TORSION, MINIMUM TIME, STRING TENSION AND ITS PHYSICAL IMPLICATIONS IN COSMOLOGY

(Letter to the Editor)

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(Received 18 June, 1991)

Abstract. We show that introducing torsion in general relativity, that is, physically, considering the effect of the spin, and linking the torsion to defects in the space-time topology we can have a minimum unit of time. In this context we have the possibility of identifying the defects in space-time topology induced by torsion as behaving like a string so that the minimum length, derived by treating the spin as an extra dimension, is related with string tension. Physical implications are considered for field theory (we can eliminate the divergence of self-energy integral without introducing any *ad hoc* cut-off), particle decay, evaporation of black holes, and information theory.

1. Introduction

In Newtonian mechanics, time and space are distinct entities since fields propagate at infinite speeds. In special relativity the finite speed of light or electromagnetic signals interlinks space and time. This gives a geometrical role for velocity of light as it is now connected to the topologically invariant signature and dimensionality of space, i.e., we have now an invariant space-time interval rather than a purely spatial interval between neighbouring events. The topologically invariant interval also carries over into general relativity (i.e., when gravitational forces are present). This also has the consequences of time dilatation and redshifting of frequencies in gravitational fields showing that time is affected by gravity.

Although this interconnection of space and time (with or without curvature!) represents considerable conceptual progress, the problems with the singular behaviour of vanishing spatial and temporal coordinates of point particles (manifested, for example, by divergences in self-energy) still persist.

In cosmology the notion of zero time associated in Big-Bang models with the instantaneous creation of matter is taken as an indication of the occurrence of an inevitable singularity suggesting a breakdown of the concepts involved.

In quantum mechanics, time is linked to energy via the uncertainty principle. Here the Planck's constant $\hbar$ (which has the units of energy $\times$ time!) plays a very fundamental role. This suggests that it might also be involved in a basic way in the concept of time at microscopical scales. Similar to a geometrical role for 'c' in special and general relativity as defining a topologically invariant interval we can think of a geometrical role

Astrophysics and Space Science **187**: 149–154, 1992.

for '$\hbar$' in the structure of space-time at very small scales. In this context we note that '$\hbar$' enters into quantum mechanics by virtue of being the basic unit of intrinsic spin and, therefore, its interaction with the underlying geometry must necessarily give rise to torsion. Since '$\hbar$' is 'energy × time', and energy gives rise to space curvature, it is suggestive that time may be linked to torsion as '$\hbar$' is also the unit of intrinsic spin which is the source for torsion.

2. Torsion, Space-Time Defects, and Minimum Time

We can directly connect this with recent works (de Sabbata and Sivaram, 1991a, b) it was suggested that torsion gives rise to defects in space-time topology. We know that in the geometrical description of crystal dislocations (in three spatial dimensions) and defects (Bilby *et al.*, 1955; Gunther, 1980, 1981), torsion plays the role of defect density (in this context we consider space-time as an elastic deformable medium in the sense of Sakharov, 1968). If we consider a small closed circuit and write

$$l^\alpha = \oint Q^\alpha_{\beta\gamma}\, \mathrm{d}A^{\beta\gamma}\,, \tag{1}$$

where $\mathrm{d}A^{\beta\gamma} = \mathrm{d}x^\beta \wedge \mathrm{d}x^\gamma$ is the area element enclosed by the loop and

$$Q^\alpha_{\beta\gamma} = \Gamma^\alpha_{[\beta\gamma]}\,, \tag{2}$$

is as usual the torsion associated with the connection $\Gamma^\alpha_{\beta\gamma}$, then l^α represents the closure failure, i.e., torsion has the intrinsic geometric meaning of the failure of the loop to close, analogous to the crystal case, l^α having the dimensions of length. In the above equation, torsion can be related to the fundamental unit of intrinsic spin $\hbar$, by postulating that defects in space-time topology at the quantum level should occur in multiples of the Planck length $(\hbar G/c^3)^{1/2}$, i.e., we have

$$\oint Q\, \mathrm{d}A \cong n(\hbar G/c^3)^{1/2}\,, \tag{3}$$

n is an integer. Note that Equation (3) can be seen to follow from the relation between the torsion tensor Q^k_{ij} and spin density tensor J^k_{ij} (which acts as the source term for torsion, analogous to matter density being source term for curvature) as

$$Q^k_{ij} \Rightarrow (G/c^3) J^k_{ij}\,, \tag{4}$$

i.e., $\overline{Q} \Rightarrow (G/c^3)\overline{J}$ (remembering that the Lagrangian contains only the axial-vector part of the torsion tensor), and if J is quantized in units of $\hbar$, Q has dimension of $(\text{length})^{-1}$, as J is $\hbar$/volume, and $(\hbar G/c^3)$ is $(\text{length})^2$. Thus Equation (3) follows once we assumed quantized spin and the Einstein–Cartan equations.

As regards the fourth component of (1), we have for temporal component:

$$t = (1/c) \oint Q\, \mathrm{d}A = n(\hbar G/c^5)^{1/2}\,, \tag{5}$$

which shows that time is defined at the quantum geometrical level through torsion, so that torsion is essential to have a minimum unit of time $\neq 0$, i.e., when $Q \Rightarrow 0$, $t \Rightarrow 0$.

In fact, this would give us the smallest definable unit of time as $(\hbar G/c^5)^{1/2} \cong 10^{-43}$ s. In the limit of $\hbar \Rightarrow 0$ (classical geometry of general relativity) or $c \Rightarrow \infty$ (Newtonian case), we would recover the unphysical $t \Rightarrow 0$ of classical cosmology. So both $\hbar$ and c must be finite to give a geometric unit for time (i.e., in this context $\hbar \Rightarrow 0$ and $c \Rightarrow \infty$ are equivalent). The fact that $\hbar$ is related to a quantized time-like vector discretizes time.

3. Relation with String Tension and Minimum Length in String Theory

Currently there is a lot of interest in the existence of a minimum physical length ($\approx L_{\mathrm{Pl}}$) in string theories (Greensite, 1991; Konishi *et al.*, 1990) minimum length which seems inbuilt in string theories is related to the inverse square root of the string tension μ as

$$l_s \propto 1/\sqrt{\mu}\,, \tag{6}$$

where

$$\mu = c^2/G = 1.6 \times 10^{28}\ \mathrm{g\,cm^{-1}} \tag{7}$$

is the superstring tension at the Planck scale. More specifically,

$$l_s = (\hbar/\mu c)^{1/2}\,. \tag{8}$$

We can possibly understand the connection between the minimum length in string theory (Equation (8)) and that given by torsion (Equation (3)), when (as was shown in Greensite, 1991) the spin-torsion interaction is regarded as the basis for the string tension and its scaling with energy. The spin-torsion interaction energy is $\overline{Q} \cdot \overline{S}$ (where $\overline{S}$ has the magnitude $\hbar/2$) with Q as given by Equation (4).

For defects characterized by a scale given by Equation (3) this spin-torsion interaction energy then implies a mass per unit length, i.e., string tension given precisely by Equation (7). So in this picture we have the possibility of identifying the defects in space-time topology induced by torsion as behaving like string characterized by a tension $\approx c^2/G$. In many string models, there is a minimum compactification radius (Konishi *et al.*, 1991) which is $\approx L_{\mathrm{Pl}}$ at which conformal or scale invariance is broken by condensation of vortices. Here space-time defects break scale invariance.

By treating spin as an extra dimension as done in de Sabbata and Sivaram (1991c) the compactification radius becomes precisely L_{Pl}. The origin of gravity, i.e., field described by a Hilbert-type of action, is related to the breaking of scale invariance at L_{Pl}. This fixes the Newtonian constant G_{N}. In fact, the Hilbert action describing the usual macroscopic properties of gravity is not scale invariant and is characterized by the dimensional Newtonian constant G_{N}. Then scale invariance must be broken at L_{Pl} to obtain such a term with a coupling G_{N}. Thus the Hilbert action for gravity can be induced by scale invariance being broken by defects in space-time topology due to torsion associated with quantized intrinsic spin. Equivalently the breaking of scale invariance induces the string tension (given by Equation (7)) giving rise to gravity and

G_N from strings. In this sense the torsion-induced defects in space-time are the more basic entities.

Also the Hagedorn limiting temperature gives a thermally-induced breaking of conformal invariance which again fixes the scale given by Equation (8) consistent with above arguments.

In the cosmological context it may be relevant to note that the 'magnetic field' associated with torsion (de Sabbata and Gasperini, 1982) given as $B \propto (G)^{1/2}(\sigma/c)$ (where σ is the spin density) break time reversal invariance by establishing a definite direction orienting signal propagation (analogous to the Faraday effect). This would give an arrow of time in cosmology when the curvature associated with the defect ($\approx L_{\text{Pl}}^{-2}$) initially caused exponential expansion with constant density (de Sabbata and Sivaram, 1991b). (Alternatively one can obtain the string tension in term of this field, as $\mu \approx (B^2/c^2)(L_{\text{Pl}})^2$, analogous to the treatment of string in QCD.)

For this inflationary expansion in the early universe we can write

$$t = H_{\text{Pl}}^{-1} \ln R \,, \tag{9}$$

in terms of the scale factors. H_{Pl} is just the inverse of the Planck time, i.e., the 'Hubble constant' characterizing this expansion. We could draw the analogy with the Boltzmann entropy relation

$$S = k_B \ln w \,, \tag{10}$$

k_B in Equation (10) is the minimum unit of entropy analogous to H_{Pl}^{-1} in Equation (9), being the minimum time unit. This would have important consequences for cosmology in the sense that neither 'cosmic times' earlier than t_{Pl} nor cosmic-scale factors that are smaller than L_{Pl} have any meaning putting severe constraints on models like chaotic inflation.

4. Physical Implications

The quantum of time also correspondingly implies a limiting frequency of $f_{\text{max}} \approx (c^5/\hbar G)^{1/2}$. This would have consequences even for perturbative QED, in estimating self-energies of electrons and other particles, i.e., the self-energy integral (in momentum space) taken over the momenta of all virtual photons. To make the integral converge Feynman in his paper on QED (Dirac, 1988) multiplied the photon propagator k^{-2} by the *ad hoc* factor:

$$-f^2/(k^2 - f^2) \,,$$

where k is the frequency (momentum) of the virtual photon. This convergence factor although it preserves relativistic invariance is *ad hoc* without any theoretical justification. Feynman considers f to be arbitrarily large without any theoretical basis. Here the presence of space-time defects associated with the torsion due to the intrinsic spin would give a natural basis for $f_{\text{max}}^2 \approx c^5/G\hbar \approx 10^{96}$ (from Equation (5) and extremely large as required by Feynman), giving finite result (not ∞!) for the self-energy. This makes f_{max}

another fundamental constant for particle physics serving as a high-frequency cut-off which is *not* arbitrary.

This also has implications for β-decay: the decay time in weak interaction β-decay scales inversely as M^5, M being the mass of the particle. Thus we have

$$M_\mu^5/M_\tau^5 = t_\tau/t_\mu \,, \tag{11}$$

where t_τ, t_μ are lifetimes for muon and τ-particle, respectively. So the existence of a minimum lifetime of 10^{-43} s would imply an upper limit to the mass of any lepton decaying by β-decay (lepton by definition can take part in only β-decay). So if $t_\mu \approx 10^{-6}$ s and $t_{\min} \approx 10^{-43}$ s, we can have an upper limit to lepton mass as

$$M_{\max} \approx M_\mu (t_\mu/t_{\min})^{1/5} \approx 10^3 \text{ TeV} \,. \tag{12}$$

Since this would be within the energy range of the next generation of accelerators (currently ≈ 40 TeV), this can be tested experimentally.

Similar limits can exist for maximum mass of pseudo-scalar meson undergoing decays like $\pi^0 \Rightarrow 2\gamma$. Here the scaling is like M^{-3}. π^0 has a decay time of $\approx 10^{-16}$ s. This, in turn, would give maximum energy of gamma photons ($\approx 10^5$ TeV) in decay with possible implications for high-energy phenomena in cosmic-ray astrophysics.

Of course implications for black-hole evaporation would suggest that black hole $< M_{\rm Pl}$ cannot decay and so must be stable. Again, as seen by an external observer, a black hole would take a finite time $(R_g/c) \ln (R_g/ct_{\rm Pl})$ to form and not ∞ as in the classical singular case.

Moreover, there could be consequences for everyday life! For instance no computer can ever process information faster than 10^{43} bits s^{-1}. In general, the theoretical limit given by Brillouin (1962) is temperature-dependent (and can be also ∞ for 0 temperature!), i.e., the maximum thermodynamically allowed processing rate is

$$I_{\max} \cong P/(k_B T \ln 2) \text{ bits s}^{-1} \,, \tag{13}$$

for $P = 1$ W at $T \cong 300$ K, room temperature, we have $I_{\max} \approx 10^{20}$ bits s^{-1}. The above absolute upper limit of 10^{43} bits s^{-1} is *independent* of temperature and of power expended, purely arising from the existence of a minimum time unit as implied by Equation (5).

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