

GENERAL RELATIVISTIC EFFECTS ON THE INDUCED ELECTRIC FIELD EXTERIOR TO PULSARS

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ABSTRACT

The importance of general relativity to the induced electric field exterior to pulsars has been investigated by assuming aligned vacuum and nonvacuum magnetosphere models. For this purpose, the stationary and axisymmetric vector potential in the Schwarzschild geometry has been considered and the corresponding expressions for the induced electric field due to the rotation of the magnetic dipole have been derived for both vacuum and non-vacuum conditions. Due to the change in the magnetic dipole field in curved spacetime, the induced electric field also changes its magnitude and direction and increases significantly near the surface of the star. As a consequence, the surface charge density, the acceleration of charged particles in vacuum magnetospheres, and the space charge density in nonvacuum magnetospheres greatly increase near the surface of the star. The results provide the most general feature of the important role played by gravitation and could have several potentially important implications for the production of high-energy radiation from pulsars.

Subject headings: magnetic fields — relativity — stars: pulsars: general

1. INTRODUCTION

The realization that pulsars are most likely to be neutron stars surrounded by a dense magnetosphere (Gold 1968; Pacini 1968) and the pioneering work by Goldreich & Julian (1969) have provided the basic physical model on which virtually all particular models have been based. At present, two general types of models are being considered in order to understand the phenomenon of high-energy radiation from pulsars, namely the polar cap model (Arons 1983) and the outer gap model (Cheng, Ho, & Ruderman 1986). Goldreich & Julian first realized that in spite of the intense surface gravity, a neutron star must possess a dense magnetosphere. Unfortunately, the actual role played by gravitation in this situation has not yet been investigated in detail.

General relativistic effects near pulsars have been explored only in the context of emission models for X-ray pulsars and gamma-ray bursts. The effect of light bending in X-ray bursts and X-ray pulsars was studied by Meszaros & Riffert (1988) and Riffert, Meszaros, & Bagoly (1989). Recently, Gonthier & Harding (1994) have examined the importance of general relativistic corrections to the production of gamma rays near the surface of neutron stars.

It is well known that the induced electric field due to the rapid rotation of the magnetic dipole of a neutron star plays the main role in determining the characteristics of the pulsar magnetosphere since this quantity determines the number density of charged particles torn off from the surface of the star, the acceleration of charged particles, and hence the production of electromagnetic radiation, and so forth. Since the magnetic field in curved spacetime differs in both magnitude and direction from that in flat spacetime, the induced electric field is also different in curved spacetime. Therefore, it is very important to include general relativity in any discussion of pulsar magnetospheres.

In this paper, the importance of general relativistic corrections to the induced electric field external to a neutron star has been investigated quantitatively by considering the simplest aligned vacuum and nonvacuum magnetosphere models.

The paper is organized in the following way: In § 2 the Schwarzschild metric and the corresponding orthonormal tetrad components along with the transformation rules are given. In § 3 we discuss the dipole magnetic field in Schwarzschild spacetime, which has been used to determine the induced electric field in § 4. In § 4, a detailed quantitative discussion on the effects of spacetime curvature on the various important quantities governed by the electric field is presented. For simplicity we have assumed the vector potential to be stationary and axisymmetric. We conclude the discussion in § 5.

2. SPACETIME METRIC

Since we are interested on the electromagnetic field exterior to a neutron star, we consider the Schwarzschild metric, as it provides a simple description of the spacetime geometry exterior to neutron stars.

The metric is given by

$$ds^2 = (1 - 2m/r)c^2 dt^2 - (1 - 2m/r)^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2, \quad (1)$$

where $m = MG/c^2$, M being the total mass of the star.

The nonzero components of the orthonormal tetrad $\lambda_{(\alpha)}^i$ of the local Lorentz frame for the Schwarzschild geometry are given by

$$\lambda_{(t)}^t = (1 - 2m/r)^{-1/2}, \quad \lambda_{(r)}^r = (1 - 2m/r)^{1/2}, \\ \lambda_{(\theta)}^\theta = 1/r, \quad \lambda_{(\phi)}^\phi = 1/(r \sin \theta).$$

If $F_{(\alpha\beta)}$ are the components of the electromagnetic field tensor in a local Lorentz frame, then the components of the electromagnetic field tensor F_{ij} are defined in the curved spacetime through

$$F_{(\alpha\beta)} = \lambda_{(\alpha)}^i \lambda_{(\beta)}^j F_{ij}. \quad (2)$$

3. DIPOLE MAGNETIC FIELD IN CURVED SPACETIME

Since the magnetic field of a neutron star is generally approximated to be that of a pure magnetic dipole, in flat spacetime we can write the components of the magnetic field

(in polar coordinates) as

$$\mathbf{B} = \frac{2\mu}{r^3} \left(\cos \theta, \frac{1}{2} \sin \theta, 0 \right), \quad (3)$$

where μ is the magnetic dipole moment.

The magnetic dipole has to be modified in curved spacetime. In Schwarzschild spacetime the electromagnetic potential A_i is given by (Pettersen 1974; Wasserman & Shapiro 1983)

$$A_i = (0, 0, A_\phi, 0),$$

with

$$A_\phi = -\frac{3\mu \sin^2 \theta}{8m^3} \left[r^2 \ln \left(1 - \frac{2m}{r} \right) + 2mr + 2m^2 \right] \quad (4)$$

Using the definition $F_{ij} = A_{j,i} - A_{i,j}$, we get the components of the magnetic field in the Schwarzschild geometry as

$$F_{r\phi} = -\frac{3\mu \sin^2 \theta}{4m^2} \left[\frac{r}{m} \ln \left(1 - \frac{2m}{r} \right) + \left(1 - \frac{2m}{r} \right)^{-1} + 1 \right], \quad (5)$$

$$F_{\theta\phi} = -\frac{3\mu r^2}{4m^3} \sin \theta \cos \theta \left[\ln \left(1 - \frac{2m}{r} \right) + \frac{2m}{r} \left(1 + \frac{m}{r} \right) \right]. \quad (6)$$

Therefore, using equation (2) we get

$$B_r(\text{Sch}) = -\frac{3\mu \cos \theta}{4m^3} \left[\ln \left(1 - \frac{2m}{r} \right) + \frac{2m}{r} \left(1 + \frac{m}{r} \right) \right], \quad (7)$$

$$B_\theta(\text{Sch}) = \frac{3\mu \sin \theta}{4m^2 r} \left[\frac{r}{m} \ln \left(1 - \frac{2m}{r} \right) + \left(1 - \frac{2m}{r} \right)^{-1} + 1 \right] \left(1 - \frac{2m}{r} \right)^{1/2}. \quad (8)$$

It should be mentioned that the above expressions are specialized to the lowest order expansion in terms of spherical harmonics. Nevertheless, they are valid in the context of neutron stars since the magnetic field is not so strong relative to the gravitational field that the spacetime geometry could be affected significantly by the magnetic field.

4. INDUCED ELECTRIC FIELD IN CURVED SPACETIME

In order to investigate the effect of curved spacetime on the electric field exterior to a rotating neutron star, we consider the two simplest models, namely, (1) the aligned vacuum rotator and (2) the aligned nonvacuum rotator. Since the effect of curvature would be the same, the results will provide the most general feature for any magnetospheric model which assumes stationary ($\partial/\partial t = 0$) and axisymmetric ($\partial/\partial \phi = 0$) behavior of the vector potential. In the present study we have ignored any effects due to magnetosphere plasma. Also, we have neglected the dragging of inertial frames due to rotation, which is insignificant at the exterior of even millisecond pulsars.

4.1. Aligned Vacuum Rotator

Since the derivations are nothing but a generalization of the Goldreich-Julian solution, we briefly discuss this solution

before going on to derive the electromagnetic field in curved spacetime—although it can be found in any text book on pulsars, e.g., Shapiro & Teukolsky (1983).

First of all, it is considered that a rotating neutron star has an aligned dipole external magnetic field given by equation (3). The stellar material is assumed to be an excellent conductor so that just inside the star an electric field will be present which satisfies

$$\mathbf{E}^{\text{int}} + \frac{\boldsymbol{\omega} \times \mathbf{r}}{c} \times \mathbf{B}^{\text{int}} = 0, \quad (9)$$

where $\boldsymbol{\omega}$ is the angular velocity of the star.

Here and afterward the superscript “int” is to be understood to represent quantities just inside the star and not far below the surface where the above condition may not be valid.

Assuming there are no surface currents, both the normal and the tangential components of \mathbf{B} are continuous across the stellar surface. Thus, just inside the surface the magnetic field can be written by taking $r = R$ in equation (3), where R is the radius of the star. Then equation (9) gives for \mathbf{E} just inside the surface

$$\mathbf{E}^{\text{int}} = \frac{2\mu\omega}{cR^2} \left(\frac{1}{2} \sin^2 \theta, -\sin \theta \cos \theta, 0 \right). \quad (10)$$

The tangential component of \mathbf{E} is continuous across the surface, so just outside the star equation (10) implies

$$\mathbf{E}_\theta^{\text{ext}} = -\frac{\partial}{\partial \theta} \left(\frac{\mu\omega \sin^2 \theta}{cR^2} \right) = \frac{\partial}{\partial \theta} \left[\frac{2\mu\omega}{3cR^2} P_2(\cos \theta) \right], \quad (11)$$

where P_2 is the Legendre polynomial of second degree.

In the present case, since the exterior is assumed to be vacuum,

$$\mathbf{E}^{\text{ext}} = -\nabla\phi,$$

where

$$\nabla^2\phi = 0. \quad (12)$$

In order to satisfy the boundary condition given by equation (11) at $r = R$, the solution of equation (12) must be

$$\phi = -\frac{2\mu\omega R^2}{3cr^3} P_2(\cos \theta),$$

and hence

$$\mathbf{E}^{\text{ext}} = -\frac{\mu\omega R^2}{cr^4} [(3 \cos^2 \theta - 1), 2 \sin \theta \cos \theta, 0]. \quad (13)$$

Now we will derive the electric components exterior to a rotating neutron star for a Schwarzschild background geometry with the basic assumptions taken unaltered from the above discussions.

Equation (9) can be written in its covariant form as

$$F_\delta^\delta u^\delta = 0, \quad (14)$$

where the u^δ are the components of the four-velocity vector.

From equation (14) we obtain

$$F_{rt} = F_{\phi r} \frac{u^\phi}{u^t}, \quad (15)$$

$$F_{\theta t} = F_{\phi\theta} \frac{u^\phi}{u^t}. \quad (16)$$

[Note that $u = (u^t, 0, 0, u^\phi)$ for the present case.]

Using equations (5) and (6) and noting that in the Schwarzschild geometry

$$E_r = F_{(rt)} = F_{rt}, \\ E_\theta = F_{(\theta t)} = F_{\theta t} / [r(1 - 2m/r)^{1/2}],$$

and taking

$$\frac{u^\phi}{u^t} = \frac{\omega}{c},$$

we obtain the induced electric field components just inside the surface ($r \simeq R$) as

$$E_r^{\text{int}} = \frac{3\mu\omega \sin^2 \theta}{4m^2 c} \left[\left(1 - \frac{2m}{r}\right)^{-1} + \frac{r}{m} \ln \left(1 - \frac{2m}{r}\right) + 1 \right], \quad (17)$$

$$E_\theta^{\text{int}} = \frac{3\mu\omega r \cos \theta \sin \theta}{4cm^3} \left[\ln \left(1 - \frac{2m}{r}\right) + \frac{2m}{r} \left(1 + \frac{m}{r}\right) \right] \\ \times \left(1 - \frac{2m}{r}\right)^{-1/2}. \quad (18)$$

If $f(r)$ and $g(r)$ are two arbitrary functions which solely determine the spacetime curvature effect, then in curved spacetime we can write

$$E_r^{\text{int}}(\text{curved}) = \frac{\mu\omega}{cr^2} \sin^2 \theta f(r), \quad (19)$$

$$E_\theta^{\text{int}}(\text{curved}) = -\frac{2\mu\omega}{cr^2} \sin \theta \cos \theta g(r). \quad (20)$$

Comparing equations (19) and (20) with equations (17) and (18), we obtain

$$f(r) = \frac{3r^2}{4m^2} \left[\left(1 - \frac{2m}{r}\right)^{-1} + \frac{r}{m} \ln \left(1 - \frac{2m}{r}\right) + 1 \right], \quad (21)$$

$$g(r) = -\frac{3r^3}{8m^3} \left[\ln \left(1 - \frac{2m}{r}\right) + \frac{2m}{r} \left(1 + \frac{m}{r}\right) \right] \left(1 - \frac{2m}{r}\right)^{-1/2}. \quad (22)$$

In the above derivations we have assumed that the metric can well describe the spacetime curvature just below the surface ($r \simeq R$) of the star; i.e., the curvature effect just below the surface is the same as that at the surface to the star.

In flat spacetime the components of the exterior electric field are given by equation (13).

Therefore, in the Schwarzschild geometry the components of the exterior electric field can be written as

$$E_r^{\text{ext}}(\text{Sch}) = -\frac{\mu\omega R^2}{cr^4} (3 \cos^2 \theta - 1) f(r) \\ = -\frac{3\mu\omega R^2}{4cm^2 r^2} (3 \cos^2 \theta - 1) \\ \times \left[\left(1 - \frac{2m}{r}\right)^{-1} + \frac{r}{m} \ln \left(1 - \frac{2m}{r}\right) + 1 \right], \quad (23)$$

$$E_\theta^{\text{ext}}(\text{Sch}) = -\frac{2\mu\omega R^2}{cr^4} \sin \theta \cos \theta g(r) = \frac{3\mu\omega R^2 \cos \theta \sin \theta}{4cm^3 r} \\ \times \left[\ln \left(1 - \frac{2m}{r}\right) + \frac{2m}{r} \left(1 + \frac{m}{r}\right) \right] \left(1 - \frac{2m}{r}\right)^{-1/2}. \quad (24)$$

Clearly, $E^{\text{ext}}(\text{curved}) \rightarrow E^{\text{ext}}(\text{flat})$ as $r \rightarrow \infty$, as desired.

Taking as an example the Crab pulsar with mass M equal to $1.4 M_\odot$, radius R equal to 10^6 cm, period P equal to 33 ms, and magnetic dipole moment μ equal to 2×10^{30} G cm³, we show in Figure 1 the r -component of the quadrupole electric field at an angle $\theta = 0^\circ$ to the axis of rotation as a function of distance from the stellar surface for flat and curved spacetimes. It is found that a significant increase in the electric field intensity near the surface is caused by the inclusion of general relativistic effect.

Now we have the functional form for the dipole magnetic field and the quadrupole electric field in flat spacetime as well as in Schwarzschild spacetime. We would like to see how the field lines differ in the two different geometries. The field lines represent constant magnetic and electric flux $b(\theta)$ and $e(\theta)$, respectively, and can be found by rotating a given field line about the z -axis in a spherical coordinate system with the magnetic dipole aligned with the z -axis. The fluxes from $\theta = 0$ to θ are given by

$$b(\theta) = \int_0^\theta \mathbf{B} \cdot d\mathbf{a} = \int_0^\theta B_r r^2 d\Omega, \quad (25)$$

$$e(\theta) = \int_0^\theta \mathbf{E} \cdot d\mathbf{a} = \int_0^\theta E_r r^2 d\Omega. \quad (26)$$

The magnetic field lines in flat spacetime and in curved spacetime have been presented by Gonthier & Harding (1994).

For an induced quadrupole electric field in flat spacetime, the field lines are

$$e(\theta) = -\frac{2\pi\mu\omega R^2}{cr^2} \sin^2 \theta \cos \theta = \text{constant}. \quad (27)$$

In curved spacetime the field lines are given by

$$e(\theta) = -\frac{3\pi\mu\omega R^2}{2m^2 c} \sin^2 \theta \cos \theta \\ \times \left[\left(1 - \frac{2m}{r}\right)^{-1} + \frac{r}{m} \ln \left(1 - \frac{2m}{r}\right) + 1 \right] \\ = \text{constant}. \quad (28)$$

Taking the Crab pulsar with a period of 33 ms as an example, we show in Figure 2a the quadrupole electric field lines in the z - x plane out to 40 stellar radii, beyond which the effect of curvature becomes insignificant. Figure 2b shows a closer view of the same field lines.

Since the normal component of the electric field is discontinuous for the case of an aligned vacuum rotator, the corresponding surface charge density can be written in flat spacetime as

$$\rho = \frac{1}{4\pi} (E_r^{\text{ext}} - E_r^{\text{int}}) = -\frac{\mu\omega}{2\pi c R^2} \cos^2 \theta. \quad (29)$$

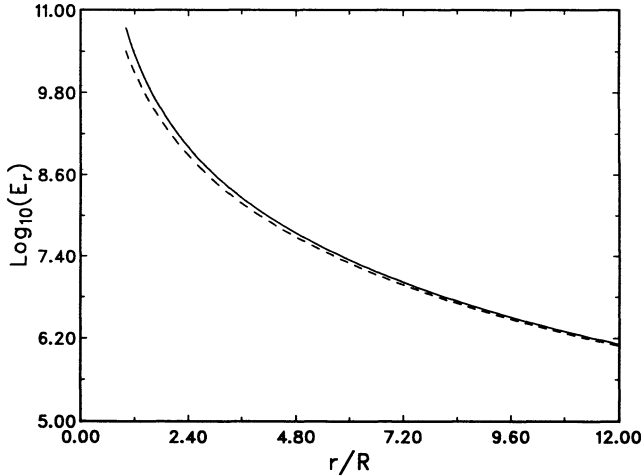


FIG. 1.—The r -component of the quadrupole electric field at an angle $\theta = 0^\circ$ to the axis of rotation for the Crab pulsar, as a function of distance from the stellar surface. The solid curve represents the field intensity for curved spacetime, and the dashed curve for flat spacetime.

In curved spacetime the same quantity can be written as

$$\rho = -\frac{3\mu\omega}{8\pi cm^2} \cos^2 \theta \left[\left(1 - \frac{2m}{r}\right)^{-1} + \frac{R}{m} \ln \left(1 - \frac{2m}{r}\right) + 1 \right]. \quad (30)$$

Since the effect of curvature increases the electric field intensity near the surface, the surface charge density in curved spacetime is much higher than that in flat spacetime. For a pulsar with mass $1.4 M_\odot$ and radius 10 km, the surface charge density at the pole in curved spacetime is twice that in flat spacetime.

The Lorentz invariant scalar product $E^{\text{ext}} \cdot B$ does not vanish. This quantity gives a measure of the force which a co-rotating charged particle feels in the direction of the magnetic field.

In flat spacetime,

$$E^{\text{ext}} \cdot B = -\frac{4\mu^2 \omega R^2 \cos^3 \theta}{cr^7}, \quad (31)$$

leading to an acceleration of a charged particle in the direction of the magnetic field:

$$a = \frac{eE^{\text{ext}} \cdot B}{m_p |B|} = -\frac{4e}{m_p c} \frac{\mu\omega R^2}{r^4} \cos^3 \theta (3 \cos^2 \theta + 1)^{-1/2}, \quad (32)$$

where m_p is the mass of the charged particle and e is its charge. The corresponding quantities in curved spacetime can be obtained by using equations (7), (8), and (24) and are given by

$$E^{\text{ext}} \cdot B = \frac{9\mu^2 \omega R^2}{8m^5 cr^2} F(r)G(r) \cos^3 \theta \quad (33)$$

and

$$a = \frac{3e}{2m_p c} \frac{\mu\omega R^2}{m^3 r^2} F(r)G(r) \cos^3 \theta \times \left[\frac{\cos^2 \theta}{m^2} G^2(r) + \frac{\sin^2 \theta}{r^2} \left(1 - \frac{2m}{r}\right) F^2(r) \right]^{-1/2}, \quad (34)$$

where

$$F(r) = \left[\left(1 - \frac{2m}{r}\right)^{-1} + \frac{r}{m} \ln \left(1 - \frac{2m}{r}\right) + 1 \right], \quad (35)$$

$$G(r) = \ln \left(1 - \frac{2m}{r}\right) + \frac{2m}{r} \left(1 + \frac{m}{r}\right). \quad (36)$$

In Figure 3 we present the variation of acceleration of charged particles with distance by considering the Crab pulsar with a rotational period of 33 ms. The acceleration has been measured at an angle $\theta = 0^\circ$ to the axis of rotation. Since gravitation greatly increases the electric field intensity near the surface of the star, the acceleration of charged particles is much higher in curved spacetime than in flat spacetime, near the

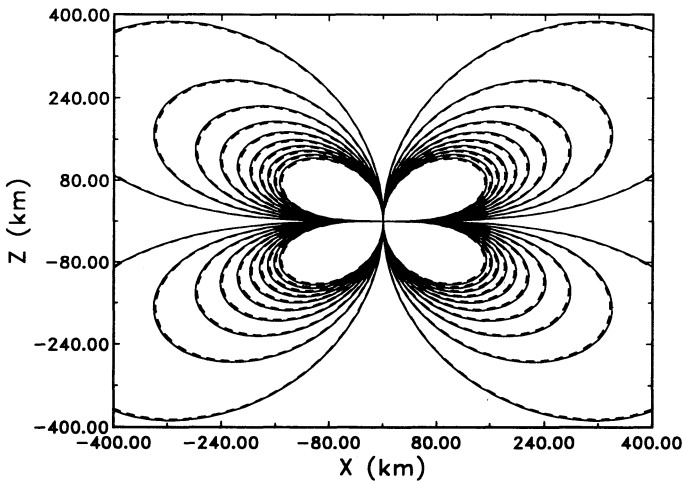


FIG. 2a

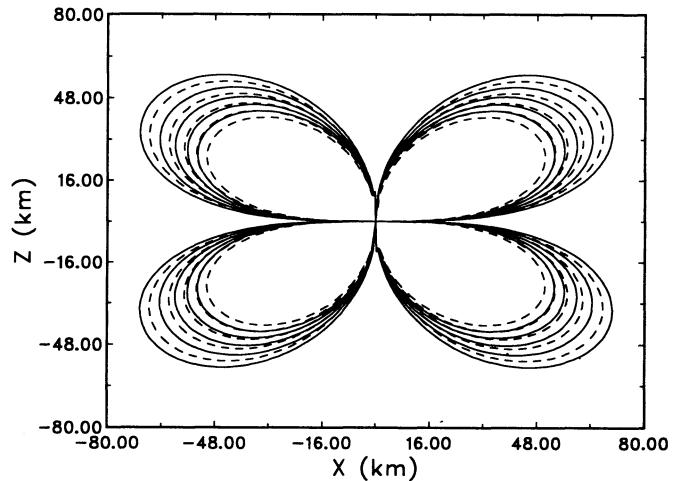


FIG. 2b

FIG. 2.—(a) Quadrupole electric field lines (induced) for the Crab pulsar with $\mu = 2 \times 10^{30} \text{ G cm}^3$. Solid curves represent field lines for curved spacetime, and dashed curves for flat spacetime. The corresponding value assigned to the field line is the same for both curved and flat spacetimes. (b) Closer view of the quadrupole electric field lines shown in (a).

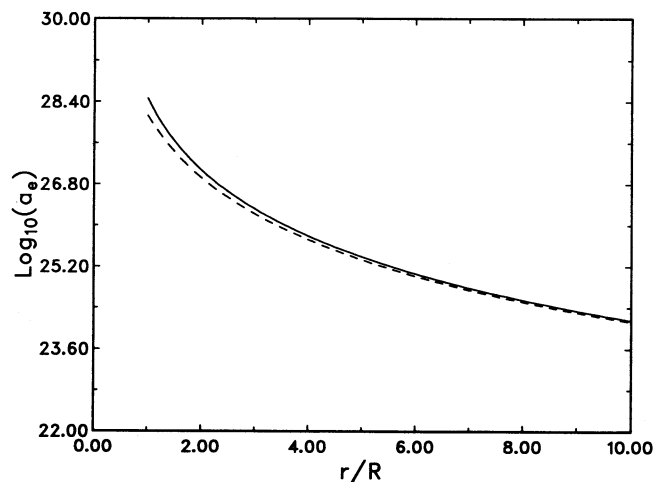


FIG. 3.—Acceleration of an electron in the direction of the magnetic field at an angle $\theta = 0^\circ$ to the axis of rotation for the Crab pulsar, as a function of distance from the stellar surface. The solid curve corresponds to curved spacetime, and the dashed curve to flat spacetime.

surface. This result indicates that the radiation spectrum of pulsars may be influenced significantly by the intense gravitational field of neutron stars and requires further attention.

Since in realistic situations the magnetosphere must be filled up with charged plasma, the various quantities discussed in this section may not represent the actual situation. However, the effect of curvature remains the same whether the region under consideration is vacuum or not, since the gravitational effect due to the magnetospheric plasma is negligible.

4.2. Aligned Nonvacuum Rotator

It is now well accepted that the magnetosphere of neutron stars must be filled up with plasma such that $E \cdot B = 0$ everywhere within the light cylinder. In the Goldreich-Julian model, the source of plasma is assumed to be free field emission from the surface.

In this section we will briefly discuss the effect of spacetime curvature on the induced electric field by considering the aligned nonvacuum rotator model.

In order to investigate the effect of curvature on the induced electric field we consider the Goldreich-Julian solution, which simply corresponds to extending the interior solution to infinity with space charge density continuing smoothly through the surface. We neglect, for simplicity, any effects due to the magnetospheric plasma. However, inclusion of these would not change the general features of the curvature effect.

Therefore, for this case, equations (17) and (18) express the components of the electric field outside the star in Schwarzschild spacetime with r being extended from just inside the surface to infinity (the superscript “int” should now be ignored).

The corresponding expression in flat spacetime can be written as

$$E(\text{flat}) = \frac{\mu\omega}{cr^2} (\sin^2 \theta, -2 \sin \theta \cos \theta, 0). \quad (37)$$

Asymptotically as $r \rightarrow \infty$, $E(\text{curved}) \rightarrow E(\text{flat})$, as it must be the boundary condition.

From equation (10) we obtain the electric field lines in flat spacetime as

$$e(\theta) = \frac{2\pi\mu\omega}{3c} (\cos^3 \theta - 3 \cos \theta + 2) = \text{constant}. \quad (38)$$

From equation (17) we obtain the electric field lines in curved spacetime as

$$e(\theta) = \frac{\pi\mu\omega r^2}{2cm^2} (\cos^3 \theta - 3 \cos \theta + 2) \times \left[\left(1 - \frac{2m}{r}\right)^{-1} + \frac{r}{m} \ln \left(1 - \frac{2m}{r}\right) + 1 \right] = \text{constant}. \quad (39)$$

Clearly, the electric field lines for flat spacetime are independent of r , whereas those in the Schwarzschild background geometry depend on r only near the surface. At a comparatively large distance, the field lines for curved spacetime asymptotically become independent of distance, and hence it is not possible to plot them at a large distance. Also, since the electric field is continuous from the interior to the exterior of the star for the nonvacuum case, the expressions are independent of the radius of the star.

Taking, as earlier, the Crab pulsar with a period of 33 ms we show in Figure 4 the induced electric field lines in curved spacetime in the x - z plane out to 4 stellar radii. Beyond this distance, the field lines almost become independent of r , and it is not possible to present them graphically.

Figure 5 shows the complete picture of the nonvacuum magnetosphere in curved spacetime with both the induced electric field lines and the dipole magnetic field lines obtained by using the expression for the magnetic field lines:

$$b(\theta) = -\frac{3\pi\mu r^2 \sin^2 \theta}{4m^3} \times \left[\ln \left(1 - \frac{2m}{r}\right) + \frac{2m}{r} \left(1 + \frac{m}{r}\right) \right] = \text{constant}. \quad (40)$$

In both the plots we have taken, for convenience, the electric and the magnetic field lines from the interior of the star and

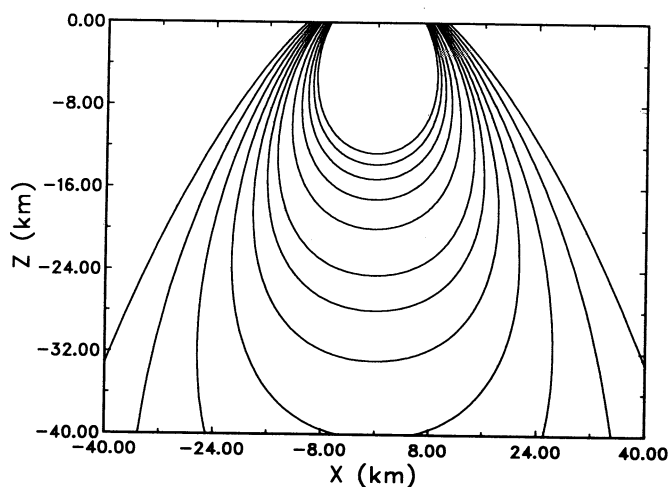


FIG. 4.—Induced electric field lines in curved spacetime for the Crab pulsar with a period of 33 ms and $\mu = 2 \times 10^{30} \text{ G cm}^3$, by considering the magnetosphere to be plasma-filled (see text).

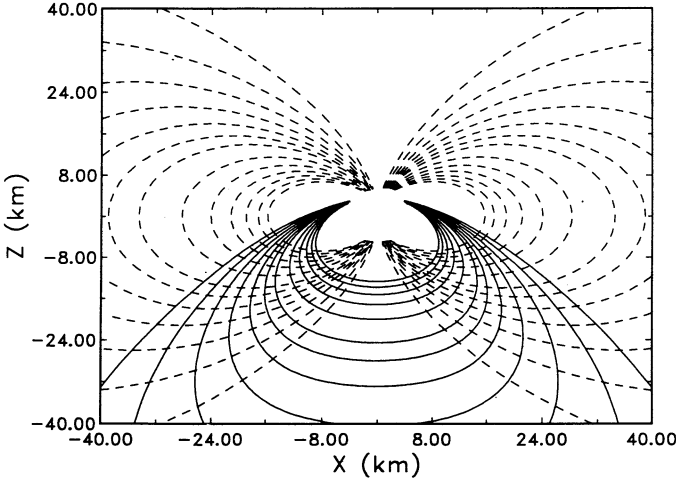


FIG. 5.—Combined dipole magnetic and induced electric field lines in curved spacetime for the Crab pulsar with $\mu = 2 \times 10^{30} \text{ G cm}^3$, by considering a plasma-filled magnetosphere. Solid curves represent the induced electric field lines, and dashed curves the dipole magnetic field lines.

extended them to the exterior. However, far below the surface, the spacetime metric considered here cannot describe the geometry of the region; hence, in the plots they are not shown at a distance far below the surface of the star.

From equations (27) and (28) we notice that the ratio between the electric fluxes in curved and flat spacetimes for the vacuum model is the same as that for the nonvacuum case and is given by

$$\frac{e(\theta)[\text{curved}]_{\text{vac}}}{e(\theta)[\text{flat}]_{\text{vac}}} = \frac{e(\theta)[\text{curved}]_{\text{non-vac}}}{e(\theta)[\text{flat}]_{\text{non-vac}}} = \frac{3r^2 F(r)}{4m^2}, \quad (41)$$

where $F(r)$ is given by equation (35).

Hence, the effect of spacetime curvature on the electric field lines remains the same in this case as well.

In flat spacetime the space charge density is given by

$$\sigma = \frac{1}{4\pi} \text{div } E = -\frac{\mu\omega}{2\pi cr^3} (3 \cos^2 \theta - 1), \quad (42)$$

which is always quadrupolar.

The corresponding expression in curved spacetime can be written as

$$\begin{aligned} \sigma = & \frac{3\mu\omega}{16\pi cm^2} \left\{ \left[\frac{4 \sin^2 \theta}{r} \left(1 - \frac{2m}{r}\right)^{-1} + \frac{3 \sin^2 \theta}{m} \ln \left(1 - \frac{2m}{r}\right) \right. \right. \\ & \left. \left. - \frac{2m \sin^2 \theta}{r^2} \left(1 - \frac{2m}{r}\right)^{-2} + \frac{2 \sin^2 \theta}{r} \right] + \frac{(3 \cos^2 \theta - 1)}{m} \right. \\ & \left. \times \left[\ln \left(1 - \frac{2m}{r}\right) + \frac{2m}{r} \left(1 + \frac{m}{r}\right) \right] \left(1 - \frac{2m}{r}\right)^{-1/2} \right\}. \quad (43) \end{aligned}$$

Figure 6 presents the space charge density as a function of distance from the surface of the Crab pulsar at an angle $\theta = 0^\circ$ to the axis of rotation. In all the figures the various quantities have been measured in cgs units and their absolute values have been taken in logarithmic scale unless mentioned otherwise.

Since gravitation increases the electric field intensity at the surface, more particles get torn off the surface. The electric field

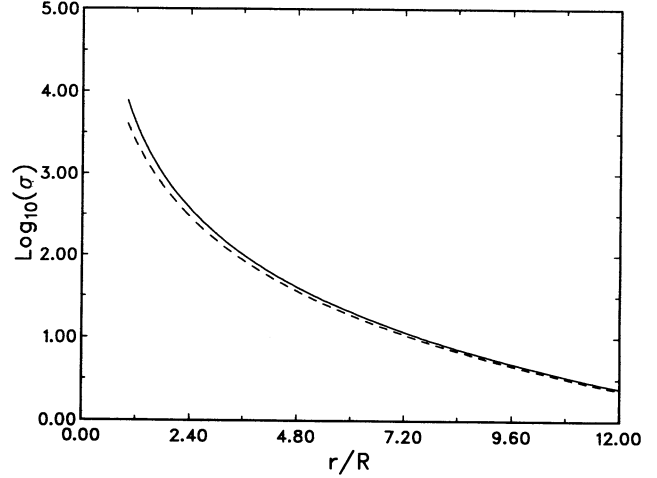


FIG. 6.—Space charge density at an angle $\theta = 0^\circ$ to the axis of rotation for the Crab pulsar, as a function of distance from the stellar surface. The solid curve corresponds to curved spacetime, and the dashed curve to flat spacetime.

required to pull ions from the molecular chains, with the ions distributed in a one-dimensional lattice along the chain and with an outer sheath of electrons, is of the order $10^{12} \text{ V cm}^{-1}$ (Ruderman & Sutherland 1975). Therefore, the effect of gravitation in some pulsars may cause the surface electric field to reach this cutoff value and hence may enable it to pull ions. However, ions may be torn off from the surface due to other effects (Michel 1982). In that case, the inclusion of gravitation would increase the number density of ions, in the same way that it increases the number density of electrons.

5. CONCLUSIONS

We have presented a quantitative discussion of the effects of spacetime curvature on the induced electric field of a pulsar. Considering a stationary and axisymmetric electromagnetic vector potential in a Schwarzschild background geometry, we have derived the components of the induced electric field that arises due to the rotation of the magnetic dipole and have compared them with those in flat spacetime. It is found that spacetime curvature increases the electric field intensity significantly near the surface of the star. As a consequence, in a vacuum magnetosphere the acceleration of charged particles along the direction of the magnetic field increases significantly near the surface. Also, the increase in the electric field intensity near the surface increases the surface charge density significantly. The quadrupole electric field lines have been presented for the Crab pulsar, as an example, in both curved and flat spacetimes and it is found that the difference in the field lines for the curved and the flat spacetimes is easily distinguishable up to 40 stellar radii.

In a plasma-filled magnetosphere, the electric field lines are independent of distance in flat spacetime, but in curved spacetime they depend on r near the surface. However, the field lines in curved spacetime asymptotically become independent of r at large distance. The induced electric field lines in curved spacetime have been presented by taking the Crab pulsar as an example, and the combined dipole magnetic field lines and the induced electric field lines near the surface of the star for curved spacetime have been presented as well.

Due to the increase in the electric field intensity near the surface, the space charge density of a plasma-filled magneto-

sphere increases significantly from that in flat spacetime. All the quantities measured in curved spacetime, however, asymptotically reach their respective values in flat spacetime as a result of the usual boundary condition. Since the ratio of the components of the electric field in curved spacetime to those in flat spacetime is the same, the effects of gravitation on various electromagnetic mechanisms remain the same for all types of magnetospheric models which assume stationary and axisymmetric vector potentials. Hence, the present results are very important in the context of radiation mechanisms in pulsars.

Though we have considered the most idealized models of the pulsar magnetosphere, the effects of spacetime curvature on the induced electric field would remain the same provided there is no dipole or gravitational radiation. The effect of inertial frame

dragging is negligible even for millisecond pulsars, but the departure from spherical symmetry due to rapid rotation of the star could alter the scenario significantly. Nevertheless, the present results would provide reasonably good insight into the effect of spacetime curvature on the induced electric field outside a rotating neutron star with its magnetic axis aligned with the rotational axis.

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