PERIODICITIES IN THE SOLAR DIFFERENTIAL ROTATION, SURFACE MAGNETIC FIELD AND PLANETARY CONFIGURATIONS

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(Received 7 July, 1994; in revised form 4 January, 1995)

Abstract. Using Greenwich data on sunspot groups during 1874–1976, we have studied the temporal variations in the differential rotation parameters A and B by determining their values during moving time intervals of lengths 1–5 yr successively displaced by 1 yr. FFT analysis of the temporal variations of B (or B/A) shows periodicities 18.3 ± 3 yr, 8.5 ± 1 yr, 3.9 ± 0.5 yr, 3.1 ± 0.2 yr, and 2.6 ± 0.2 yr at levels $\geq 2\sigma$. This analysis also shows five more periodicities at levels $1-2\sigma$. The maximum entropy method is used to set narrower limits on the values of these periods. The reality of the existence of all these periodicities of B (or B/A) except the one at 2.8 yr is confirmed by analyzing the simulated time series of B and B/A with values of A and B randomly distributed within the limits of their respective uncertainties. Four of the prominent periods of B agree, within their uncertainties, with the known periods in the the large-scale photospheric magnetic field. The deviations from the average differential rotation are larger near the sunspot minima. On longer time scales, the variations in the amount of sunspot activity per unit time are well correlated to the variations in the amplitudes of the 'torsional oscillation' represented by the 22-yr periodicity in B. All the periods in B found here are in good agreement with the synodic periods of two or more consecutive planets. The possibility of planetary configurations providing perturbations needed for the Sun's MHD torsional oscillations is speculated upon and briefly discussed.

1. Introduction

The origins of sunspot activity and the solar cycle have been studied in great detail both theoretically and observationally (e.g., Rosner and Weiss, 1992, and references therein). So far, no theory has successfully explained all the observed properties of the solar cycle. Among all the theories the turbulent dynamo theory is the most developed one. In this theory, the differential rotation is an important component of the flows that drive the solar dynamo to generate the magnetic field of the Sun for the solar activity. But the exact role of the differential rotation in the cyclic variation of the solar activity is not yet clear (e.g., Levy, 1992; Gilman, 1992). The study of the temporal variations of the Sun's differential rotation is therefore vital for understanding the physical process responsible for the solar activity and the solar cycle.

The Sun's differential rotation is well approximated by the formula

$$\omega(\lambda) = A + B \sin^2 \lambda \, , \tag{1}$$

where $\omega(\lambda)$ is the solar rotation rate at latitude λ, the parameter A represents the equatorial rotation rate and B is a measure of the latitude dependence of the rotation rate.

Solar Physics **158:** 173–195, 1995.

Variations in the Sun's differential rotation with the phase of the sunspot cycle have been investigated by a number of authors. Balthasar and Wöhl (1980) analyzed the Greenwich data on sunspot groups during the years 1940–1968 and found that the equatorial rotation is faster during the sunspot minima than during the maxima. Arévalo *et al.* (1982), using the same data for the years 1874–1902, found a similar variation. Lustig (1983) analyzed sunspot drawings of Kanzelhöhe Observatory for the years 1947–1981 and found a higher equatorial rotation rate and slightly more rigid rotation during the sunspot minima. Gilman and Howard (1984) analyzed Mt. Wilson sunspot data for the years 1921 to 1981 and found strong peaks of residual rotation rate near the sunspot minima and somewhat weaker ones near the sunspot maxima. Balthasar, Vázquez, and Wöhl (1986) analyzed the complete sample of the spot group data compiled from Greenwich Photoheliographic Results (GPR) during the years 1874–1976. They found relatively high values of A around the sunspot minima, secondary maxima of A just before the main sunspot maxima, and the lowest values of A after the sunspot maxima. Kambry and Nishikawa (1990) analyzed sunspot drawings obtained at the National Astronomical Observatory of Japan during the years 1954 to 1976 and found that at the beginning of the cycle, the value of A is high and it decreases subsequently. They did not find systematic variations in B. LaBonte and Howard (1982) analyzed Mt. Wilson full-disk velocity data measured during 1967–1980 and found that the ratio B/A varies with the phase of the solar cycle. They called this a 'torsional oscillation' of period 11-yr and wave number 1 hemisphere^{-1}. (The ratio is used to avoid systematic effects in the Mt. Wilson velocity data.) The torsional oscillation found by Howard and LaBonte (1980) appears as alternate bands of latitudes rotating faster (or slower) than the average rotation which move from polar regions to the equator in 22 yr. Similar properties were noticed by Tuominen, Tuominen and Kyröläinen (1983) from the the latitudinal and longitudinal motions of recurrent sunspot groups from GPR during the years 1874 to 1976. (Ward (1965) from the study of the rotation of spot groups in the Greenwich data during 1935–1944, suggested the possible existence of a wave phenomenon in the solar atmosphere which resembles a torsional oscillation.) Snodgrass (1991) analyzed Mt. Wilson magnetograms and found magnetic patterns similar to the torsional oscillations. Recently Komm, Howard and Harvey (1993) analyzed high-resolution magnetograms taken with the Vacuum Telescope on Kitt Peak from 1975–1991. They found, in the rotation rates of both small and large scale magnetic fields, the same general cycle dependence and similar torsional oscillation patterns as those obtained from Doppler measurements.

Attempts were also made in the past to investigate the cycle-to-cycle variations of differential rotation. From comparison between cycles 12 and 19 Clark *et al.* (1979) found that A was independent of sunspot activity. The same conclusion was arrived at by Hanslmeier and Lustig (1986) from the data during cycles 18 to 21. Tuominen and Kyröläinen (1981) found in the variation of A, a significant dip from cycle 13 to cycle 14. Balthasar, Vázquez, and Wöhl (1986) found that the only significant variation in A was the same dip in A and that there was no significant variation of A from cycles 14 to 20. So they concluded that there was no hint of the 22-yr cycle in the rotation velocity. Kambry and Nishikawa (1990) found that the smaller the amplitude of the solar cycle, the larger is the value of A. Recently Yoshimura and Kambry (1993a) found a monotonic increase in the value of A from cycle 17 to cycle 20. They also found a good correlation between the modulation of the sunspot cycle during 1948–1987 and the mean angular momentum density at the surface (Yoshimura and Kambry, 1993b). Gilman and Howard (1984) noticed cycle-to-cycle differences in the yearly values of the residual rotation rates of sunspots. Lustig (1983) and Balthasar, Vázquez, and Wöhl (1986) found that there were no systematic variations in the yearly values either of A or of B. Kambry and Nishikawa (1990) found approximately systematic variations in the yearly values of A, but not in B. Savernò *et al.* (1991) analyzed Greenwich data on spot groups of age $\leq$ 3 days during the years 1874–1976 and obtained 16.1-yr and 6.2-yr periodicities in A.

The 11-yr sunspot cycle is a manifestation of the 22-yr solar magnetic cycle (e.g., Stenflo and Vogel, 1986; Gokhale *et al.*, 1992). The 22-yr magnetic cycle may itself be a torsional magnetohydrodynamic (MHD) oscillation as first considered by Walén (1944) as a mechanism for the solar cycle. However, a 22-yr periodicity has not been detected so far in the differential rotation rate (see previous two paragraphs). Hence, in this paper we determine the significant periodicities in B (or in B/A).

In order to see if these periodicities are those of torsional MHD oscillations, we compare the periodicities in B (or B/A) with the known periodicities in the photospheric magnetic field. To see if the torsional MHD oscillations are energetically related to the sunspot activity, we determine the correlation between the long-term variations in the sunspot activity and in the time-dependent part of the coefficient B.

The maintenance of torsional MHD oscillations will need internal or external perturbations (either continuous or episodic), with appropriate periodicities. So far, no source of such perturbations has been identified (Rosner and Weiss, 1992). In this context we recall the works of Jose (1965), Wood and Wood (1965), Blizard (1981), Fairbridge and Shirley (1987), relating the solar activity to the changes in the Sun's angular momentum about the center of mass of the solar system (the barycenter). It seems that the changes in the solar rotation caused by the inertial torques due to the Sun's motion about the barycenter cannot be ignored. This motion of the Sun is controlled by the changes in the planetary configurations. Hence we compare the periodicities of B/A determined in the present analysis with the synodic periods of sets of two or more planets.

In Section 2, we describe the data and determination of A and B. In Section 3.1, we give the values of A and B during the whole period 1874–1976 and compare them with the corresponding values in Balthasar, Vázquez, and Wöhl (1986). In Section 3.2 we show the computed temporal variations in both A and B. In

TABLE I

The values of the parameters A and B (in deg day^{-1}) and their uncertainties ΔA and ΔB derived from the data during the entire period 1874–1976

	A	ΔA	B	ΔB	N
Northern hemisphere	14.524	±0.006	−2.757	±0.063	42482
Southern hemisphere	14.544	±0.007	−2.797	±0.067	39926
Whole sphere	14.533	±0.005	−2.772	±0.046	82448

Section 4.1 we show the periodicities in the ratio B/A obtained from the FFT analysis, and in Section 4.2 we confirm the existence of these periodicities by analyzing the simulated series of B/A. In Section 4.3 we determine the values of the periods of B/A by the maximum entropy method (MEM). In Section 4.4 we verify that the the periodicities found in B/A are not an artifact of the variation in the relative amounts of data coming from the young and the old spot groups. In Section 4.5 we confirm the periodicities in B/A from the analysis of the effect of variation in the latitudinal distribution of sunspot activity. In Section 5 we compare these periodicities of B/A with those in the solar magnetic cycle. In Section 6.1 we show the variations of the absolute value of the time-dependent part of B during individual sunspot cycles and in Section 6.2 we show the long-term correlation between the amount of sunspot activity and the absolute value of the time dependent part of B. In Section 7 we show the periods of B/A agree with the synodic periods of sets of planets in which the planets are consecutive. In Section 8 we summarize the conclusions and give a combined interpretation of the results.

2. Data and Determination of A and B

A magnetic tape of the data on sunspot groups compiled from Greenwich photoheliographic results for the years 1874–1976 was kindly provided by H. Balthasar. The data consists of the observation time (the date and the fraction of the day), heliographic latitude (λ) and longitude (L) for each spot group on each day of its observation. For period 1940–1976 the data for groups beyond a central meridian longitude exceeding 58° are not available. Hence in the present analysis such groups are omitted also from the data during the period 1874–1939. The sidereal rotation velocities (ω) have been computed for each pair of consecutive days in the life of each spot group. Using the method of least-square fit we computed the values of the parameters A and B in Equation (1) during intervals of a specified length successively shifted by 1 yr with different choices (1 to 5 yr) for the length of the intervals. All the least-square fits were computed by excluding the data corresponding to the 'abnormal' motions, viz., displacement exceeding 3° day^{-1}

in longitude or 2° day^{-1} in latitude. This reduces the data sample by about 3% but guards against recording errors and against making errors in identifying small spot groups from one day to the next (Ward, 1965, 1966). This helps to reduce the uncertainties in the values of A and B considerably.

3. Analysis

3.1. VALUES OF A AND B AND THEIR UNCERTAINTIES: COMPARISON WITH EARLIER RESULTS

In Table I we give the values of the parameters A, B and their respective uncertainties ΔA, ΔB derived from the data in the northern and the southern hemispheres separately and in combination, during the whole period 1874–1976. In the same table we also give the number of data points (N) from the respective hemispheres and from the whole Sun. Comparing with the corresponding values of N, A, and B derived by Balthasar, Vázquez, and Wöhl (1986), we find the following:

(1) Our number of data points N, is less by about 3%.

(2) Our values of A are slightly smaller but the uncertainties ΔA are smaller by about 17%.

(3) Our absolute values of B are smaller by about 3% but the uncertainties ΔB are smaller by about 23%.

(4) The north–south asymmetries of A and of the amount of activity N remain the same. The amount of activity is about 6% higher in the northern hemisphere. The value of A is little higher in the southern hemisphere than that expected from the anticorrelation between A and N (Sakurai, 1976; Kambry and Nishikawa, 1990).

(5) The absolute value of B is slightly larger in the southern hemisphere than in the northern hemisphere. The last result agrees with the result obtained by Hathaway and Wilson (1990) from the data for individual sunspots. However in our analysis the north–south difference in B is smaller than the uncertainty in B.

3.2. TEMPORAL VARIATIONS OF A AND B

Figures 1(a) and 1(b) show respectively the variations in the annual values of A and B. In these figures we find that many of the variations in A and B are larger than their respective uncertainties. In B this is particularly true in the latter half of the diagram. These properties could also be seen in the corresponding diagrams in Balthasar, Vázquez, and Wöhl (1986). However in the present analysis the uncertainties are smaller. Our reduction of uncertainties is more effective during the years near the sunspot minima, where it is important for determining the long periodicities in B or B/A. Figures 2(a) and 2(b) show respectively the variations in the values of the parameters A and B during 5 yr long intervals successively

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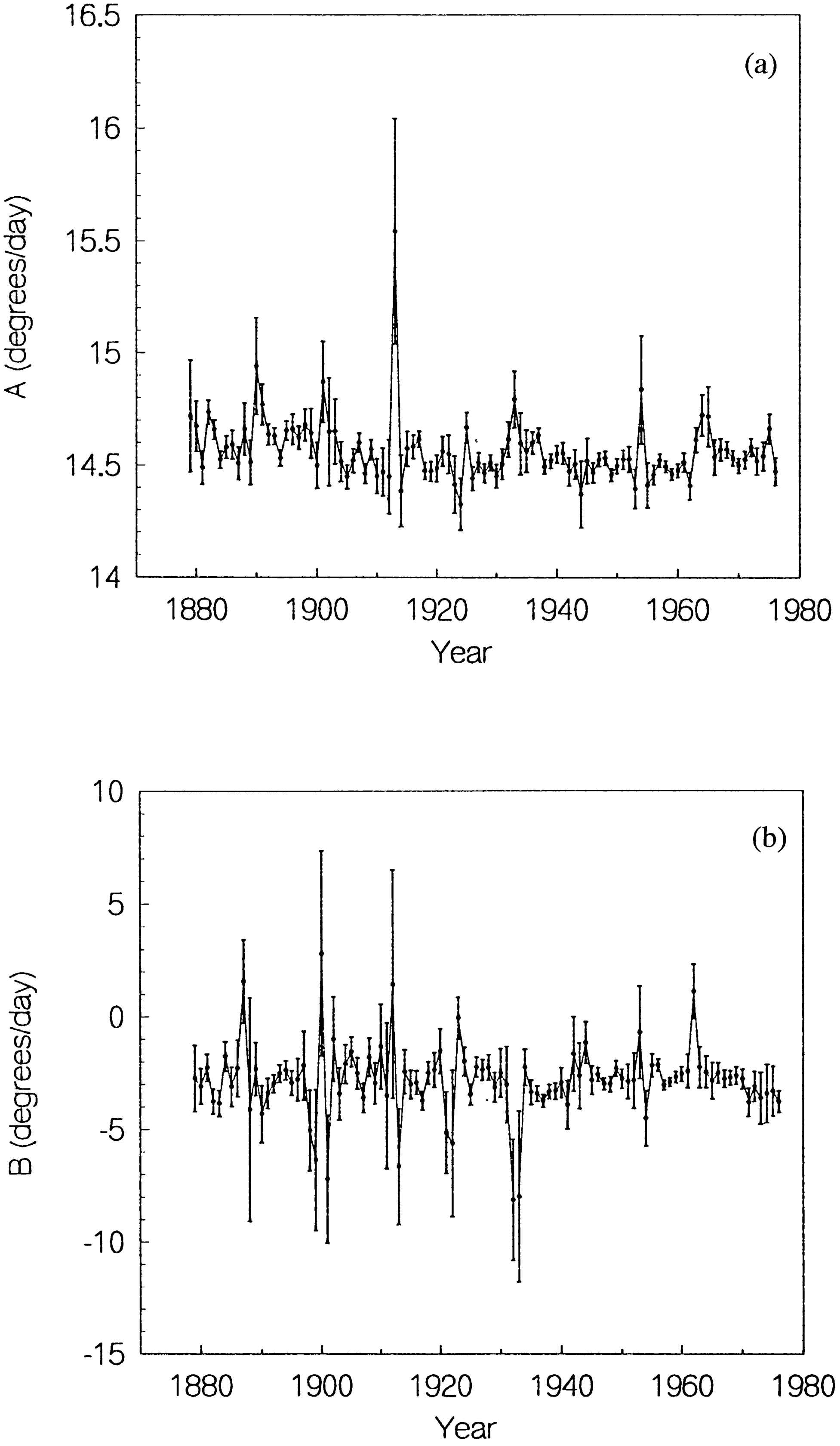

Fig. 1a–b. Variation of the coefficients A and B in Equation (1) represented by their yearly values from 1879 to 1976.

displaced by 1 yr. In these figures it can be seen that the uncertainty bars in A and B are sufficiently small for us to determine the variations of A and B on time scales > 5 yr.

Figure 3 shows the variation in the ratio B/A derived from time series with intervals of length 1 yr, 2 yr, and 3 yr. We have used the ratio B/A because it would reduce the overall effect of systematic errors in the variations of A and B due to the absence of data in latitudes beyond $\sim 35°$ and also due to the dependence of solar rotation deduced from sunspot data on the type of the spot groups included (Howard and LaBonte, 1986). As a result of variation in the amount of data, the errors in both A and B will be large during the sunspot minima and small during the sunspot maxima. Hence the relative errors in B/A will be smaller than that in B alone. In Figures 1(b), 2(b), and 3 it can be seen that the variations in B and B/A are almost the same. This is because in the present analysis we have used the sidereal values of ω and hence the variations in A are small. So whatever we discuss hereafter holds for B as well as for B/A. (*Note*: in the determination of periodicity in B/A or B, the rotation rates of the years 1874–1878 have not been included because of a large uncertainty in B during 1878.)

4. Periodicities in B or B/A

4.1. PERIODICITIES IN B OR B/A FROM THE FFT ANALYSIS

The continuous curves in the Figures 4(a) and 4(b) represent the FFT power spectra of the temporal variations in the ratio B/A, obtained from sequences of non-overlapping intervals of length 1 yr and 2 yr, (respectively, during 1879–1976. In Figures 4(c) and 4(d) are the power spectra obtained from sequences of 3-yr and 5-yr intervals each successively displaced by 1 yr. In all these figures the units of the ordinates are left arbitrary, since we do not compare the power in any one figure with that in any other. While computing the FFT on any sequence of values, the mean value is subtracted from each value and then the size of the sequence is extended to the next power of 2 by taking zero values. This removes the power at zero frequency and reduces the leakage between the repeated data segments (Brault and White, 1971).

(We also repeated the FFT calculations by subtracting a first order polynomial and masking the first and the last 10% of the data by a cosine bell function. No changes in the positions of the peaks were found for the frequency resolution permitted by the data length.)

It should be noted here that as the interval length increases, the uncertainties in the parameters B and B/A reduce. However in the power spectrum derived from larger interval lengths, the peaks corresponding to the shorter periods are washed out and the remaining peaks get broader. In the yearly values of A and B, the uncertainties are large, and hence in the spectrum of B/A or B the noise level is

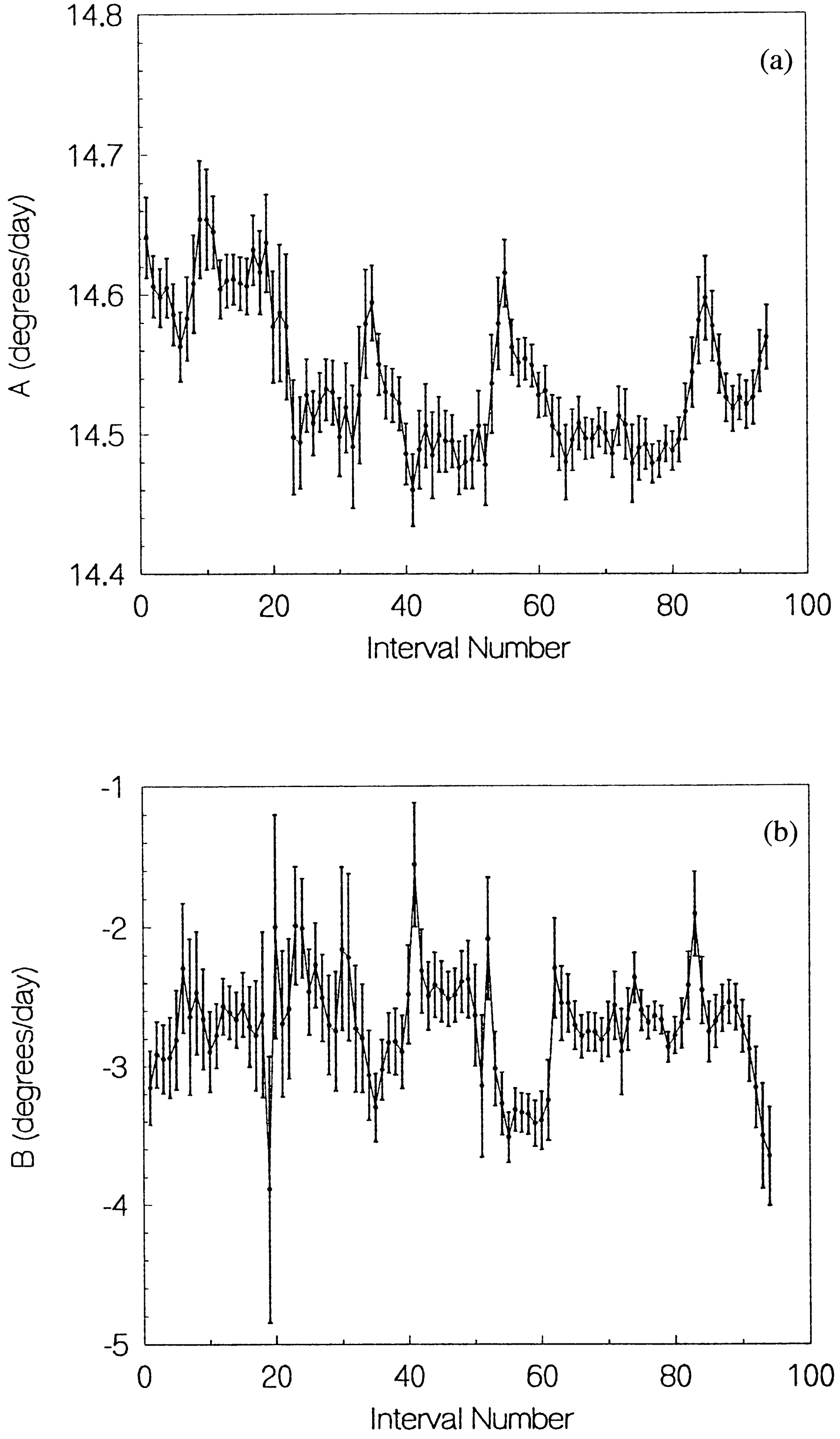

Fig. 2a–b. Variation of A and B obtained from the sequence of 5-yr intervals successively displaced by 1 yr.

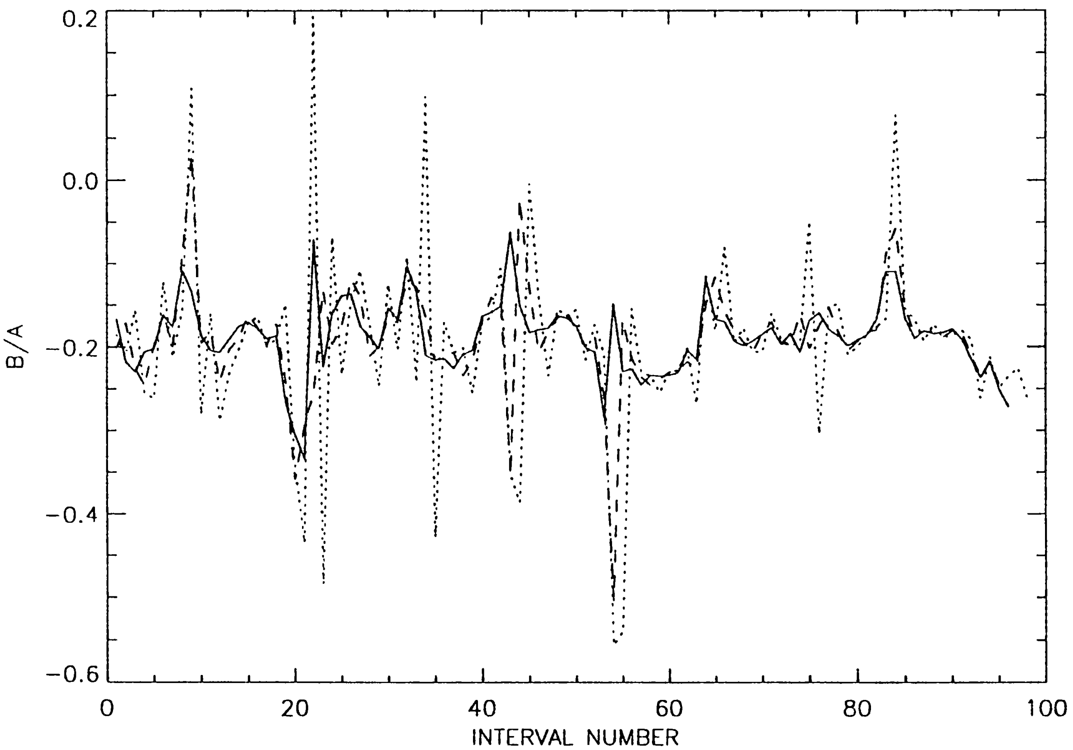

Fig. 3. Variation of the ratio B/A represented by its yearly values (dotted curve) and as obtained from sequences of 2-yr and 3-yr intervals successively displaced by 1 yr (broken and continuous curves respectively).

slightly high, but the peaks are well separated and their levels of significance are also high. (We have also verified the periodicities in B/A for the true frequency resolution allowed by the data by doing the Fourier analysis without extending the number of data points by zero values. Except for a slightly larger broadening in the peaks, the power spectra obtained by this method are found to be the same as the FFT power spectra).

In Table II, we give the results obtained from the power spectra of the ratio B/A (continuous curves in Figures 4(a–d). In each spectrum the level of significance of each periodicity peak is determined in the units of the standard deviation of the values of the power over the whole frequency range. From this table, we conclude that there exist 18.3 ± 3, 8.5 ± 1, 3.9 ± 0.5, 3.1 ± 0.2, and 2.6 ± 0.2 yr periodicities in the ratio B/A with confidence levels $\geq 95\%$. There is also a possibility of the existence of periodicities at 42 ± 10, 3.7 ± 0.05, 2.8 ± 0.1, 2.3 ± 0.1, and 2.06 ± 0.05 yr with confidence levels between 67% and 95%. There are weak signals at 6.0 ± 0.2 and 4.8 ± 0.2 yr. The spectra of B/A do not show any periodicity near 11 yr.

4.2. CONFIRMATION FROM SIMULATED TIME SERIES OF B/A

We have generated simulated time series, A' and B' respectively for A and B, where in each interval the value of A' is a random number between $A - \Delta A$ and

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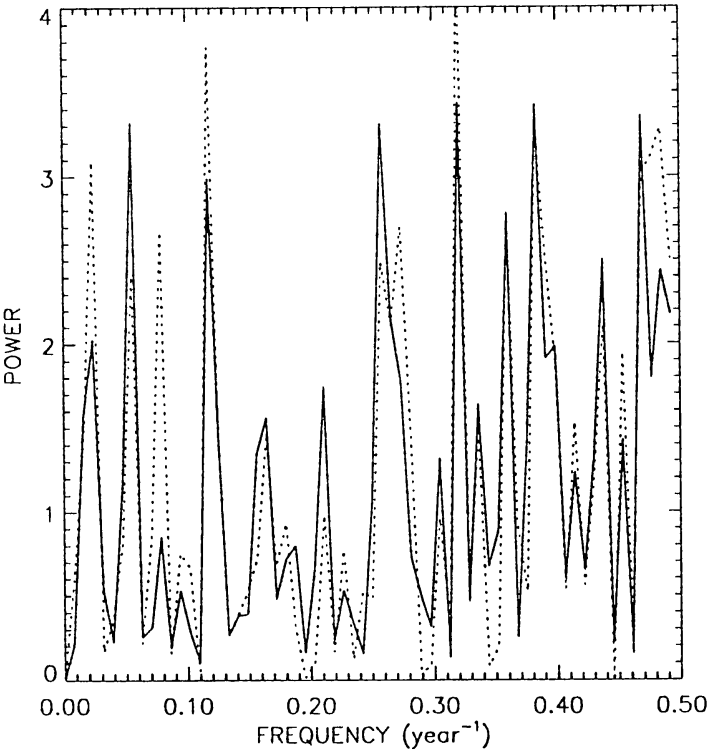

Fig. 4a.

Fig. 4a–d. FFT power spectra of B/A and of B'/A' (simulated variation) represented by continuous and dotted curves, respectively. These are obtained from (a) yearly values, (b) values during successive 2-yr intervals, (c) values during 3-yr intervals successively displaced by 1 yr, and (d) values during 5-yr intervals successively displaced by 1 yr. In all these figures the units of the ordinates are arbitrary.

$A + \Delta A$, and B' is a random number between $B - \Delta B$ and $B + \Delta B$. The dotted curves in Figures 4(a–d) represent the FFT spectra of the simulated time series of the ratio B'/A'. It can be seen that the peaks in these spectra coincide with the peaks in the corresponding spectra obtained from the real time series of the ratio B/A. The levels of significance are also as high as those given by real time series. This indicates that the periodicities in Table II are not artifacts of errors in the values of A and B.

(The '12-yr' peak in the simulation B' or B'/A' of the annual series must be an artifact of the 11-yr periodic variation of the uncertainty ΔB which is inversely proportional to the square root of the amount of sunspot activity. This is supported by the fact that this periodicity is not present in the spectra of simulated values obtained from intervals of lengths $>$ 2 yr. The origin of this peak in the variation of ΔB is also obvious from the fact that it is dominant in the spectra obtained from the extreme yearly values $B - \Delta B$ or $B + \Delta B$.)

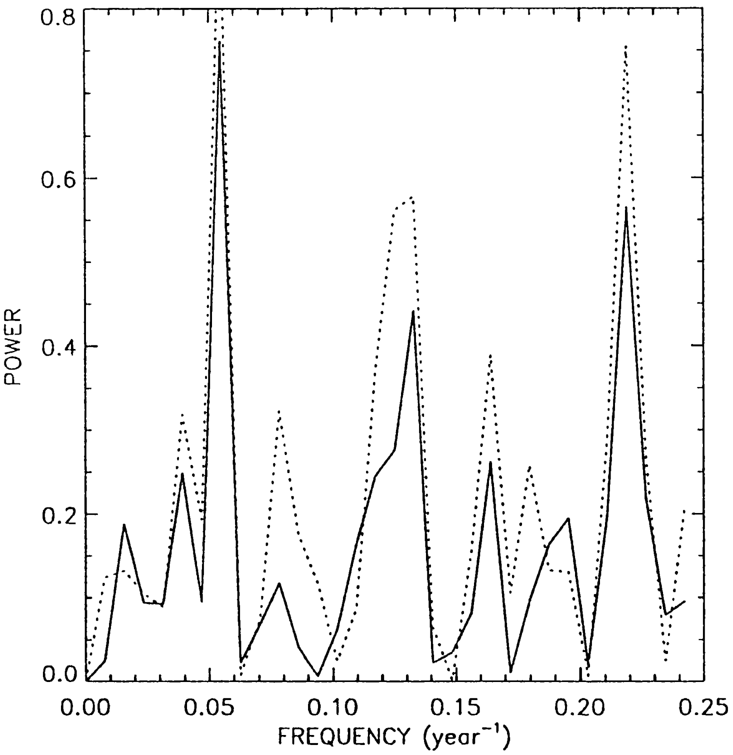
0.8
0.6
0.4
0.2
0.0
POWER
0.00
0.05
0.10
0.15
0.20
0.25
FREQUENCY (year⁻¹)

Fig. 4b.

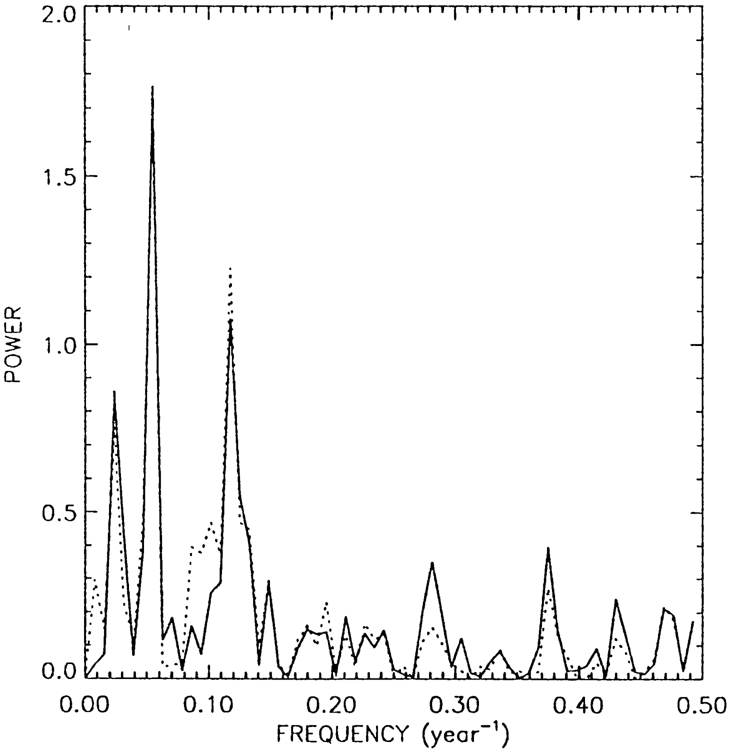
2.0
1.5
1.0
0.5
0.0
POWER
0.00
0.10
0.20
0.30
0.40
0.50
FREQUENCY (year⁻¹)

Fig. 4c.

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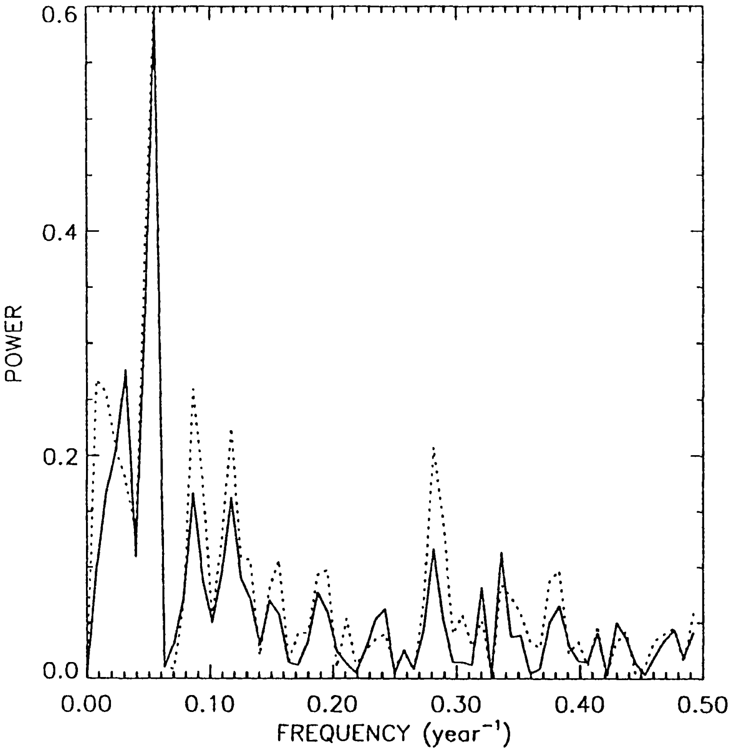

Fig. 4d.

4.3. DETERMINATION OF THE PERIODS IN B/A OR B FROM MAXIMUM ENTROPY ANALYSIS

To determine the values of the periodicities in B/A with better accuracy, we have computed the spectra of B/A by maximum entropy analysis (MEM) for autoregressive (AR) process order $m = n/2$ and $m = n/3$, where n is the number of intervals in the analyzed series (Ulrych and Bhishop, 1975). In Figures 5(a–c) we show the MEM power spectra of B/A for $m = n/2$ obtained from sequences of non-overlapping successive intervals of lengths 1 yr and 2 yr respectively, and from the sequence of 2-yr intervals successively shifted by 1 yr. The spectra of B/A for $m = n/3$ are similar and hence are not presented here. The MEM power spectra show the existence of periodicities: 18.9 ± 1.1 yr, 8.3 ± 0.8 yr, 6.1 ± 0.2 yr, 4.8 ± 0.2 yr, 3.9 ± 0.1 yr, 3.7 ± 0.05 yr, 3.1 ± 0.1 yr, 2.8 ± 0.05 yr, 2.6 ± 0.1 yr, 2.3 ± 0.1 yr, and 2.06 ± 0.05 yr in B/A (or B).

Figure 5(d) shows MEM power spectra of B'/A', viz., simulated annual time series B/A. This figure confirms all the periodicities determined above except the one at 2.8 yr. The MEM spectra of B'/A' obtained from longer intervals (not shown here) also reproduce the peaks given by the real series of B/A.

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TABLE II

Periods of B/A (or B) with their uncertainties derived from FFT analysis of temporal variations represented by values during different sequences of time intervals. Below the value of each period the level of significance, in units of standard deviation (σ), is given in brackets.

The sequence of	Periods (in years) and levels of significance				
1 yr successive intervals	43 ± 10	18.3 ± 2.0	8.5 ± 1.0		
(yearly values)	(1.0)	(2.2)	(1.9)		
	3.9 ± 0.15	3.7 ± 0.05	3.1 ± 0.1		
	(2.2)	(1.0)	(2.3)		
	2.8 ± 0.05	2.6 ± 0.1	2.3 ± 0.05		
	(1.6)	(2.3)	(1.4)		
	2.06 ± 0.05				
	(1.3)				
2 yr successive intervals	18.3 ± 3.0	7.5 ± 1.0	4.6 ± 1.0		
	(3.6)	(1.7)	(2.5)		
2 yr intervals successively	43 ± 10	18.3 ± 2.0	8.5 ± 1.0		
shifted by 1 yr	(1.9)	(5.6)	(3.3)		
3 yr intervals successively	~ 43	~ 32	18.3 ± 2.0	8.5 ± 1.0	
shifted by 1 yr	(2.5)	(1.0)	(5.7)	(3.3)	
4 yr intervals successively	~ 43	~ 32	~ 21	18.3 ± 2.0	8.5 ± 1.0
shifted by 1 yr	(1.8)	(1.3)	(1.5)	(6.2)	(2.3)
5 yr intervals successively	~ 64	~ 43	~ 32	~ 21	18.3 ± 2.0
shifted by 1 yr	(1.1)	(1.5)	(2.3)	(2.9)	(5.8)
	~ 12	8.5 ± 1.0			
	(1.1)	(1.0)			

4.4. CHECKING OF PERIODICITES IN B/A FROM THE ANALYSIS OF ROTATION RATES OF THE YOUNG AND THE OLD SUNSPOT GROUPS

Balthasar, Vázquez, and Wöhl (1986) pointed out that the variation of the rotation rate during the 11-yr sunspot cycle comes mainly from the variation in the rotation rates of young sunspot groups (and not from the variations in the percentage of the young groups in the whole sample). To verify whether the results given in Table II are true for both young and old spot groups we have analyzed separately the data corresponding to the spot groups of age ≤ 3 days and those of age > 3 days. For each of these data sets, the coefficients A and B are determined during the intervals of length 3 yr successively displaced by 1 yr. The following results are obtained.

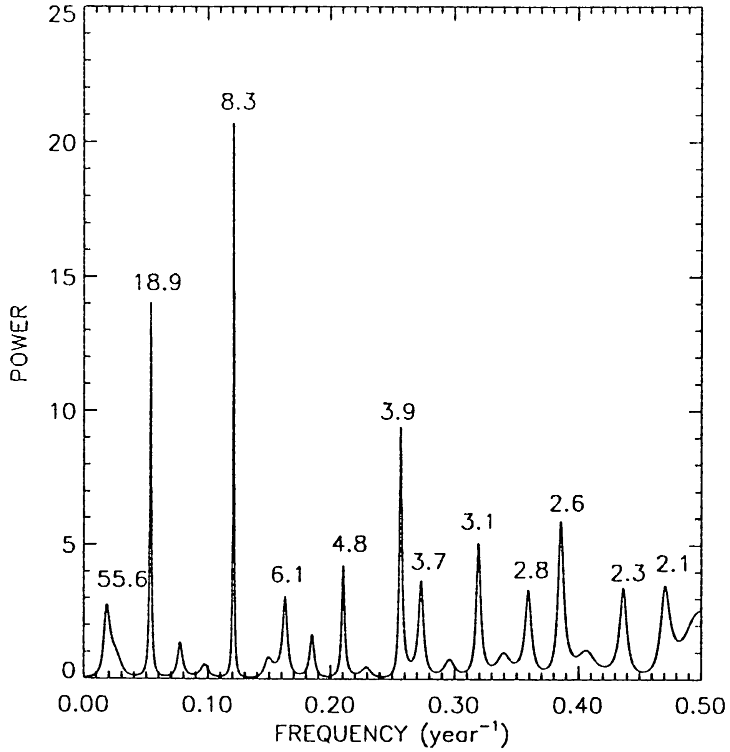

Fig. 5a.

Fig. 5a–d. MEM power spectra of B/A (for AR process order $m = n/2$, where n is the length of the time series). These are obtained from (a) yearly values, (b) values during successive 2-yr intervals, (c) values during 2-yr intervals successively displaced by 1 yr and (d) MEM power spectrum of B'/A', i.e., simulated yearly variation of B/A. In all these figures the units of the ordinates are arbitrary.

(i) The average power in the FFT spectrum of B/A is higher in the case of the young spot groups than in case of the old spot groups.

(ii) The FFT spectrum obtained from the young groups shows peaks at $\sim$ 43 yr, 18.3 ± 3 yr, $\sim$ 14.3 yr, and 8.5 ± 1 yr, at levels of 3.7σ, 3.4σ, 2.3σ, and 1.0σ, respectively.

(iii) The FFT spectrum obtained from old groups shows peaks at 18.3 ± 3 yr and 8.5 ± 1 yr at levels 3.7σ, and 4.0σ, respectively. (The peak at $\sim$ 43 yr is found at $< 1\sigma$ level.)

These results suggest that the periodicities in Table II are present in the real variation of the Sun's differential rotation and are not coming from the variation in the relative amount of data from the young groups. [The result (i), the dominance of the $\sim$ 43 yr period obtained from the young groups and that of the shorter periods obtained from the old groups may be due to the young groups being anchored to deeper layers than the old groups (Schüssler, 1987).]

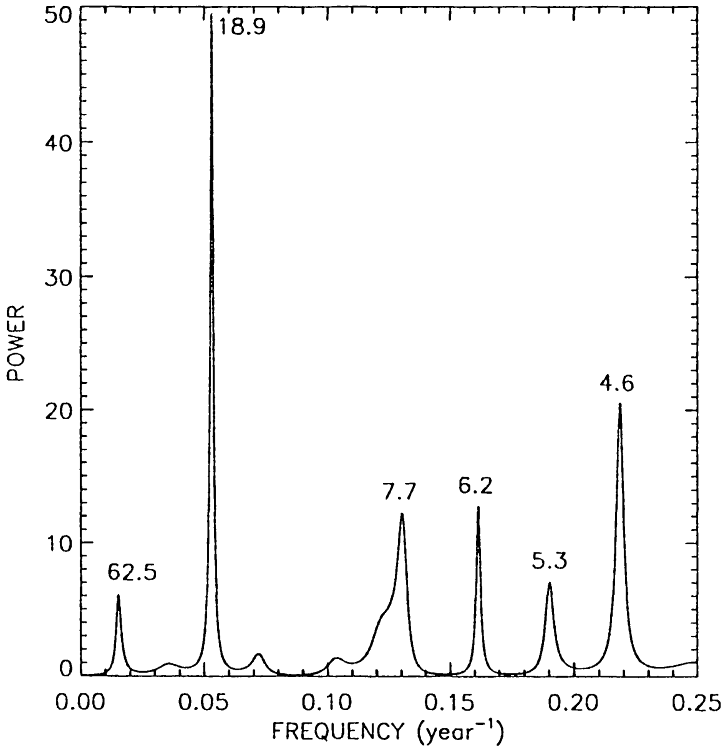
POWER
50
40
30
20
10
0
0.00
0.05
0.10
0.15
0.20
0.25
FREQUENCY (year^{-1})
18.9
62.5
7.7
6.2
5.3
4.6

Fig. 5b.

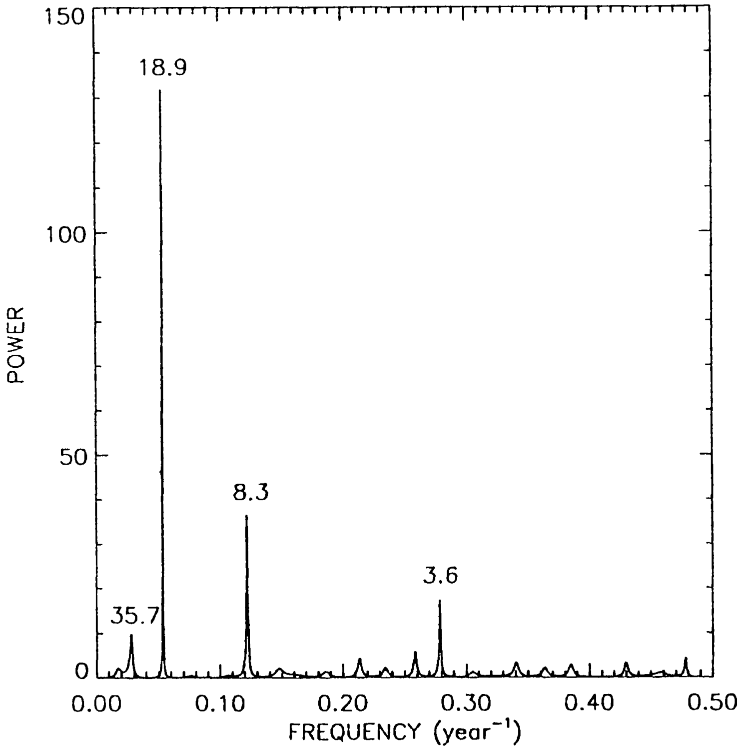
POWER
150
100
50
0
0.00
0.10
0.20
0.30
0.40
0.50
FREQUENCY (year^{-1})
18.9
8.3
3.6
35.7

Fig. 5c.

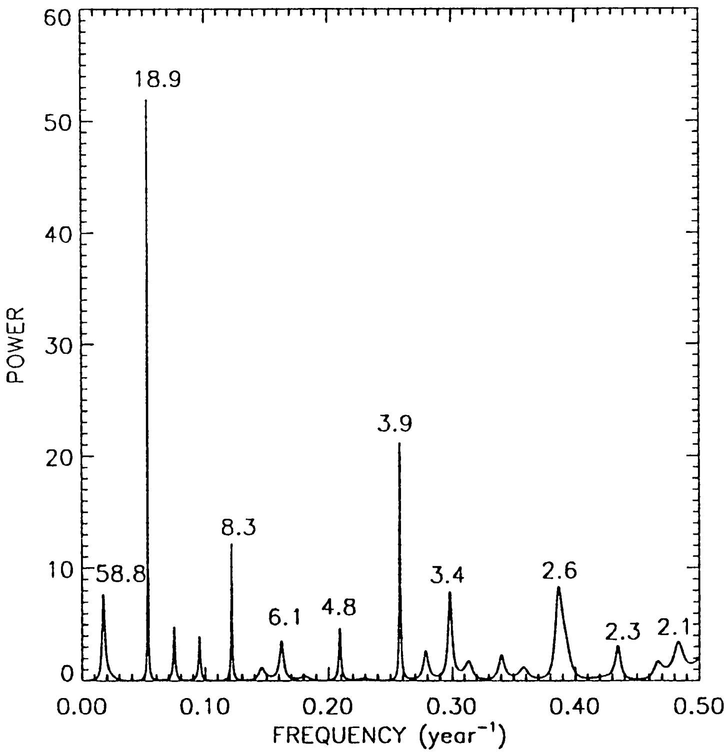

Fig. 5d.

4.5. CHECKING OF PERIODICITIES IN B/A FROM THE ANALYSIS OF THE EFFECT OF VARIATION IN THE LATITUDINAL DISTRIBUTION OF SUNSPOT ACTIVITY

Near the latitude of 35°, sunspots appear only in the beginning of the solar cycle and the mean latitude of all sunspots is $\sim 15°$. Hence, as in Balthasar, Vázquez, and Wöhl (1986), we have fitted the rotation values to the form

$$\omega(\lambda) = G + H(\sin^2 \lambda - \sin^2 15°) .$$

Obviously $H = B$ and $G = A + H \sin^2 15°$. As in their work, we find that the variation and the uncertainties in G are slightly smaller than those in A. Importantly, we find that the periodicities in H/G are same as those in B/A. Thus the periodicities in B/A are not artifacts of the variation in the latitudinal distribution of the spot groups.

5. Comparison with Periodicities in the Photospheric Magnetic Field

The '18.9-yr' period of B/A determined here approximately matches with the '22-yr' band in the surface magnetic field determined by Stenflo and Vogel (1986) from the analysis of magnetogram data during 1960–1985, and by Gokhale *et al.* (1992) from the analysis of distribution of magnetic field inferred from sunspot data

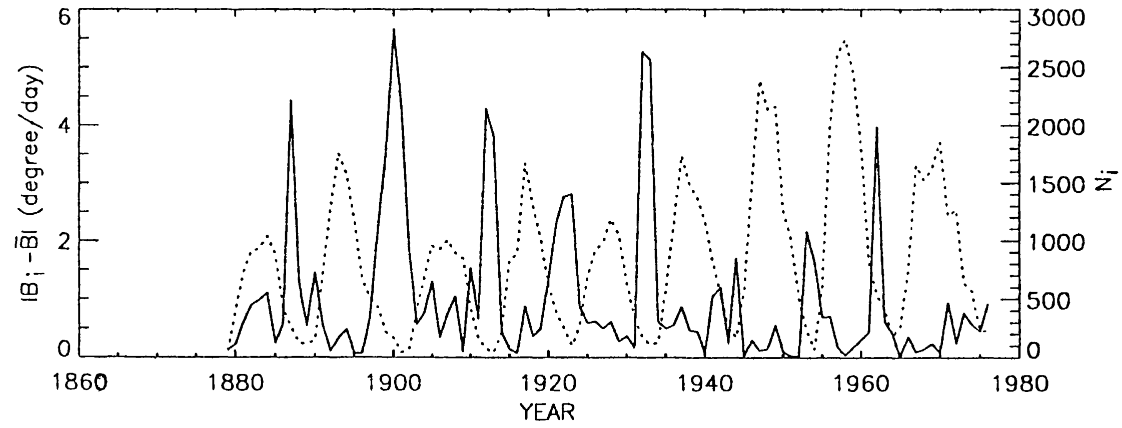

Fig. 6a.

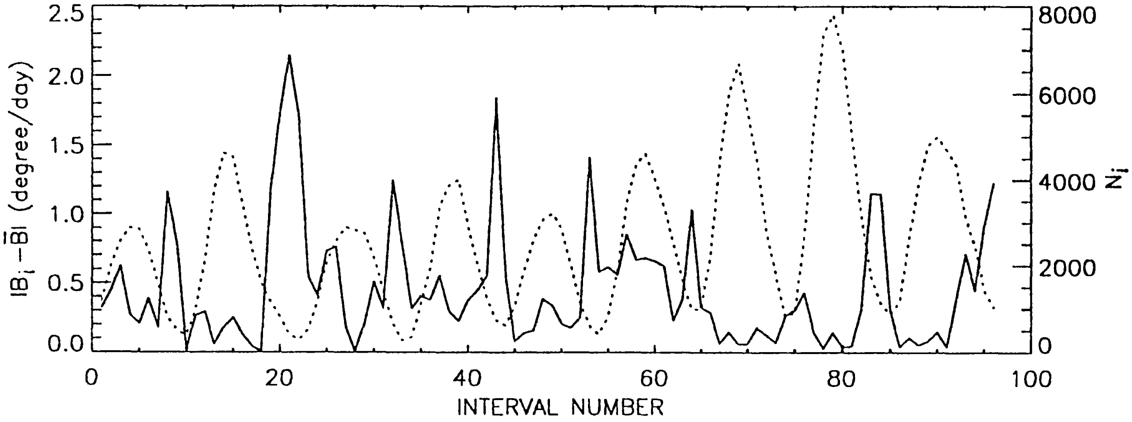

Fig. 6b.

Fig. 6a–b. Variation of $|B_i - \overline{B}|$, where i is the interval number in a sequence and B is the average of all B_i's (continuous curves) and of the amount of sunspot activity per unit time, represented by the value of N_i, during the interval i (dotted curves), as obtained from sequences of (a) yearly values and (b) values during 3-yr intervals successively displaced by 1 yr.

1874–1976. The'8.3'-yr period of B/A approximately matches with the $\sim$ 22/3-yr period, viz., the 'third harmonic of the 22-yr band', determined by Bracewell (1988) from the analysis of annual mean sunspot numbers with signs attached, and by Gokhale and Javaraiah (1990, 1995) in the magnetic field inferred from the sunspot data. The 3.9 ± 0.4 and 2.6 ± 0.2-yr periods match with the 4.19 and 2.55-yr periods found in the photospheric magnetic field by Csada (1974) using the magnetograms (1959–1967). Since the differential rotation is essentially a torsion, the agreement between the periods in the magnetic field and in B/A strongly suggest that the'solar magnetic cycle' essentially consists of 'torsional MHD oscillations' as suggested by LaBonte and Howard (1982), (though the torsional oscillations which they found have 11-yr periodicity), and argued by us (Gokhale and Javaraiah, 1995).

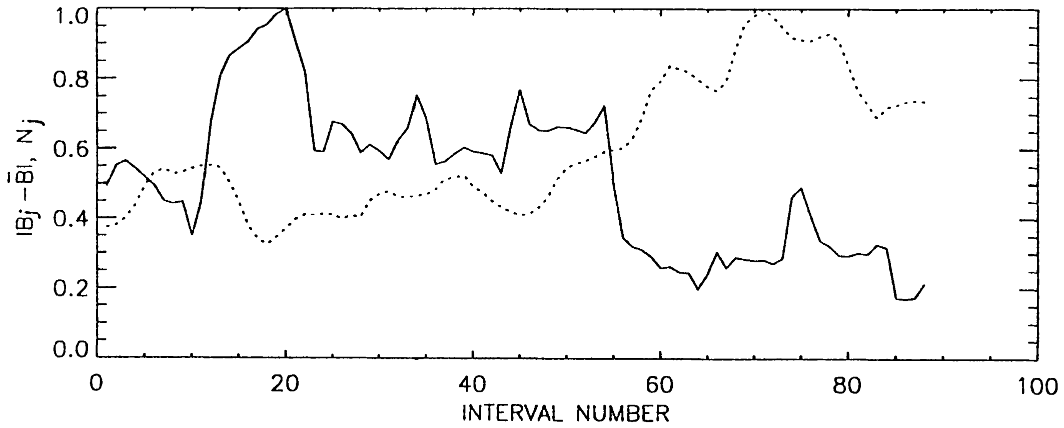

Fig. 7a.

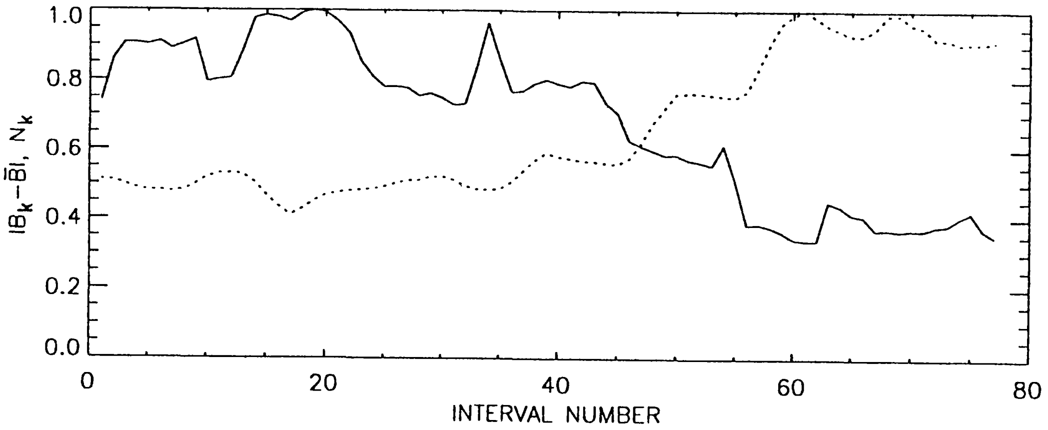

Fig. 7b.

Fig. 7a–b. Similar to Figure 6, but obtained from (a) 11-yr running averages of the yearly values of $|B - \overline{B}|$ and N, and (b) 22-yr running averages of the yearly values of $|B - \overline{B}|$ and N.

6. Relation between the Variations in the Magnitude of B and in the Amount of Sunspot Activity per Unit Time

6.1. VARIATIONS DURING INDIVIDUAL SUNSPOT CYCLES

Since we found that the prominent periodicity in B is '18.3 ± 3 yr', we expect a prominent periodicity of 7.6–10.6 yr in the absolute value $|B - \overline{B}|$, viz., in the time-dependent part of B, where $\overline{B}$ is the average over the whole time series. We compare here the variations in $|B - \overline{B}|$ with the annual amount of sunspot activity during each sunspot cycle. In Figure 6(a) we show the variation of $|B_i - \overline{B}|$ w.r.t. N_i (the measure of the sunspot activity) during the successive intervals 'i' of length 1 yr, where $\overline{B}$ is the average value of all the 98 values of B_i. The variation of $|B_i - \overline{B}|$ determined from sequence of 3-yr intervals successively displaced by 1 yr, is shown in Figure 6(b). From these figures we see that the value of $|B_i - \overline{B}|$ peaks just before the minimum at the end of each sunspot cycle. Such a peak at the end of cycle number 17 (Waldmeier number) is low. There are also small peaks just after the minima at the beginnings of the cycle numbers 13 and 18. The peak at the end of cycle number 20 seems to be small. In most of the cycles there are very weak peaks around the sunspot maxima which are not as well defined as those near the sunspot minima. Near the maximum of the large amplitude sunspot cycle ('number 19') there is no peak in $|B_i - \overline{B}|$ and in fact the value of $|B_i - \overline{B}|$ reached is quite low. The low amplitude cycles, number 12 and 14 have a slightly larger peak (or peaks) near the maxima. There is also some suggestion that the large (small) amplitude sunspot cycles follow the low (high) amplitude peaks of $|B_i - \overline{B}|$ (especially in Figure 6(b)). In general, there are noticeable differences in the variations of $|B_i - \overline{B}|$ within each sunspot cycle and also from one sunspot cycle to another. Gilman and Howard (1984) already noticed many of the above mentioned characteristics in the residual rotation rates of sunspots. Recently Komm, Howard, and Harvey (1993), from the analysis of magnetograms over 17 yr during 1975–1991, have suggested that solar rotation is more differential during sunspot minimum and more rigid during the sunspot maximum. This result is confirmed in the present analysis for the 98-yr long time series. The correlation between $|B_i - \overline{B}|$ and N_i as determined from yearly values (Figure 6(a)), and also from the sequence of 3-yr intervals (Figure 6(b)), is -47%. This anticorrelation is marginally increased when the values of N_i are shifted backward by 1 yr.

6.2. CORRELATION BETWEEN THE LONG-TERM VARIATIONS OF $|B - \overline{B}|$ AND THE AMOUNT OF SUNSPOT ACTIVITY PER UNIT TIME

Figure 7(a) shows the temporal variations of $|B_j - \overline{B}|$ and N_j, the 11-yr running averages of the yearly values of $|B - \overline{B}|$ and N, respectively. We find that the correlation between $|B_j - \overline{B}|$ and N_j is -79%. The correlation between $|B_j - \overline{B}|$ and N_{j+q} reaches a maximum value of -81% for $q = 2$. Figure 7(b) shows the temporal variations of $|B_k - \overline{B}|$ and N_k, the 22-yr running averages of the yearly values of $|B - \overline{B}|$ and N, respectively. We find that the correlation between $|B_k - \overline{B}|$ and N_k is -96%. The correlation between $|B_k - \overline{B}|$ and N_{k+q} reaches a maximum value of -97% for $q = 1$. [*Note*: there is absolutely no correlation between yearly values of $|B|$ and N. The maximum correlation between these quantities is found only 40% (negative), and that too in the 17–19 yr running averages.]

The magnitude of the anticorrelation between $|B - \overline{B}|$ and N improves gradually up to 96% when running averages are taken over longer and longer time intervals up to 22 yr and remains at the same level for intervals longer than 22 yr. This shows that the amount of sunspot activity per unit time is related to the amplitude of the $\sim$ 22 yr oscillation in B rather than the yearly values of B. The negative sign of the above correlations confirms that $|B - \overline{B}|$ and the 'magnetic field', (associated with sunspot activity) vary in nearly opposite phases in the 22-yr variation. The magnitude of the negative correlation increases by about 1% when the sequence of

N values is shifted backward by 1–2 yr. (This suggests that the modulations in N are caused by that in B.)

The trends in Figures 7(a) and 7(b) suggest the existence of an $\sim$ 90-yr periodicity in $|B - \overline{B}|$ as well as in the sunspot activity (i.e., an $\sim$ 180-yr periodicity in B and in the solar magnetic field). These periodicities are already known to be present in the variation of sunspot activity (e.g., Wolf, 1976).

In order to determine the dependence of the amount of activity, N, on the amplitude of $|B - \overline{B}|$ we computed their correlations by considering the sequences of non-overlapping intervals (of lengths > 2 yr). The magnitude of the negative correlation between $|B - \overline{B}|$ and N improves gradually for longer and longer time intervals. For intervals of 8–10 yr and 22–23 yr, the negative correlation has values $\sim 80\%$ and $\sim 98\%$, respectively. From this we conclude that the cycle-to-cycle variation in the amount of sunspot activity depends substantially upon the cycle-to-cycle modulation of the amplitude of the '22-yr' torsional oscillation.

7. Comparison with Synodic Periods of Planetary Configurations

If 'X' and 'Y' represent two planets (sets of planets containing one or more planets) with orbital (synodic) periods P_X and P_Y, respectively, then the relative positions of these planets (configurations), repeat over the mutual synodic period

$$P(X, Y) = |P_X^{-1} - P_Y^{-1}| \, . \tag{2}$$

Thus, for pairs of planets in consecutive orbits, we have $P(U, N) = 171.39$ yr, $P(S, U) = 45.83$ yr, $P(J, S) = 19.86$ yr, $P(C, J) = 7.52$ yr, $P(Ma, C) = 3.18$ yr, $P(E, Ma) = 2.14$ yr, $P(V, E) = 1.6$ yr, and $P(M, V) = 0.396$ yr, where the planets' names are denoted by their first letters, except for Mars which is denoted by Ma. The letter C represents the largest asteroid Ceres. In view of the small mass of Ceres, the pair (Ma, J) can also be considered as consecutive. In Table III, it can be seen that each of the periodicities in B/A determined in Section 4 agrees, within its uncertainties, with one or the other among the above synodic periods or their modifications given by inclusion of one or more neighbouring planets. Also, the '43 ± 10-yr' periodicity in B/A given in Table II matches with $P(S, U)$ and the '180 yr' periodicity suggested in Section 6.2 matches with the 178-yr period of conjunction of all the planets (because of their large uncertainties we have not included these in Table III). On the other hand, periods in B/A which are < 2 yr (if any) cannot be determined in the present analysis. Thus each periodicity in B/A found in the present analysis matches with the synodic period of a set of two or more consecutive planets.

TABLE III

Periods in B/A (or B) and synodic periods of planetary configurations. The planets names are denoted by their first letters, except for Mars which is denoted by *Ma*. The letter C represents the largest asteroid Ceres. The sequence of brackets denotes successive use of Equation (2)

Synodic periods (in years)	Periods of B/A (in years)
$P(J,S) = 19.86$	18.9 ± 1.1
$P(C,J) = 7.52$	8.3 ± 0.8
$P[(C,J),(S,U)] = 9.01$	
$P\{[(E,Ma),C],J]\} = 6.00$	6.1 ± 0.2[a]
$P\{[(Ma,C),J],S]\} = 5.09$	4.8 ± 0.2[a]
$P[(E,Ma),C] = 3.97$	3.9 ± 0.1
$P[(Ma,C),(J,S)] = 3.79$	3.7 ± 0.05
$P[(E,Ma),(C,J)] = 2.98$	
$P(Ma,C) = 3.18$	3.1 ± 0.1
$P[(V,E),(Ma,C)] = 3.22$	
$P\{[(E,Ma),J],S\} = 2.85$	2.8 ± 0.05
$P[(E,Ma),J] = 2.60$	2.6 ± 0.1
$P[(E,Ma),(J,S)] = 2.39$	2.3 ± 0.1
$P(Ma,J) = 2.23$	
$P[(Ma,J),S] = 2.08$	2.06 ± 0.05
$P(E,Ma) = 2.14$	

[a] These have levels of significance $< 1\sigma$ but are included for the sake of completeness.

8. Conclusions and Discussion

From the analysis of the rotation rates of sunspot groups we have drawn the following conclusions:

(1) The temporal variations of B (or B/A) have the periodicities 18.9 ± 1.1 yr, 8.3 ± 0.8 yr, 3.9 ± 0.1 yr, 3.1 ± 0.1 yr, 2.6 ± 0.1 yr, and may have periodicities also at ~ 180, 43 ± 10, 6.1 ± 0.2, 4.8 ± 0.2, 3.7 ± 0.05, 2.8 ± 0.05, 2.3 ± 0.1, and 2.06 ± 0.05 yr.

(2(a)) The periodicities 18.9 ± 1.1 yr, 8.3 ± 0.8 yr, 3.9 ± 0.1 yr, and 2.6 ± 0.1 yr in B/A match with similar periodicities in the photospheric magnetic field.

(2(b)) The solar rotation is more differential during the sunspot minima than during the maxima. Thus $|B - \overline{B}|$ and 'magnetic field' vary nearly in opposite phases.

(2(c)) From (a) and (b) it is concluded that the variations in the differential rotation and in the magnetic field are two aspects of torsional MHD oscillations. (The phase difference of 1–2 yr indicates that the modulation of N may be controlled by that of B.)

(3) The long-term modulation of the amount of sunspot activity per unit time is well correlated to the modulation of the 22-yr torsional oscillation.

(4) Each of the periodicities found in B/A are in good agreement with the synodic period of a set of two or more consecutive planets.

From the conclusions 2(a) and 4 we speculate that some specific configurations of consecutive planets may provide perturbations needed for exciting torsional MHD oscillations. It will be interesting to see if this effect comes through the changes in the Sun's angular momentum about the center of mass of the solar system (Jose, 1965; Wood and Wood, 1965; Blizard, 1981).

The main doubts about any role of the solar system dynamics in solar activity come from (i) the near absence of sunspot activity during the Maunder minimum (1640–1715) and (ii) the variation of stellar magnetic activity along the H–R diagram and with age. Attempt to explain the prolonged activity minima on the basis of Sun's orbital motion has been made by Fairbridge and Shirley (1987). However, the role of the solar system dynamics in the basic mechanism of solar activity is still a matter of speculation.

Acknowledgements

We are thankful to the unknown referee for the detailed comments which helped us to improve our presentation substantially. We thank Dr A. V. Raveendran for providing MEM Fortran program and Mr J. S. Nathan who helped in making plots on the computer.

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