{1-\varepsilon}{\phi(X,r)}$$
$$\times\int_{-\infty}^{+\infty}\mathrm{d}x'\int_{-1}^{+1}\mathrm{d}\mu' R_{\mathrm{III}}(X',X)I(x',\pm\mu,r)+\varepsilon B(r), \quad (7)$$

where ε (the thermalization parameter) is the probability per scatter that a photon is destroyed by collisional de-excitation, written as

$$\varepsilon=\frac{C_{21}}{C_{21}+A_{21}[1-\exp(-h\nu/k\theta)]^{-1}}; \quad (8)$$

here, C_{21} is the rate of collisional de-excitation from level 2 to level 1, A_{21} is Einstein's spontaneous emission probability for transition $2\rightarrow1$, h is Planck's constant, ν is the photon frequency, k is Boltzmann's constant and θ is the temperature.

Under the assumption of complete redistribution, the line source function is written as

$$S_l(r)=\frac{1-\varepsilon}{2}\int_{-\infty}^{+\infty}\mathrm{d}x\int_{-1}^{+1}\mathrm{d}\mu\phi(X,r)I(x,\pm\mu,r)+\varepsilon B(r), \quad (9)$$

where $\phi(X, r)$ is taken to be a Voigt profile with damping parameter set equal to 10^{-3}.

The normalization condition for the profile function at each radial point is

$$\int_{-\infty}^{+\infty}\mathrm{d}x\phi(X,r)=1. \quad (10)$$

In a differentially expanding medium, due to the presence of velocity gradients, one has to compute, at all radial points, all four redistribution functions appearing in the scattering integral (equation 7), namely

$R[x'+V(r)\mu, x+V(r)\mu]$.

$R[x'-V(r)\mu, x-V(r)\mu]$,

$R[x'+V(r)\mu, x-V(r)\mu]$

and

$R[x'-V(r)\mu, x+V(r)\mu]$,

in order to evaluate the diffuse radiation field (Peraiah 1978, 1979).

I have used a linear velocity law given by Peraiah (1978), which gives a radially increasing velocity, with minimum velocity (V_a) at the innermost boundary and maximum velocity of expansion (V_b) at the outer boundary of the stellar envelope. The envelope is divided into N shells, each of equal radial thickness; $n=1$ defines the outermost shell ($\tau=0$) and $n=N$ defines the innermost shell ($\tau=T$). The total optical depth (T) of the envelope is taken to be 10^3, and I set $N=10$ in these computations. The optical depth (τ) through the envelope varies as r^{-2}. V_b is set equal to 1 and 2 in units of the mean thermal velocity of the gas. The rest-frame calculation is limited to low expansion velocities, because of numerical difficulties. The following boundary conditions are used for the transfer equation:

$$U_1^+(x,\tau=0,\mu)=0 \quad \text{and} \quad U_{N+1}^-(x,\tau=T,\mu)=0 \quad \text{if } \varepsilon>0, \quad (11)$$

$$U_1^+(x,\tau=0,\mu)=0 \quad \text{and} \quad U_{N+1}^-(x,\tau=T,\mu)=1 \quad \text{if } \varepsilon=0, \quad (12)$$

where

$$U_n^\pm(x,\tau,\mu)=4\pi r_n^2 I[x,\mu,\tau(r_n)], \quad (13)$$

$I[x, \mu, \tau(r_n)]$ being the specific intensity. + specifies a ray directed towards the bottom of the envelope ($\tau=T$), and $-$ specifies a ray directed out of the envelope. The Planck function is set equal to 1 throughout the medium. Flux conservation is maintained to an accuracy of the order of 10^{-10} in double precision for a purely scattering medium, i.e. when $\varepsilon=0$ (Peraiah 1984).

To solve the above equations, I have used the discrete space theory method described in detail by Peraiah (1978).

3 RESULTS

Fig. 1 gives the source function S as a function of optical depth τ through a spherically symmetric and differentially expanding envelope, where $\varepsilon=\beta=10^{-6}$. We recover the known results that the increases in velocity and extent (b/a) suppress the source function. We also see that the effect of the expansion velocity is greater at larger extents. The source

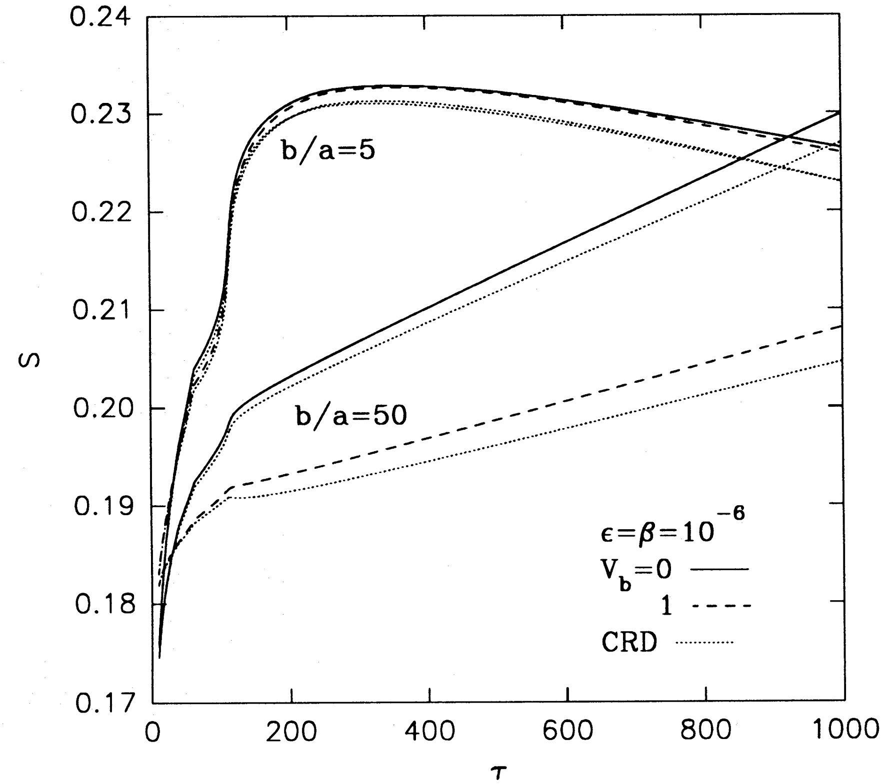

Figure 1. Plots of source function S as a function of optical depth τ through a spherically symmetric and differentially expanding stellar envelope having expansion velocity $V_b=0$ and 1, extent $b/a=5$ and 50, and $\varepsilon=\beta=10^{-6}$. The total optical depth $T=10^3$. Within the medium, the optical depth τ varies as r^{-2}. Results from complete redistribution (CRD) are shown by dotted lines.

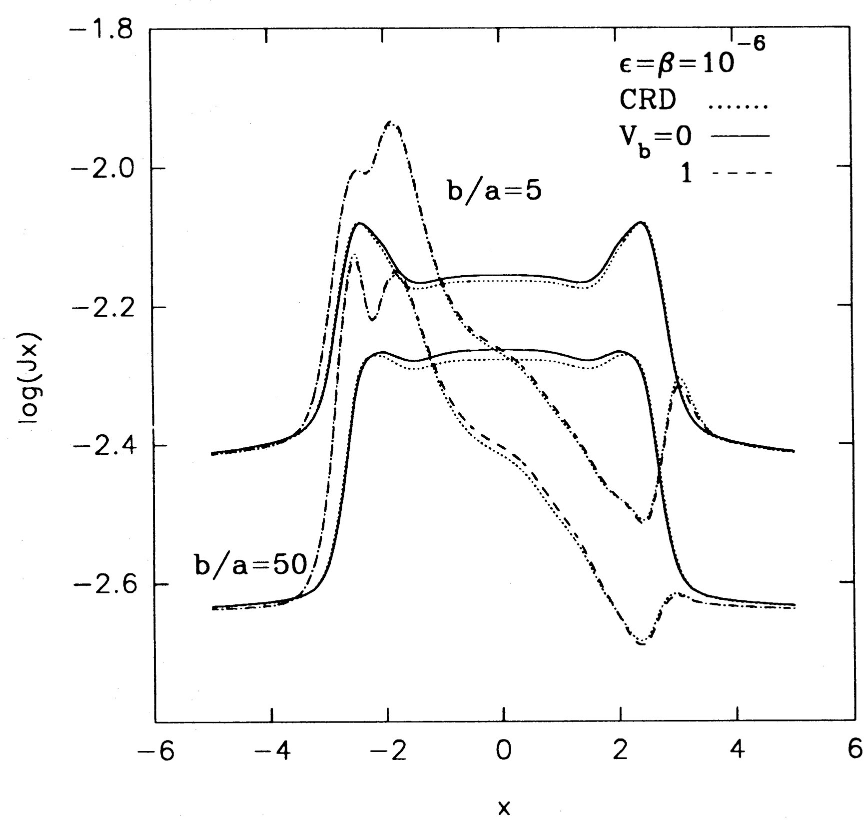

Figure 2. Mean intensity profiles J_x corresponding to the source functions in Fig. 1. Here, $x=(\nu-\nu_0)/w$, w being the standard Doppler width.

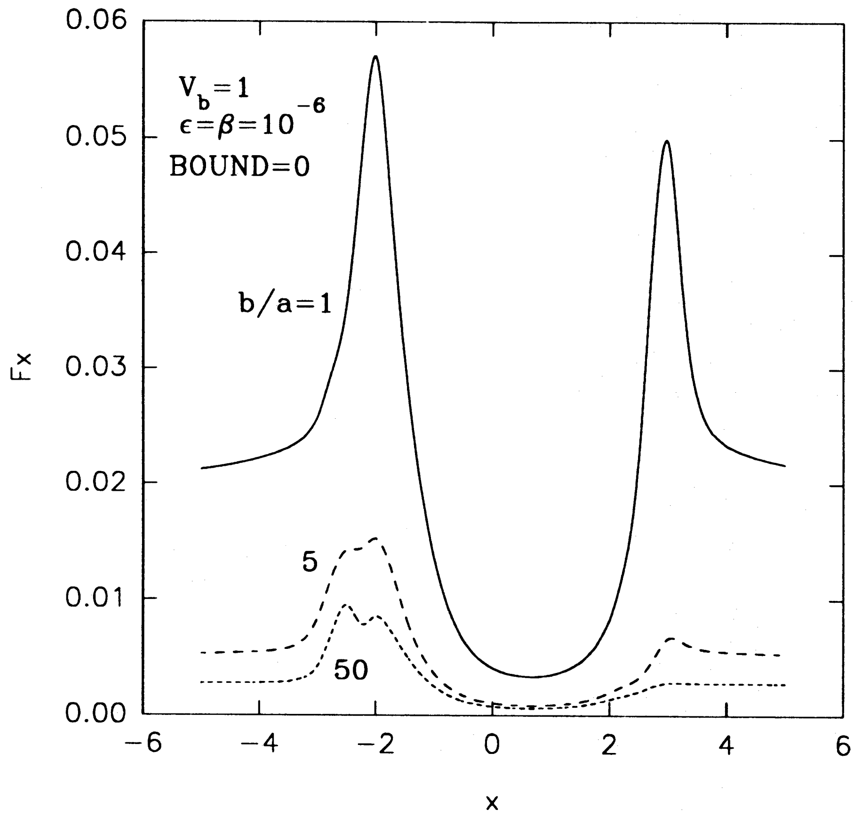

Figure 3. Emergent flux profiles F_x for a spherically symmetric and differentially expanding envelope having $V_b = 1$, $b/a = 1$ (i.e. plane-parallel case), 5 and 50, $\varepsilon = \beta = 10^{-6}$, and total optical depth $T = 10^3$. Within the envelope, the optical depth τ varies as r^{-2}.

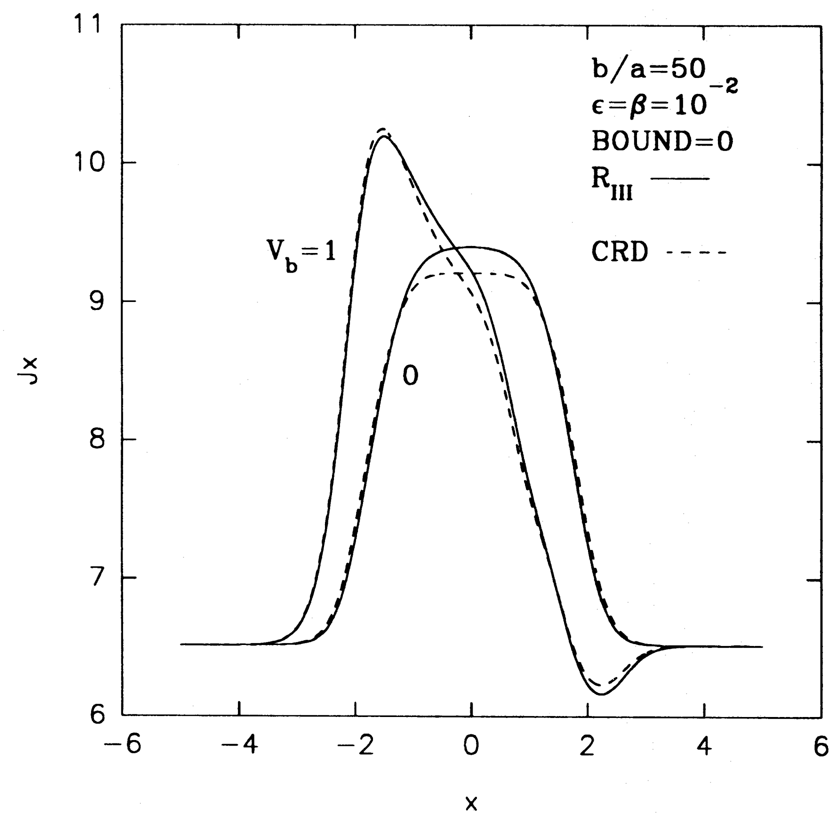

Figure 4. Emergent mean intensity profiles J_x resulting from angle-averaged R_{III} and CRD, for static ($V_b = 0$) and differentially expanding ($V_b = 1$) spherically symmetric stellar envelopes having extent $b/a = 50$ and $\varepsilon = \beta = 10^{-2}$.

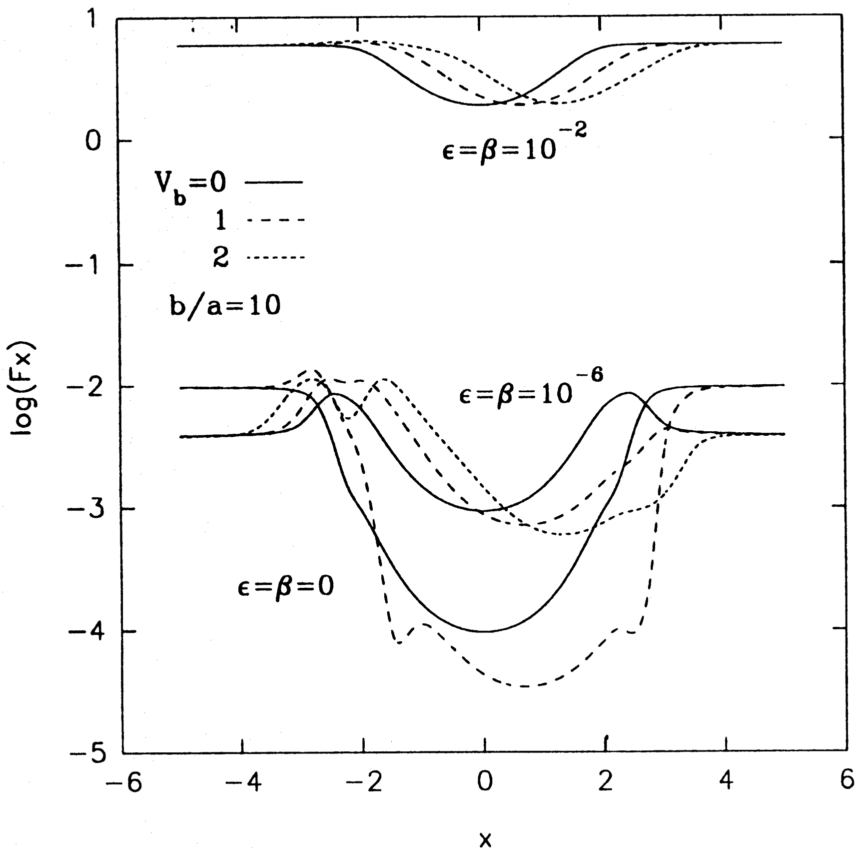

Figure 5. Plots of emergent flux profiles $\log(F_x)$ for a spherically symmetric stellar envelope having extent $b/a=10$ and expansion velocity $V_b=0$ (i.e. static case), 1 and 2, for different values of $\varepsilon=\beta=0$ (i.e. pure scattering without background continuum), 10^{-6} and 10^{-2}.

functions arising from CRD are smaller than those resulting from $R_{\rm III}$.

Fig. 2 gives the mean intensity profiles J_x coresponding to the source functions shown in Fig. 1. It can be seen that, at any given velocity of expansion, the mean intensity in the line is lower for larger extents. The profiles are asymmetric for $V_b=1$ (expressed in terms of the mean thermal velocity of the gas). More important is the result that the difference between the CRD and $R_{\rm III}$ profiles is below 5 per cent near the line centre and negligible in the wings. This is much smaller than reported for planar geometries by Finn (1967) and Vardavas (1976a, b).

In Fig. 3 are shown emergent flux profiles for a differentially expanding ($V_b=1$) envelope at different values of the extent. The emergent line flux from a plane-parallel ($b/a=1$) atmosphere is considerably higher than from spherically symmetric and extended ($b/a=5$, 10) atmospheres. We again recover the known result that the higher the extent, the lower the emergent line flux.

In Fig. 4 I have compared the emergent mean intensity (J_x) profiles arising from CRD and $R_{\rm III}$ for static ($V_b=0$) and differentially expanding ($V_b=1$) spherically symmetric envelopes having extents of $b/a=50$ and $\varepsilon=\beta=10^{-2}$. Although the differences between the CRD and $R_{\rm III}$ profiles are slightly greater than in Fig. 2 where a lower value of $\varepsilon=\beta=10^{-6}$ was used, the differences are still within 5 per cent, again much lower than those reported for planar geometries by Finn (1967) and Vardavas (1976a, b).

Fig. 5 shows the effect of ε and β on emergent flux (F_x) profiles arising from $R_{\rm III}$ for static ($V_b=0$) and differentially expanding spherically symmetric extended envelopes. It is seen that the profiles have greater absorption depths and more asymmetry at lower values of ε and β. The same is also true for CRD, as shown by Kunasz & Hummer (1974).

4 CONCLUSION

The differences between the emergent mean intensity profiles resulting from complete redistribution and from angle-averaged $R_{\rm III}$ are found to be much smaller in spherically symmetric and differentially expanding stellar atmospheres than those reported for plane-parallel (static and moving) atmospheres.

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