

# A photoionization model of the nebula around the WC11 star M4–18

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## ABSTRACT

A photoionization code has been developed incorporating provision for the presence of dust mixed with gas. M4–18 has been modelled utilizing optical CCD spectra and other observations ranging from UV to radio wavelengths.  $T_e$  and  $N_e$  from the model are 6600 K and  $6050 \text{ cm}^{-3}$ , respectively. The abundances of C, N and Ne are found to be the same as the solar values while O is overabundant by a factor of two and S is lower than the solar value by  $\sim 90$  per cent. A monosized amorphous carbon grain dust model explains the *IRAS* fluxes satisfactorily but emits poorly in the 1- to  $10\text{-}\mu\text{m}$  region. M4–18 is a protoplanetary nebula with properties similar to CPD – 56° 8032 and V348 Sgr.

**Key words:** radiative transfer – stars: Wolf–Rayet – dust, extinction – planetary nebulae: individual: M4–18 – infrared: ISM: continuum.

## 1 INTRODUCTION

M4–18 is a low-excitation planetary nebula, whose central star is a cool Wolf–Rayet star that has been classified as WC10 by Webster & Glass (1974) and later as WC11 by van der Hucht (1981). This rare group of late Wolf–Rayet central stars initially consisted of M4–18, He 2–113, CPD – 56° 8032 (hereafter CPD) and V348 Sgr. Their spectra comprise mainly emission lines of He I, C III and numerous lines of C II, in addition to low-excitation nebular lines of [O II], [S II] and [N II]. Lines of [O III] are either absent or weakly present. Large amounts of IR excess radiation is a prominent characteristic of all these objects. Four more objects have been added to this class of WC11 (Cohen & Jones 1987; Menzies & Wolstencroft 1990; Hu & Bibo 1990; Hu & Dong 1992). V348 Sgr, known as a hot RCB star, shows characteristic large light variations while CPD shows small-amplitude light variations. It is not known if others vary in light.

Most of these objects show dust emission features at 7.7, 8.7 and  $11.3 \mu\text{m}$  which are attributed to polycyclic aromatic hydrocarbon (PAH) molecules (Aitken & Roche 1984; Cohen et al. 1986; Cohen & Jones 1987). CO (1–0) and (2–1) emission have been detected in He 2–113, 21282 + 5050 and CPD (Likkell et al. 1988; Knapp et al. 1989). 07027 – 7934 has recently been found to show strong 1612-MHz OH maser as well as weak CO emission (Zijlstra et al. 1991).

Rao (1987) compared the nebular and stellar properties of He 2–113, M4–18, CPD and V348 Sgr and indicated the possibility that their evolution might proceed from left to right in the ( $\text{Log } L\text{-Log } T$ ) diagram, towards the asymptotic giant branch for a second time and they might form a link with RCB stars. In order to understand the nature of these objects and study their low-excitation nebulae, a photo-

ionization code has been developed and applied to the nebula around M4–18.

The model code is described in Section 2, and the observations are presented in Section 3. The nebular reddening and distance are discussed in Sections 4 and 5. The observational inputs for the central star are analysed in Section 6 and the nebular model is presented and discussed at length in Section 7.

## 2 THE PHOTOIONIZATION CODE

The model nebula is assumed to be spherically symmetric, static and in steady state. The basic theory is enunciated in Osterbrock (1989). The diffuse radiation field is treated in the OTS approximation. Dielectronic recombination and charge transfer reactions have been included in the ionization equilibrium where appropriate. However, the latter is not considered in the equation for hydrogen for the sake of simplicity. The code provides for inclusion of dust along with gas. The treatment of dust is similar to the approach of Harrington et al. (1988) and Hoare & Clegg (1988). Atomic parameters are taken from the compilation made by Mendoza (1982) and from literature up to 1991. The code has been cross checked for accuracy with results from a known code for a test nebula (Köppen 1991, private communication). A detailed description of the code and atomic parameters are given in Surendiranath (1992).

## 3 OBSERVATION AND REDUCTION

### 3.1 Optical and UV spectra

Spectra of M4–18 in the wavelength range  $\sim 5500 \text{ \AA}$ – $1 \mu\text{m}$  were obtained with the 1- and 2.3-m telescopes at the Vainu Bappu Observatory, Kavalur. At the 1-m telescope a Zweiss

Universal Astronomical Grating Spectrograph (UAGS) provided with a Photometrics CCD system containing a Thomson CSF Th7882 chip, was used at the Cassegrain focus with a (2 pixel) resolution of  $\sim 14 \text{ \AA}$ . At the 2.3-m Vainu Bappu Telescope (VBT) a Boller and Chivens spectrograph provided with a CCD camera having a GEC P8603 chip was used and a resolution of  $5 \text{ \AA}$  could be achieved. Table 1 gives the log of observations. The slit was kept N-S in an effort to reduce errors in spectrophotometry due to atmospheric dispersion (Filippenko 1982).

The *International Ultraviolet Explorer* (IUE) observations (Figs 1 and 2) were obtained on 1991 August 24, with a

large-aperture camera in low-resolution mode covering  $\lambda 1250\text{--}3100 \text{ \AA}$ .

### 3.2 Data reduction

All the CCD optical spectra were reduced using the software package RESPECT (Prabhu, Anupama & Giridhar 1987; Prabhu & Anupama 1991). The line equivalent widths were measured by fitting Gaussian profiles and blends were treated as superposed Gaussians. Flux calibration was done by observing the standards He 3 (=EG50) and Feige 34. The fluxes for these are from Barnes & Hayes (1984) and

Table 1. Journal of observations.

Date	Object	Wavelength region $\text{\AA}$	Resolution ( $\text{\AA}$ )	Exposure time (Min)	Number of exposures	Sky & seeing
1990 Jan 19/20	M 4-18	5550-7150	14	40	2	Clear sky 2" - 2.5"
"	He 3 (= EG 50)	"	"	"	1	
1990 Dec 21/22	M 4-18	8300-9950	"	"	4	Fair (cirrus all over) poor seeing.
1991 Feb 10/11	M 4-18	"	15	"	3	Very good ~ 2"
"	Feige 34	"	"	"	3	
1991 Feb 11/12	M 4-18	7050-8600	"	"	4	Very good ~ 2"
"	Feige 34	"	"	"	3	
1991 Feb 28	M 4-18	5630-6955	5	40	1	Very good 1.2"
1991 Aug 24	M 4-18	LWP 21064 (IUE)	9	100	1	-
"	"	SWP 42313 (IUE)	5	200	1	-
1993 Jan 4/5	M 4-18	9000-10000	4	40	4	Clear sky ~ 2"

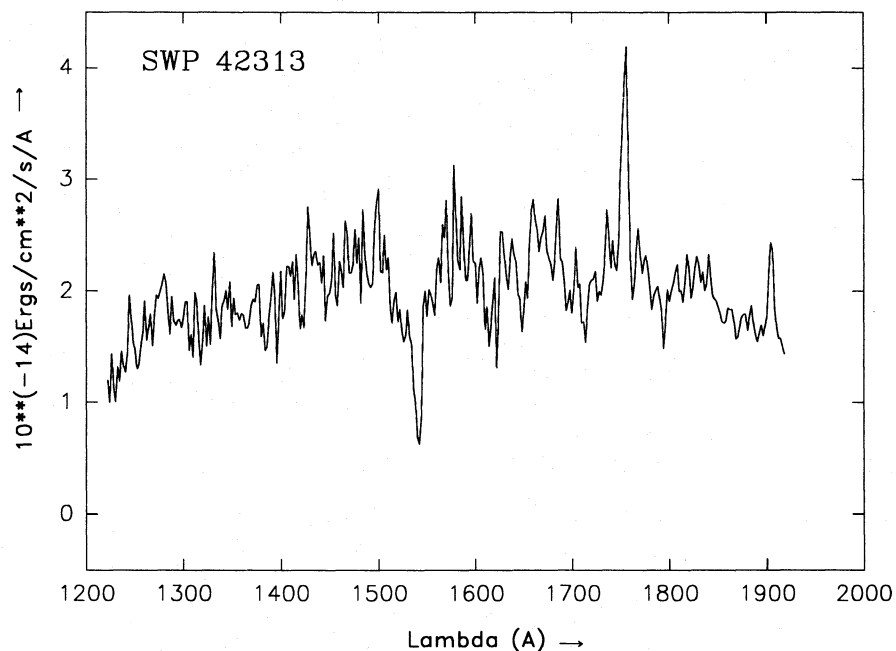
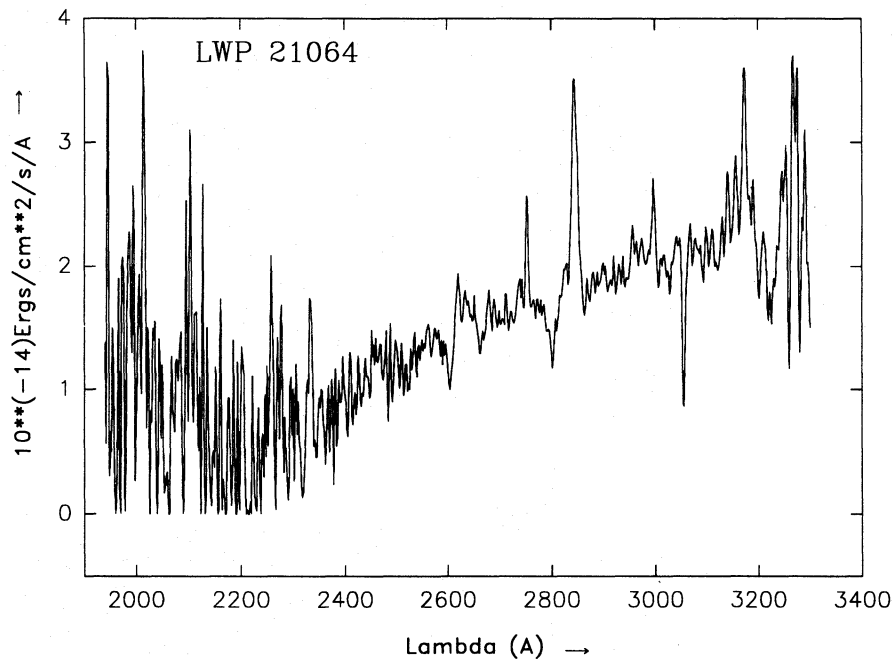


Figure 1. Dereddened IUE spectrum (SWP) of M4-18 observed on 1991 August 24, with the large aperture camera in low-resolution mode.



**Figure 2.** Dereddened *IUE* spectrum (LWP) of M4-18 observed on 1991 August 24, with the large aperture camera in low-resolution mode. +: saturation.

Massey & Gronwall (1990) respectively. Our absolute calibration error is estimated to be a factor of two. The CCD spectra from the 1-m telescope are shown in Fig. 3(a) to (e), and those from the 2.3-m telescope in Fig. 3(f) to (h) and Fig. 4. From a comparison of the measured continuum between our 1-m observations and those of Goodrich & Dahari (1985, hereafter GD) in the wavelength regions common to both, it was noticed that the continuum level of our 1-m spectra was lower; this was more so in the region 7000–9200 Å. GD corrected line fluxes from their red scan in order to make them consistent with the fluxes from their blue and IR scans, and they have ignored small difference between their blue and IR scans. Since no variation of the stellar light has been observed, this continuum level of GD is adopted and extended up to  $\sim 7700$  Å. The observed *V* magnitude of Shaw & Kaler (1985) is consistent with this adopted continuum. The line fluxes shown in Table 2 are the product of the measured equivalent width and this adopted continuum. For the region 7700–9000 Å, the lines seen are identified and no fluxes are given due to a poor S/N ratio. In the range 9000 Å–1  $\mu\text{m}$ , our flux calibration is uncertain and the [S III] line ( $\lambda 9069$ ) flux was calculated by ratioing to the Paschen line ( $\lambda 9015$ ) for which the flux could be computed. Observations were made with a 2-arcsec slit admitting only a part of the nebula ( $\sim 4$ -arcsec diameter) and consequently the absolute fluxes might not represent the whole nebula; however, relative fluxes are expected to be unaltered.

The UV line fluxes from our longer exposure *IUE* spectra are also shown in Table 2. Comparing our measured fluxes with those listed by GD (from archival spectra), our fluxes are lower by a factor of three for C II 1760 Å and C III] 1908 Å, while for C II 2747 Å there is agreement. The line C II 2837.3 Å was completely saturated in our spectrum. We dereddened the measured fluxes using Seaton's (1979) extinction curve.

### 3.2.1 Final adopted fluxes

Our high-resolution spectrum of M4-18 shows a P-Cygni type profile for the lines C II 6462 Å and He I 6678 Å. The measured radial velocities (from the blueshifted absorption dips) are  $390 \pm 25$  and  $330 \pm 25$  km s $^{-1}$  respectively indicating stellar wind. The nebular contribution to the He I line is not easy to separate out. However, in CPD and He 2-113, the H I lines as seen on high-resolution ( $\lambda/\Delta\lambda \sim 35\,000$ ) spectra obtained by one of us (NKR) with the *ESO* 1.5-m telescope, are much sharper relative to C II and He I lines, indicating that H I lines originate from the nebula. For M4-18 we assume that a similar situation prevails and the H I lines are nebular. Some of the nebular lines are blended with the stellar lines e.g. H $\beta$  could have been contaminated by stellar C II (Revised Multiplet Table, 10.01 at  $\lambda\lambda 4862, 4867$ ) at our low resolution and the nebular [S II] lines  $\lambda\lambda 4068$  and 4076 are contaminated (see table 2 of GD) by stellar C II and C III as well as O II lines at  $\lambda\lambda 4072$  and 4076. Some of the collisional lines used for the nebular diagnostics are unresolved e.g. [O II]  $\lambda\lambda 3727, 3729$ . Although this line was observed by GD, its flux is uncertain because of its proximity to the UV cut-off of the image tube response curve. The [Ne II]  $\lambda 12.8$ - $\mu\text{m}$  line flux, as observed by Aitken & Roche (1982, hereafter AR) with a wider aperture which admits the whole nebula, has been included in our analysis.

## 4 NEBULAR REDDENING AND EXTINCTION

GD used an  $E(B - V)$  value of  $0.90 \pm 0.09$  to correct their measured line fluxes for interstellar extinction. It has been pointed out by Rao, Giridhar & Nandy (1990) that the use of 10-GHz radio flux (Purton et al. 1981) and absolute H $\beta$  flux (as given by GD and assumed to be of nebular origin) leads to a value of  $E(B - V)$  between 0.41 and 0.56. There-

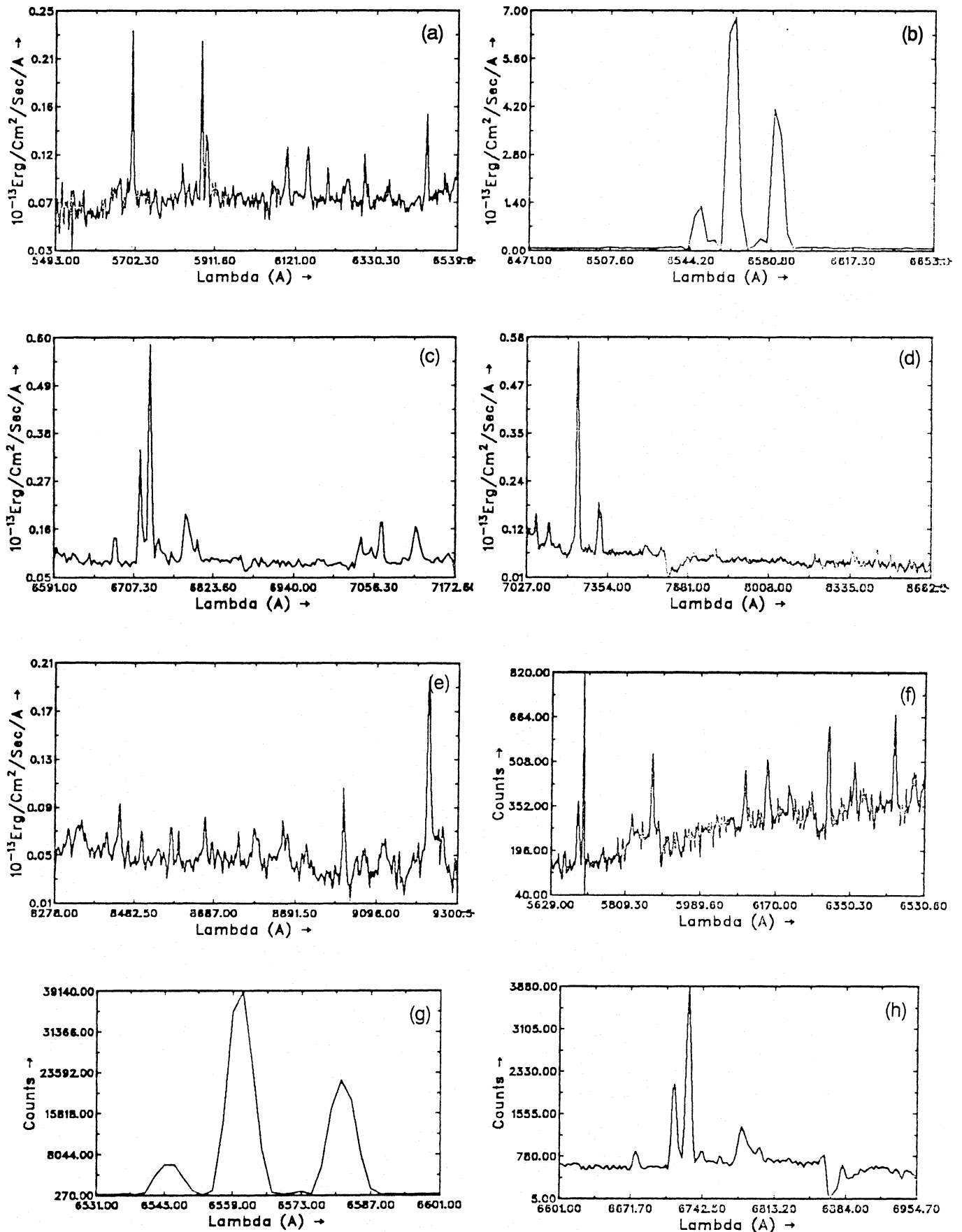


Figure 3. CCD spectra of M4-18 obtained at VBO, Kavalur. (a)–(e) are from UAGS at the 1-m telescope. (f)–(h) are the higher resolution spectra from the B and C spectrograph at the VBT.

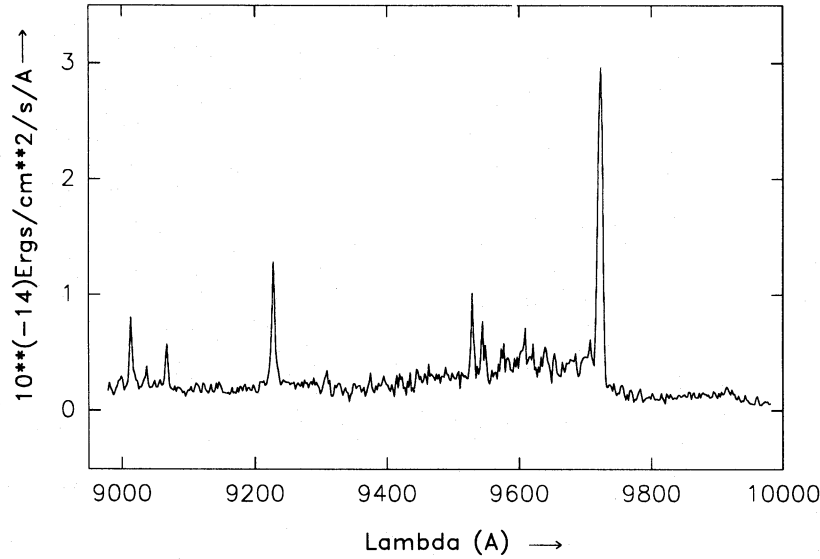


Figure 4. CCD spectrum of M4-18 in the far red (VBT observation).

Table 2. Measured line fluxes.

Measured Wavelength (Å)	Laboratory Wavelength (Å)	Ion	EW(Å)		Measured flux ( $10^{-15} \text{ ergs cm}^{-2} \text{ s}^{-1}$ )		Dereddened flux ( $10^{-15} \text{ ergs cm}^{-2} \text{ s}^{-1}$ )	
			New	New	New	GD		
1335		C II		13.3		672		
1760		C II		59.3		1860	5800	
1908		C III]		40.7		1500	4440	
2297		C III]		$\leq 6.33$		$\leq 329$		
2326		C II]		64.7		2870		
2471		[O II]		$\leq 14.7$		$\leq 393$		
2747		C II		49.3		762	967	
2837		C II		Saturated			2770	
2993		C II		36.7		434		
3140		?		19.3		204		
3154		?		25.3		265		
3165		C II		80.7		835		
5693.7	5695.9	C III]	10.8	102.2		390.3	399.4	
	5694.3	C II						
5756.3	5754.6	[N II]	1.30	11.92		44.7	55.2	
5826.1	5826.4	C III]	1.24	11.1		40.8	54.0	
	5823.1	C II						
	5827.8							
5843.7	5843.6	C II	1.47	13.1		47.8	30.6	
5862.2	5856.0	C II	1.46	12.9		46.9	46.2	
	5863.2	C III]						
5877.1	5875.6	He I	11.4	100.4		363.6	338.3	
	5876.0							
5889.3	5889.3	C II	6.4	56.3		203.1	225.4	
	5889.8							
	5891.6							
6098.9	6095.3	C II	6.3	52.6		179.2	176.4	
	6098.5							
	6102.6							
6152.8	6151.4	C II	6.3	52.2		175.2	161.2	
6204.6	6205.6	C III]	2.8	22.6		74.6	30.8	
6244.3	6246.6	C II						
	6250.7							
	6253.8							

Table 2 – continued

Measured Wavelength (Å)	Laboratory Wavelength (Å)	Ion	EW(Å)		Measured flux ( $10^{-15} \text{ergs cm}^{-2} \text{s}^{-1}$ )		Dereddened flux ( $10^{-15} \text{ergs cm}^{-2} \text{s}^{-1}$ )	
			New	New	New	GD		
6256.3	6256.5 6257.2 6259.6	C II						
6300.8	6300.3	[O I]	4.1	33.9		109.5	170.9	
6307.9			1.3	10.7		34.5		
6332.3								
6362.6	6363.8	[O I]	0.6	5.0		15.9	57.9	
6461.5	6462.0	C II	5.4	45.0		139.7	161.4	
6507.3			1.36	11.2		34.4		
6549.0	6548.1	[N II]	95.2	786.4		2391.7	2763.3	
6563.2	6562.8	H I	587.8	4855.2		14715.5	17024.0	
6582.3	6583.6	[N II]	327.3	2703.5		8154.3	8969.0	
6593.1			0.53	4.39		13.2		
6617.9			2.4	19.8		59.2	41.0	
6641.7			1.0	7.9		23.5	37.5	
6679.2	6678.1	He I	4.2	34.3		101.2	124.2	
6717.0	6716.4	[S II]	20.2	167.1		489.0	580.0	
6731.0	6730.8	[S II]	38.8	319.8		933.1	1132.3	
	6727.4	C II						
	6731.0	C III						
6743.0	6742.4	C III	6.2	51.2		148.9	67.2	
	6744.4							
6751.5	6750.6	C II	2.9	24.0		69.7	10.7	
6763.1	6762.2	C III	2.3	19.0		55.0	41.7	
6781.5	6779.9	C II	2.91	24.0		69.2	121.5	
	6773.4	C III						
	6774.9							
6785.8	6783.9	C II	12.43	102.7		296.0	228.3	
	6787.2							
6793.2	6791.5	C II	1.85	15.3		44.0	102.1	
6801.0	6798.1	C II	3.61	29.8		85.6	104.3	
	6800.7							
7037.5	7037.2	C III	5.32	43.6		119.2	103.3	
7045.4	7046.3	C II	1.23	10.1		27.5	43.2	
7052.6	7053.1	C II	3.91	32.0		87.3	56.6	
7066.3	7063.7	C II	9.7	80.1		217.7	249.2	
	7065.2	He I						
	7065.7							
7100.4			1.3	10.7		28.9	28.6	
7113.5	7112.5	C II	7.4	60.7		163.3		
	7113.6							
7116.8	7115.6	C II	2.87	23.5		63.2		
7121.4	7119.9	C II	6.05	49.6		133.3	139.8	
	7125.7							
7133.4	7132.4	C II	1.18	9.7		26.0	42.2	
	7134.1							
7147.7								
7211.3	7210.5	C III						
	7212.3							
7234.24	7236.4	C II						
	7237.2							
7278.31	7281.3	He I	1.7	14.0		36.4	56.0	
7318.0	7318.6	[O II]	18.8	155.2		401.3	371.6	
	7319.4							
7328.5	7329.9	[O II]	13.3	109.8		283.4	332.4	
	7330.7							
7485.4	7486.5	C III	1.1	9.1		22.8	16.3	
7505.3	7505.3	C II						
7506.3	7508.9	C II						

Table 2 – continued

Measured Wavelength (Å)	Laboratory Wavelength (Å)	Ion	EW(Å)	Measured flux	Dereddened flux	
			New	( $10^{-15} \text{ ergs cm}^{-2} \text{ s}^{-1}$ ) New	( $10^{-15} \text{ ergs cm}^{-2} \text{ s}^{-1}$ ) New	GD
7525.6	7519.5	C II				
	7519.9					
7704.5	7707.4	C III				
7794.5						
8194.0	8196	[Cl III]?				
8311.2						
8355.0	8357.9	C III				
	8358.7					
8389.7	8392.4	P 20?				
8402.2						
8412.0	8413.4	C II				
	8414.5	C II				
	8412.0	P 19?				
8432.8	8432.8	P 18?				
8445.3		O I				
8466.2	8467.2	P 17?				
8500.9	8500.3	C III				
	8500.9	P 16?				
8542.6	8545.4	P 15?				
8556.5						
8576.7						
8596.2	8598.4	P 14?				
8665.0	8663.6	C III				
8682.8	8682.5	C II				
8696.7	8696.7	C II				
8715.7						
8731.0						
8748.8	8750.5	P 12?				
8787.0						
8796.0	8793.8	C II				
	8799.9					
8851.3						
8860.0	8862.8	P 11?				
8871.8						
8910.0						
8919.1						
9013.1	9014.9	P 10				
9036.9						
9066.5	9068.9	[S III]				
9227.0	9229.0	P 9				
	9229.2	C II				
	9229.8					
9528.8		[S III]				
9544.5		P 8				
9548.6						
9573.0						
9577.2						
9609.0						
9621.1						
9638.5						
9707.0	9701.1	C III				
	9705.4					
	9706.4					
9722.6		C II				

fore an  $E(B-V)$  value of 0.48 is adopted in our analysis and the extinction law as given by Seaton (1979) has been used. Shaw (1984) estimates extinction of M4-18 on the basis of detection of Ca II K line in absorption; it could be circumstellar in origin and need not be interstellar. Our value is also consistent with the estimate of  $E(B-V)$  of 0.40 by GD using the strong 2175-Å interstellar absorption feature. The dereddened line fluxes are shown in column 6 of Table 2. GD's equivalent widths have been converted to fluxes and dereddened with  $E(B-V)$  of 0.48. The dereddened H $\beta$  flux from GD's observations is  $5.27 \times 10^{-12}$  erg cm $^{-2}$  s $^{-1}$ . Shaw & Kaler's (1985) value (from filter photometry) was  $\log F_{H\beta} = -11.880$ . Correcting this with  $E(B-V)$  of 0.48 yields an absolute H $\beta$  flux of  $6.68 \times 10^{-12}$  erg cm $^{-2}$  s $^{-1}$ . The mean of these two values (i.e.  $5.97 \times 10^{-12}$  erg cm $^{-2}$  s $^{-1}$ ) is adopted for the present analysis.

## 5 DISTANCE

Nebular mapping with VLA at 5 GHz showed the angular diameter to be 4 arcsec (Zijlstra et al. 1989) which refers to the ionized region. We adopt this as the nebular size although there is a possibility of neutral gas beyond this. With a mean 5-GHz radio flux (mean of Zijlstra et al. 1989 and Aaquist & Kwok 1990) of 20.25 mJy, the distance computed by Daub's (1982) expression is 4.44 kpc. By the modified Shklovskii method the estimated distance is 3.88 kpc; while Cudworth's (1974) scale gives  $d=6.47$  kpc, the scale of Mallik & Peimbert (1988) gives  $d=7$  kpc. The galactic longitude and latitude of M4-18 are 146° and 7°. Neckel & Klare (1980) give the visual extinction  $A_V$  as a function of distance for different galactic longitude and latitude zones. Although the zone of  $b^{\text{II}}$  around 7° is not covered in their results, it is of interest to estimate the extinction distance from the nearest zone. From their map for the zone 146+3° [their Fig. 6b (34)], we get  $d=0.63$  kpc. Similarly, the  $(E(B-V), R, Z)$  distribution given by Fitzgerald (1968) yields  $d=0.65$  kpc [from his fig. 3g (42)] for the longitude range of 140°-150°,  $R$  in the range 100-800 pc and  $Z = \pm 400$  pc. Pottasch et al. (1984) have shown a clear correlation between nebular radius and dust temperature  $T_D$ , where  $T_D$  represents the blackbody temperature estimated from *IRAS* fluxes. For M4-18, a blackbody fit of the *IRAS* 12-, 25-, 60- and 100- $\mu$ m flux densities gives a value of  $T_D = 230$  K, which is in agreement with 229 K as given by Zhang & Kwok (1990, hereafter ZK). With this  $T_D$ , the nebular radius is estimated as  $\sim 1.6 \times 10^{16}$  cm ( $\sim 0.0052$  pc). Since the radio angular radius is 2 arcsec, we obtain a distance of 0.6 kpc. Kaler, Mo Jing-Er & Pottasch (1985) have derived an empirical relation that gives distance as a function of stellar luminosity, temperature, visual magnitude and colour excess. This method gives  $d=5.63$  kpc for a star assumed to be a blackbody of 23 000 K and radius  $r=1.8 \times 10^{11}$  cm ( $\sim 2.5 R_{\odot}$ ).

Statistical distances, in general, give very poor estimates when applied to individual nebulae (see Gathier 1984). The extinction distances derived above should also be viewed with caution because extinction varies widely even in directions only 0.2° away from the object; also, extinction could be very patchy. Since the zone diagrams used in the distance estimates are only neighbourhood representations of the actual direction, there could be an error of a factor of two or three. Therefore a value of 1 kpc has been adopted as

a starting value for modelling M4-18; this is also consistent with the estimate from the dust temperature method. With this adopted distance and an angular diameter of 4 arcsec, the outer radius of the nebula is  $2.992 \times 10^{16}$  cm (0.0097 pc).

## 6 THE CENTRAL STAR

Since the spectrum of the star alone is not available, properties like temperature and radius have to be deduced in other ways.

### 6.1 Temperature

GD derived a temperature of 22 000 K using the Stoy (1933) energy-balance method, as formulated by Preite-Martinez & Pottasch (1983). By considering the excitation conditions in the nebula as well as that of similar nebular spectra seen in CPD and He 2-113, the above temperature estimate seems to be consistent. The C III/C II line ratio appears to be larger in CPD relative to M4-18.  $T_*$  of CPD was estimated to be  $\sim 26$  000 K by Rao et al. (1990) as well as Aitken et al. (1980). The Zanstra hydrogen temperature for M4-18 is 26 000 K and  $T_{\text{BB}}$ , by blackbody fit to the dereddened energy distribution in UV and optical, is 23 000 K. Therefore, a value of 23 000 K seems to be appropriate for the central star of M4-18.

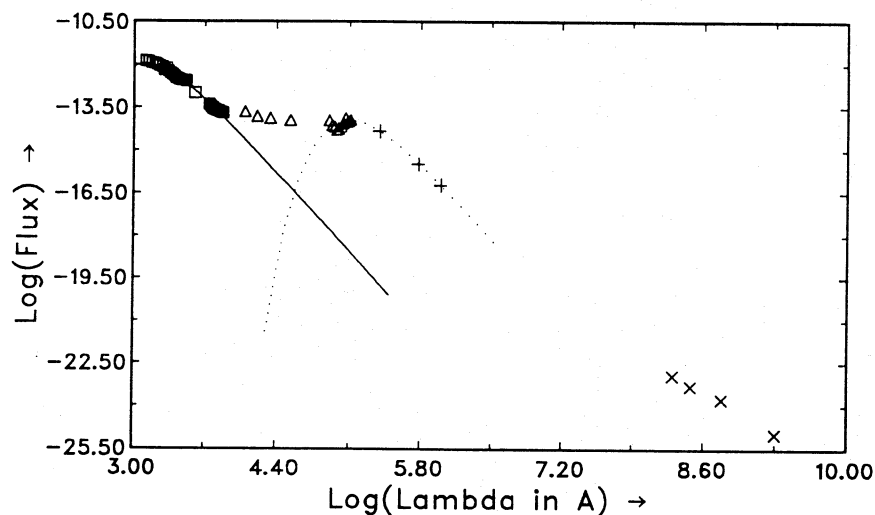
### 6.2 Radius

It has been shown that the core masses of central stars lie in the range 0.546 to  $\sim 0.8 M_{\odot}$  by Schönberner (1981). Rao (1987) has deduced that the stellar mass is about  $0.56 M_{\odot}$  for the WC11 group, based on its estimated position in the HR diagram. From its position in the (radio brightness temperature, optical depth at 60  $\mu$ m) diagram, ZK infer that M4-18 is likely to have a low-mass central star. Assuming that M4-18 represents the minimum mass for a star evolving from AGB to PN we adopt  $0.546 M_{\odot}$ . From the ( $\log L, \log T$ ) diagram for a  $0.546 M_{\odot}$  star (Schönberner & Weidemann 1981) and the above estimate of  $T_*$ , an estimate of  $R_* \sim 2.5 R_{\odot}$  was adopted for M4-18. GD obtained  $3.5 R_{\odot}$  from the bolometric correction method. Bolometric corrections are not easy to derive for a star whose spectral type and luminosity class are not well defined. However the value of  $L$  obtained by the adopted distance, the observed  $m_v$  (dereddened), and the bolometric correction (for blackbodies) for  $T_* \sim 23$  000 K leads to a much lower luminosity. There is a possibility of neutral extinction occurring around the star, but this has not been taken into account.

### 6.3 The total energy emitted

The observed UV, optical and IR fluxes ( $J, H, K$  and  $L$  from ZK, the *IRAS* fluxes from the Point Source Catalogue and the 8- to 13- $\mu$ m spectrum from AR) along with the radio flux (Purton et al. 1981; Zijlstra et al. 1989; Aaquist and Kwok 1990) in the GHz range are corrected for reddening and plotted in Fig. 5. A blackbody fit for stellar temperature of 23 000 K as well as the 230-K blackbody representing the IR emission by dust are also shown. The observed luminosity of the star (by integration over the range 0.01 to 350  $\mu$ m) is





**Figure 5.** Observed continuum from M4-18. Squares: our new *IUE* and optical observations. Triangles: 8–13  $\mu\text{m}$  spectrum from AR. Pluses: *IRAS* band fluxes. Crosses: VLA observations in the GHz range by Aaquist & Kwok (1990), Purton et al. (1981) and Zijlstra et al. (1989). Solid line: scaled BB (23 000 K) fit. Dotted line: scaled BB (230 K) fit.

$164.1 d^2 L_{\odot}$ , where  $d$  is the distance in kpc. The total far-IR luminosity alone is  $97.1 d^2 L_{\odot}$  (after subtracting the 23 000-K scaled BB emission of  $67 d^2 L_{\odot}$ ) which is about 60 per cent of total flux.

## 7 THE MODEL

### 7.1 Physical conditions in the nebula

The diagnostic lines to estimate nebular electron temperature ( $T_e$ ) and electron density ( $N_e$ ) are rather limited. The [O III]  $\lambda 5007$  line is not seen in the spectrum. The [N II]  $\lambda 6584$  line is very likely contaminated by stellar C II  $\lambda 6582$  as is the case in CPD. Since our high-resolution spectrum shows that the [N II] lines are well resolved, the observed line flux of [N II]  $\lambda 6548$  was used to compute the expected flux in  $\lambda 6584$ . The ratio  $(I_{6548} + I_{6584})/I_{5755}$  gave an estimate of  $T_e = 8500$  K from the  $[\log T_e, \log N_e]$  diagram for [N II]. The [O II] line ratios used by GD to estimate  $T_e$  may not be reliable, since  $\lambda 3727$  of [O II] is close to the UV cut-off in the response curve of their image tube. The [N II] line ratios of GD are also unreliable because it is uncorrected for blending in  $\lambda 6584$ . Kaler (1986) has derived a relation between  $T_e$  and the ratio of [C III]  $\lambda 1908$  to C II  $\lambda 4267$ . For M4-18, he gives  $T_e = 6400 \pm 500$  K using this method. It is likely that the  $\lambda 1908$  line seen in the *IUE* spectrum of M4-18 is of stellar origin, similar to CPD (Rao et al. 1990). We therefore adopt an initial estimate of  $T_e = 8500$  K for M4-18.

Although GD state that  $N_e$  is well determined from the [S II] line ratio of  $(I_{6731}/I_{6717})$ , the line  $\lambda 6731$  is contaminated by stellar C II  $\lambda 6727.4$  and C III  $\lambda 6731.0$ . Since it is difficult to quantify the extent of contamination, we avoid using [S II] lines for determining  $N_e$ . Therefore, by using Gathier's (1984) expression relating  $N_e$  to observed radio continuum flux at 5 GHz,  $T_e$ , nebular radius and distance, we derive  $N_e = 7600 \text{ cm}^{-3}$ . GD derive  $T_e = 5600$  K and  $N_e = 1.1 \times 10^4 \text{ cm}^{-3}$  from uncorrected line ratios.

### 7.2 Nebular abundance

With our derived values of  $T_e$ ,  $N_e$  and line fluxes, abundances of  $\text{N}^+$ ,  $\text{O}^+$ ,  $\text{O}^{\circ}$  and  $\text{S}^+$  were computed by  $n$ -level statistical equilibrium equations. The resulting abundances are [with  $\log N(\text{H}^+) = 12$ ],  $\log N(\text{N}^+) = 7.6 \pm 0.3$ ,  $\log N(\text{O}^+) = 8.1 \pm 0.1$ ,  $\log N(\text{O}^{\circ}) = 6.9 \pm 0.1$ ,  $\log N(\text{S}^+) = 6.3 \pm 0.1$  and  $\log N(\text{S}^{++}) = 5.2 \pm 0.1$ . For comparison, we recall that in the case of the Sun,  $\log N(\text{N}) = 8.05 \pm 0.04$ ,  $\log N(\text{O}) = 8.93 \pm 0.04$  and  $\log N(\text{S}) = 7.21 \pm 0.06$ . The abundances given by GD [ $\log N(\text{N}^+) = 8.4 \pm 0.3$ ,  $\log N(\text{O}^+) = 9.7 \pm 0.6$ ,  $\log N(\text{O}^{\circ}) = 7.9 \pm 0.4$  and  $\log N(\text{S}^+) = 7.1 \pm 0.3$ ] are different from our values due to the fact that the electron density and colour excess derived by them are abnormal as shown earlier.

### 7.3 Dust in the nebula

The dust model, as envisaged in this paper, pertains to the intermixed dust in the ionized region alone. As for the nature of this dust, amorphous carbon grains were considered, since in CPD, Rao et al. (1990) have shown that the feature around  $2500 \text{ \AA}$  could be explained as being due to amorphous carbon. Roche & Aitken (1986) have also suggested that the dust continuum could be due to amorphous carbon in carbon rich PNe. It is likely that the nature of dust particles is similar in all WC11 objects.

The dust modelling formalism adopted in our code follows, in general, the procedure of Harrington et al. (1988), but restricted to monosized grains. Stellar photons and Ly  $\alpha$ -line photons were assumed to heat the dust. The mean  $Q/a$  values of amorphous carbon (mean of 'BE' and 'XY') from Bussoletti et al. (1987) were used in the wavelength range  $2000 \text{ \AA} \leq \lambda \leq 300 \mu\text{m}$ . For  $\lambda < 2000 \text{ \AA}$  we computed  $Q_{\text{abs}}$  from the optical constants of Rouleau & Martin (1991), using the Mie theory computer program of Shah (1977).

### 7.4 Filling factor

The nebula is compact and relatively unevolved; also the VLA 5-GHz contour map indicates a fully-filled nebula. Therefore the filling factor was set at unity (although our code could accommodate fractional values).

### 7.5 Results

The best matched model parameters are given in Table 3. The mean electron temperature  $\bar{T}_e$  and the mean electron density  $\bar{N}_e$  are 6600 K and  $6050 \text{ cm}^{-3}$ , respectively. The ionization structure of H and He are shown in Fig. 6. Fig. 7

Table 3. Parameters of the final model for M4-18.

Parameter	Value
<i>CSPN (BB)</i>	
$T_{eff}$	22 500 K
Radius	$0.934 R_{\odot}$
<i>Nebula</i>	
Const. density	$N_H = 6 620 \text{ cm}^{-3}$
Abundance	
	H He C N
	12.00 10.99 8.56 8.079
	O Ne S Ar
	9.23 8.09 6.00 6.56
Ang. radius	2 arc sec
Distance	0.9 kpc
Filling factor	1
<i>Dust (Amorphous Carbon)</i>	
$m_d/m_H$	0.1 (inner rad. - 0.0025 r)
	$8.0d-2$ (0.0025 r - 0.025 r)
	$5.5d-2$ (0.025 r - 0.13 r)
	$2.0d-4$ (0.13 r - outer rad.)
Size distribution	Single size
Size	$0.02 \mu m$

shows the same for prominent ions of other elements. Table 4 gives the model and observed line fluxes. The model flux in [S III]  $\lambda 9069$  was matched with observation by reducing sulphur abundance. For the [Ne II] line at  $12.8 \mu m$ , the agreement between model and observation is good, although we did not change the original abundance which was taken as solar. The model predicts a weak C II  $\lambda 2326$  line, and the observed flux in this line must be mostly stellar. [O II]  $\lambda 2471.1$  almost agrees with the upper limit derived from our *new IUE* spectra. Within the error limits of observation the agreement between model and observed fluxes is fairly good. The continuum fluxes from UV to far-IR of the model and observation are plotted in Fig. 8. The *B* and *V* fluxes of Shaw & Kaler (1985) are also dereddened and plotted. The nebular abundances are nearly solar (Table 3) except oxygen, which is higher by a factor of two, and sulphur, which is lower by a factor of  $\sim 10$ , compared with the Sun. If we assume that the abundances of  $N^+$ ,  $O^+$  and  $S^+$  obtained by the line-ratio method represent N, S, and O (since they are mostly present in singly-ionized state throughout the nebula (Fig. 7)), then these are in fairly good agreement with model values but with a large difference in oxygen. When we recalculated the abundances with the model  $\bar{T}_e$  of 6600 K,  $\log N(O^+)$  is  $9.03 \pm 0.1$  for  $\bar{N}_e = 6050 \text{ cm}^{-3}$ , in agreement with the model. We do find an overabundance of oxygen, although not to the extent claimed by GD. As a check we ran a model with  $T_*$ ,  $R_*$  and abundances obtained by GD for a nebula without dust. This model produced  $\bar{T}_e = 5550 \text{ K}$  and  $\bar{N}_e = 8825 \text{ cm}^{-3}$ , close to their quoted value. However, it failed in the following respects: (i) the model stellar luminosity was very high i.e.,  $2588 L_{\odot}$ ; (ii) the absolute  $H\beta$  flux was  $\sim 1.5 \times 10^2$  times what was observed; (iii) the model could not reproduce the observed line fluxes of [O II] 3727 and 3729 Å, and produced very high fluxes in [S III] 9069 and 9532 Å. We ran a model with the same distribution of dust as our model. This also failed in many respects *vis-à-vis* obser-

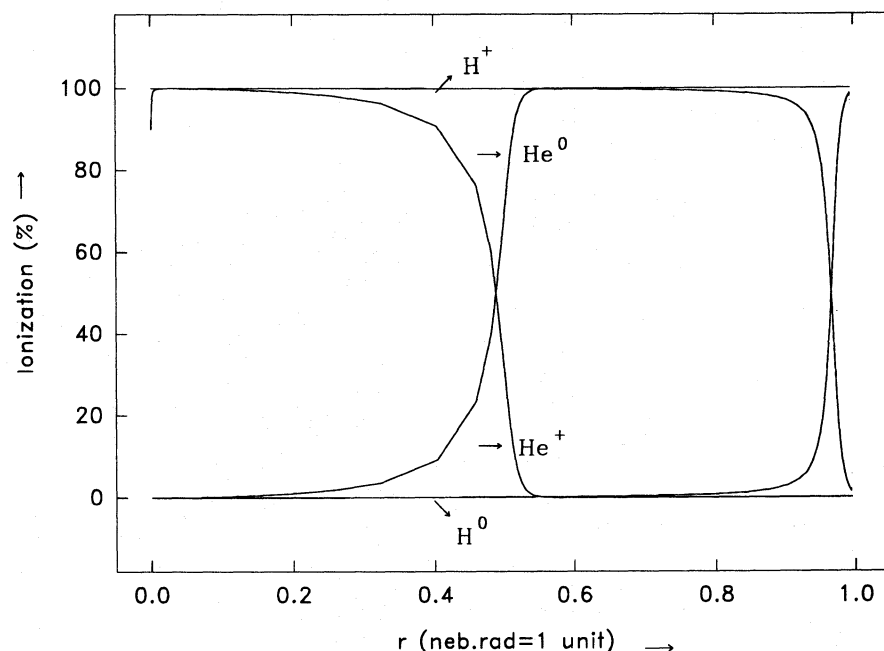


Figure 6. Ionization structure of H and He in M4-18.

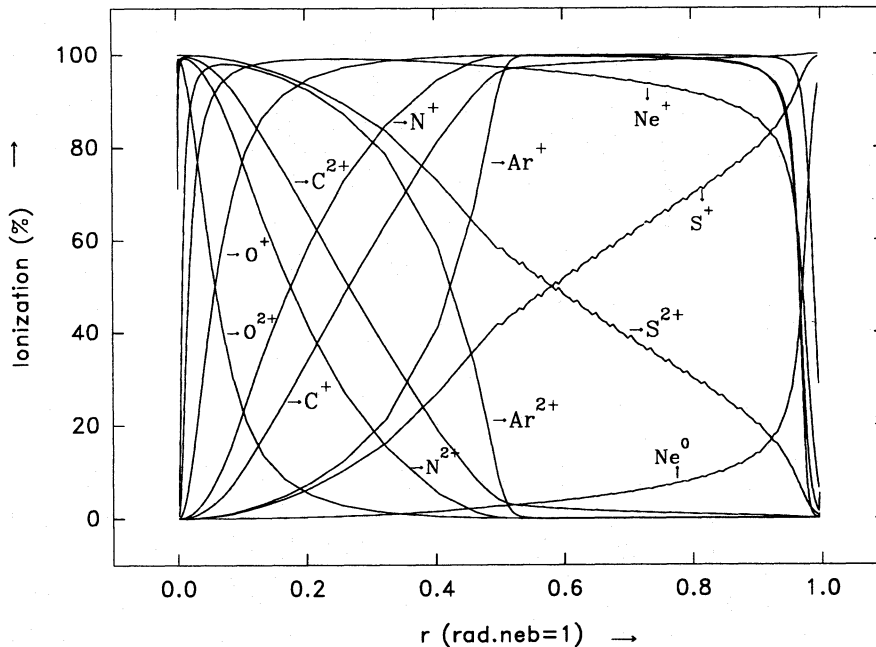


Figure 7. Ionization structure for some of the dominant ions of heavy elements in M4-18.

vations. We checked the sensitivity of our model results for variations in central star temperature, radius and distance of the nebula. The ‘spread’ in  $T_*$ ,  $R_*$  and  $d$  are generally less than  $\pm 200$  K,  $\pm 0.1 R_\odot$  and  $\pm 0.1$  kpc respectively.

### 7.6 Dust radiation

Table 5 gives the model and observed fluxes in the range 1–10  $\mu\text{m}$  and in the *IRAS* bands. The frequencies represent Gaussian abscissae points and the fluxes in the last column are the observed *J*, *H*, *K*, *L* (from ZK) and 10.5- $\mu\text{m}$  (from AR) fluxes given against the closest wavelength of column 1. The model fluxes have been convolved with the system response function of the *IRAS* bands by adopting a procedure similar to Harrington et al. (1988) and the flux densities are compared. Initial models had constant  $m_d/m_H$  ratios across the nebula but the resultant IR fluxes did not compare well with observations for various sizes of the grain. We therefore envisaged a step function for the variation of  $m_d/m_H$  across the nebula as given in Table 3. The best-fitting grain size was found to be 0.02  $\mu\text{m}$ . Looking at Table 5, it is clear that this model could match the *IRAS* band fluxes satisfactorily, except for the 60- $\mu\text{m}$  band; but emission in the 1–10  $\mu\text{m}$  region is rather low compared with what is observed (see Fig. 9). The model produces 51.5  $L_\odot$  in the IR (1–300  $\mu\text{m}$ ) compared with the observed 97.1  $d^2 L_\odot$  (i.e. 78.6  $L_\odot$  for  $d=0.9$  kpc). The deficit of 27.1  $L_\odot$  is not accounted for by the present model. Ly  $\alpha$  emission as per the model is 3.34  $L_\odot$ . Thus, Ly  $\alpha$  makes only a small contribution to the heating of dust. The stellar luminosity from the model is 201.64  $L_\odot$ . This is higher than what is observed (132.92  $L_\odot$  for 0.9 kpc). The observed absolute H $\beta$  flux indicates a luminosity of 127  $L_\odot$  for a 22 500 K blackbody. Our model nebula could absorb less than 180  $L_\odot$ , leaking out about 21  $L_\odot$ . Fig. 10 shows the variation of grain temperature with position. We experimented with larger grains and

higher dust temperatures, and found that if the dust temperature at the inner edge of the nebula was allowed to reach  $\sim 3000$  K, then for a 0.04  $\mu\text{m}$  grain and specific distribution of  $m_d/m_h$  along  $r$ , we could obtain a satisfactory fit with the observed fluxes in the 1–10  $\mu\text{m}$  region as well as *IRAS* bands. This model had a central star luminosity of about 172  $L_\odot$ . It emitted 50  $L_\odot$  in the IR and produced the observed H $\beta$  flux. But grains will sublimate at those high temperatures and will not remain stable. Very small grains (0.005  $\mu\text{m}$ ) have a  $Q_{\text{abs}}$  value (computed using Mie theory) peaking at wavelengths shorter than 912  $\text{\AA}$ , and this causes skewed absorption by gas and results in insufficient ionization. Our choice of grain size was also guided by the experimental extinction curve (in the region of 500 to 1500  $\text{\AA}$ ) given by Colangeli et al. (1993, see Fig. 2). At around 0.02 to 0.04  $\mu\text{m}$ , our computed  $Q_{\text{abs}}$  values mimic the trend of the aforementioned experimental curve.

ZK try to explain the IR radiation in M4-18 in terms of a mixture of SiC and graphite. They needed a fraction of 0.21 of SiC and 0.79 of graphite. The feature seen by AR at 11.25  $\mu\text{m}$  was attributed by them to SiC. They also found a 8.65- $\mu\text{m}$  feature and a rise towards a 7.7- $\mu\text{m}$  feature in their spectrum. In their subsequent paper (Roche & Aitken 1986) they suggested that the continuum emission could be due to amorphous carbon grains. On the other hand, ZK attributed the emission in 1–10  $\mu\text{m}$  region as free-free radio continuum. This latter interpretation may not be valid on the ground that the free-free radio emission (extrapolated to the IR region) is two orders of magnitude lower than the observed value. Also, it may be noted that their extracted *IRAS* low-resolution spectrum in the 8–13  $\mu\text{m}$  region is very noisy and has a poorer S/N ratio than the observations of AR.

The present calculations do not take into account effects of scattering, grain growth, destruction etc. A fully-fledged explanation of IR radiation must take into account all of these aspects and modelling with a distribution of grain sizes may have to be explored.

**Table 4.** Model and observed line fluxes relative to  $H\alpha = 100$  units.

Ion	Wavelength $\text{\AA}$	Model flux ratio	Obsd. flux ratio
C I	8727	0.01	-
"	9823	0.03	-
"	9850	0.10	-
C II	1335	0.30	-
C II]	2326	2.99	$\leq 19.5^*$
C III	2297	0.16	-
[N II]	5755	0.29	0.62
"	6548	18.00	16.20
"	6584	52.99	55.40
[O I]	6300	2.95	0.74
"	6363	0.97	0.11
[O II]	3727	62.74 ]	93.60
"	3729	24.60 ]	
"	7321.8	0.50 ]	2.70
"	7322.4	1.52 ]	
"	7332.2	0.83 ]	1.92
"	7332.8	0.82 ]	
"	2471.0	0.63 ]	$\leq 2.70^*$
"	2471.1	2.43 ]	
[O III]	4363	0.00	-
"	4959	0.13	-
"	5007	0.39	-
[Ne II]	128000	22.5	24.40
[S II]	4068	0.18	-
"	4076	0.06	-
"	6717	0.48	3.32
"	6731	0.91	6.34*
[S III]	9069	0.40	0.40
"	9532	0.98	-
"	187000	0.63	-
[Ar III]	7135	0.30	-
		( <i>Ergs cm<sup>-2</sup> sec<sup>-1</sup></i> )	
Abs. $H\beta$ flux		5.70d-12	5.97d-12
Abs. $H\alpha$ flux		1.68d-11	1.47d-11

\*Blended with stellar line.

## 7.7 Discussion

### 7.7.1 Evolutionary status

The luminosity of the model central star is given by  $\log(L/L_{\odot})=2.30$ , which is lower than the luminosity expected from the evolutionary track for  $0.545 M_{\odot}$  star (Schönberner & Weidemann 1981). Initially, the stellar radius was estimated as  $2.5 R_{\odot}$  on the basis of the  $(\log L, \log T_e)$  diagram. This gave rise to a much higher luminosity. To achieve agreement with the observed nebular spectrum and dust emission, we require  $R_*$  equal to  $0.934 R_{\odot}$  and  $d$  equal to  $0.9$  kpc. The value for  $T_{*BB}$  is consistent with the excita-

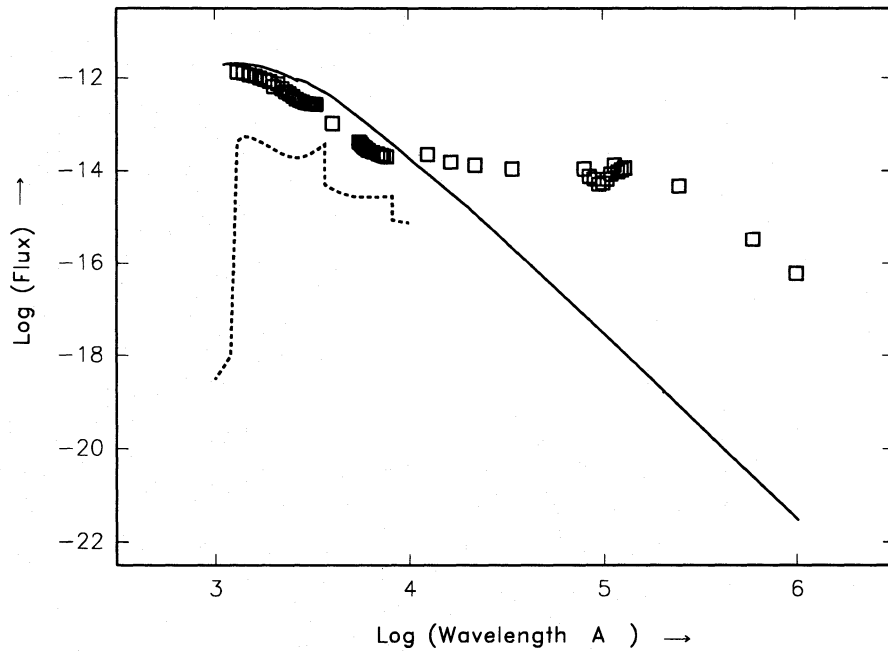
**Table 5.** Dust radiation in the infrared region.

Wavelength $\mu\text{m}$	Frequency Hz	Model flux ( <i>Ergs/cm<sup>2</sup>/sec/Hz</i> )	Obs. flux
1.063	2.8200D+14	1.74D-26	-
1.123	2.6701D+14	2.61D-26	-
1.242	2.4142D+14	5.16D-26	7.92D-25
1.445	2.0752D+14	1.25D-25	-
1.781	1.6831D+14	3.48D-25	1.10D-24 <sup>1</sup>
2.355	1.2729D+14	1.06D-24	1.83D-24 <sup>2</sup>
3.404	8.8078D+13	3.45D-24	3.96D-24
5.534	5.4177D+13	1.19D-23	-
10.484	2.8594D+13	4.16D-23	2.43D-23
<b>IRAS band fluxes (Jy)</b>			
12	-	6.29	5.21
25	-	8.35	8.97
60	-	7.40	3.49
100	-	1.82	1.97

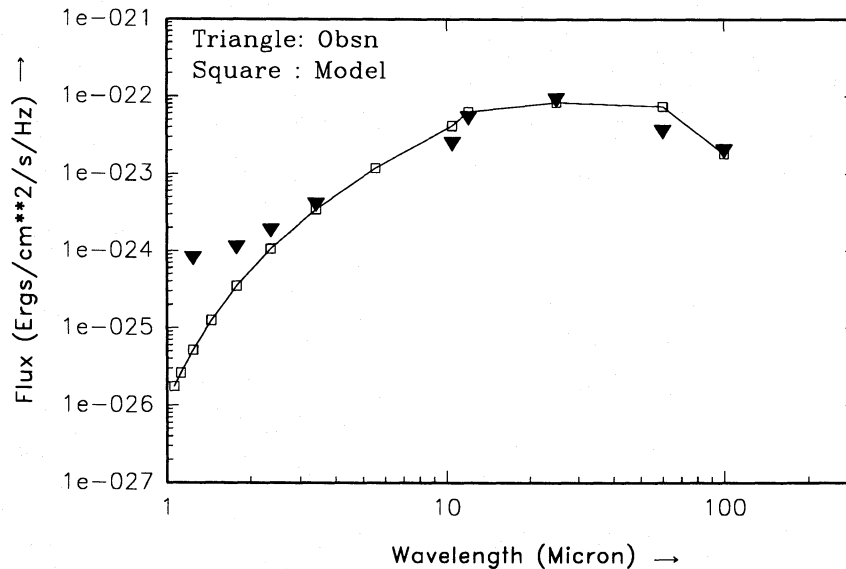
<sup>1</sup>Flux at  $1.65 \mu\text{m}$ <sup>2</sup>Flux at  $2.20 \mu\text{m}$ .

tion conditions in the nebula. Even then, the model stellar luminosity of  $201.64 L_{\odot}$  is higher than the estimated value ( $132.92 L_{\odot}$ ) for  $d=0.9$  kpc. Fig. 8 shows the model continuum (UV and optical) along with what is observed. Assuming uniform expansion, the nebular  $V_{\text{exp}}$  of  $7.5 \text{ km s}^{-1}$  (Sabbadin, Bianchini & Hamzaoglu 1984) gives an age of  $\sim 1300$  yr. Thus, M4-18 is a protoplanetary nebula (PPN). Pollacco et al. (1992) recently discovered photometric variations ( $\Delta V=1$ ) in CPD (compared to a difference of several magnitudes in V348 Sgr). Lawson & Jones (1992) confirmed this from their visual observations and suggest that the general nature of the fade and recovery is similar to the declines of RCB stars. If we consider the angular diameter there is a progression of age from CPD (1.3 arcsec) through M4-18 (4 arcsec) to V348 Sgr (30 arcsec). We find a decrease in the degree of excitation of the emission-line spectrum as we go along the sequence He 2-113, CPD, M4-18 and V348 Sgr, which is indicative of a change in  $T_{*BB}$  from higher to lower values. Since CPD exhibits continuum variability (Rao et al. 1990), M4-18 may also show a similar phenomenon. It is not clear in what way these objects differ from other PPN. In a normal PPN, the cooler the star, the more dense (both particle and electron) is the nebula which expands with age. On the other hand, the WC11 group show that objects with cooler central stars have lower  $N_e$  (and may be  $N_H$ ), but larger nebulae.

The dust in M4-18 absorbs more than  $\sim 60$  per cent of the stellar radiation whereas in the case of CPD it is more than 80 per cent. A proper understanding of the nature, distribution and heating mechanism of dust is essential to fully delineate the nature of the central star. In the case of 07027-7934 the stellar optical flux is  $\sim$  seven times lower than the IR flux (Menzies & Wolstencroft 1990). At present the nature of these WC11 stars and their relationship to RCB stars is uncertain. M4-18 seems to possess properties intermediate between CPD and V348 Sgr.



**Figure 8.** Continuum radiation of model and observation of M4-18. Solid line: stellar continuum. Squares: observed continuum from UV to IR. Dotted line: nebular continuum.



**Figure 9.** IR fluxes from model and observation.

### 7.7.2 Conclusions

Our analysis of M4-18 brings out several significant points.

(i) The nebular emission-line spectrum can be well reproduced, with  $T_{*BB} = 22\,500$  K,  $R_* = 0.934 R_\odot$ ,  $d = 0.9$  kpc and a constant density  $n_H = 6620$  cm $^{-3}$ .

(ii) For cool central stars, the dust absorbs mostly stellar continuum radiation and Ly  $\alpha$  heating is very small. The

thermal and ionization structure of the nebula is not drastically affected by well-mixed dust until  $m_d/m_H$  is  $\sim 10^{-2}$  or higher.

(iii) The dust in M4-18 is likely to be amorphous carbon.

(iv) The properties of M4-18 are similar to CPD and V348 Sgr.

Detailed modelling based on high-resolution spectra and proper treatment of the dust is needed for more WC11 objects to clarify many uncertain aspects.

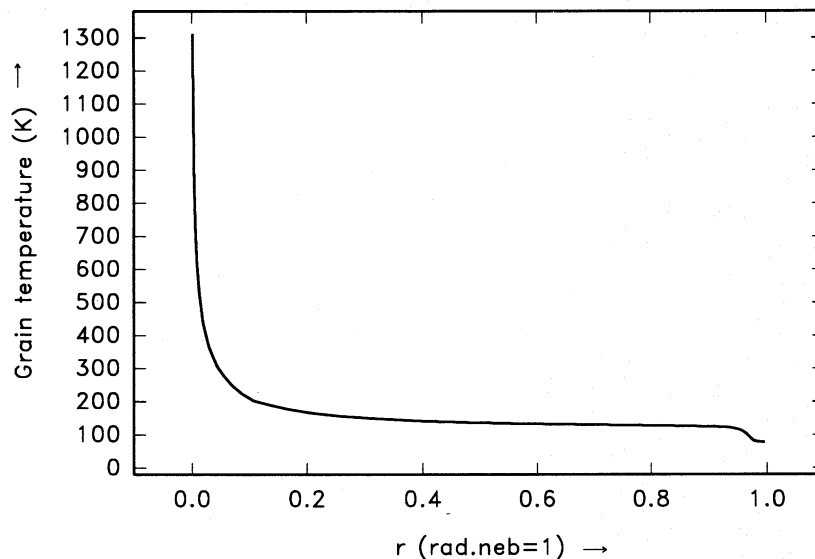


Figure 10. Grain (amorphous carbon,  $0.02 \mu\text{m}$ ) temperature shown as a function of position across the nebula.

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