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Dissipative Collapse of a Spherical Cluster of Gas Clouds

K. Indulekha[1], G.V. Vijayagovindan[1], S. Ramadurai[2,*]
[1]*School of Pure and Applied Physics, Mahatma Gandhi Unviersity, Kottayam 686 560, India*
[2]*Tata Institute of Fundamental Research, Homi Bhabha Road, Colaba, Mumbai 400 005, India*

Abstract. We investigate the time scale for the dissipative collapse of a spherical cluster of gas clouds by supplementing the scalar virial equation with an evolution equation for the energy. We find that collapse times are more than doubled even for low filling factors $f \sim 10^{-2} - 10^{-3}$, for which support by supernovae, usually considered for galactic structure formation models is ineffective.

Keywords : Galaxies; formation.

1. Introduction

Two component models for galaxies envisage longer than free fall time scales for the formation of the structures in the Galaxy (see [5] for a discussion). We examine the dissipative evolution of a spherical cluster of gas clouds with an isotropic velocity distribution using the scalar virial equation supplemented with an evolution equation for the energy, to obtain the time for collapse impeded by virialized mass motions. For a detailed discussion see [4]. The gravitational binding energy released during collapse feeds the random where the influence of dark matter can be ignored [9], our model shows that two component galaxies could have taken longer than a free fall time to collapse, even for low filling factors $\sim 10^{-2} - 10^{03}$, for which support from supernovae, which is usually considered, is less effective.

2. The Model

We consider a spherically symmetric cluster of mass M and radius R, consisting of N individual, equal mass clouds of radius R_c and mass M_c distributed uniformaly. The clouds have a one dimensional r.m.s. velocity v_{rms}. The gas clouds are in pressure balance with an intercloud medium.

*e-mail: durai@tifr.res.in

The mass in the intercloud medium is taken to be too small to affect the dynamics and the contribution from it to the virial as well as the dissipation will be ignored throughout [6]. The scalar virial equation for the system is [1] $\frac{1}{2}\frac{d^2I}{dt^2} = 2T + \Omega$ where $I = 4\pi \int \rho(r) r^4 dr$ is like the moment of inertia of the system about the centre and $\rho(r)$ is the density at a distance r from the origin. T is the kinetic energy associated with mass motions, and Ω is the self gravitational potential energy of the cluster. We may use the expression for I and Ω corresponding to a homogeneous distribution of matter. See [4] for a discussion. The velocity of the clouds may be separated into two parts, a random part and a mean motion for homologous collapse with respective kinetic energies $\frac{3}{2}Mv_{rms}^2$ and $\frac{3}{10}M(\frac{dR}{dt})^2$. Introducing all these the virial equation becomes

$$\frac{d^2R}{dt^2} = \frac{5v_{rms}^2}{R} - \frac{GM}{R^2}. \tag{1}$$

The total energy of the system evolves due to cloud collisions at a rate $\frac{dE}{dt}|_{\text{dissipative}} = -\nu\frac{Mv_{rms}^3}{R}$. The parameter $\nu N^{1/3} f^{2/3}$ is proportional to the number of collisions in a free fall time, for virial theorem random motions in the system. With $E = T_R + T_M + \Omega$ and $\frac{dE}{dt} = \frac{dE}{dt}|_{\text{dissipative}}$ we get the evolution equation for the energy in the form

$$\frac{dv_{rms}^2}{dt} = \frac{-2v_{rms}^2}{dt} - \nu\frac{2v_{rms}^3}{3R}. \tag{2}$$

We consider three different cases for the filling factor f. One is to keep f constant as the collapse proceeds. We also consider two cases of varying ν as follows. For clouds at a constant temperature T_c, in pressure equillibrium with an intercloude medium at the virial temperature ($= \frac{GMm_H}{Rk}$) of the cluster, $f = (1 + \frac{GM_{ic}m_H}{RkT_c})^{-1} \approx \frac{RkT_c}{GM_{ic}m_H}$. Here M_{ic} is the mass in the interclump medium, k is the Boltzmann constant, and m_p is the proton mass. For T_c a constant ν which is proportional to $f^{2/3}$, decreases as $R^{2/3}$ as the collapse proceeds. For clouds which keep a constant radius, ν proportional to R^{-2}, increases as the collapse proceeds.

For $v_{rms}^2 \approx \frac{GM}{R_0}$ corresponding to virial equilibrium, the collision time t_c is $\sim 1/\nu$ in units of the free fall time $t_{ff} \sim \sqrt{(\frac{R_0^3}{GM})}$, where R_0 is the initial radius. For dissipation by collisions, the dissipation time is also seen to be $\sim 1/\nu$. For the system to be collisional the mean free path of the clouds should be less than twice the radius of the cloud. This yields the condition $N^{1/3} f^{2/3} > 1/6$.

3. Results and Discussion

The evolution of the cluster has been considered under the conditions discussed below. We define $K = v_{rms}^2(t = 0) = \frac{2}{5}$ the initial random kinetic energy/the modulus of the initial potential energy. For a system starting from virial equilibrium $K = 0.2$ initially. For a constant filling factor f and hence constant $\nu^{-1} = t_c$, we consider the cases (a1) with $K = 0.2$, (a2) with $K = 0.15$, and (a3) with $K = 0.1$. Case (b) has $\nu = \nu_0 R^{2/3}$, and case (c) has $\nu = \nu_0 R^{-2}$. For comparison with earlier works, which examine the evolution of virial theorem random motions under collisional

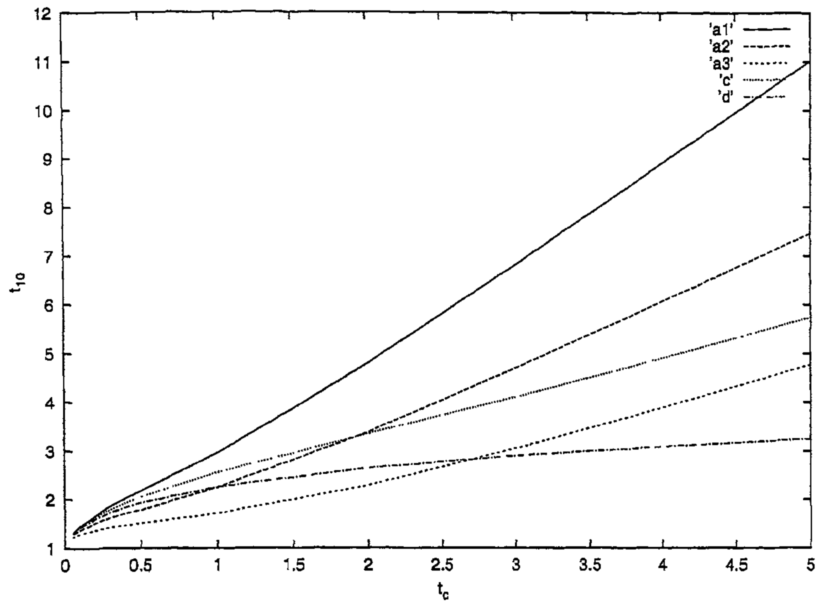

Figure 1. t_{10} as a function of t_c for cases (a1), (a2), (a3), (c) and (d). Both times are in units of t_{ff}.

dissipation, we consider as case (d) the evolution of the cluster starting with $K = 0.2$ but with no gravitational feeding of the random kinetic energy. Equations (1) and (2) were solved for various values of the only free parameter $\nu = 1/t_c$ where t_c is the collision time for virial theorem random motions in units of t_{ff}. (ν_0^{-1} was taken as the free parameter for cases (b) and (c)). The equations were normalized using R_0 as the unit of length and $\sqrt{\frac{R_0^3}{GM}}$ as the unit of time where R_0 is the initial radius of the cluster, and integrated with a constant time step. Desired accuracy was achieved by using a time step which was appropriately small. It was checked that with zero dissipation the change in the total energy was much less than one percent over 10 units of normalized time.

In Fig. 1 we show t_{10} the time for collapse to one-tenth size in units of t_{ff}, as a function of ν^{-1} which we have designated as t_c the collision time, for the cases (a1), (a2), (a3), (c), and (d). The various cases are marked in the figure. For case (c) the curve shown in the figure is for $K = 0.2$, and in this case the values given on the x axis are for $t_{c0} = \nu_0^{-1}$. Case (b) is not shown in the figure. For case (b) we consider t_{c0} increasing as the collapse proceeds, the condition for the system to be collisional would cease to be met within R greater than one-tenth the starting radius. In all the cases including case (b) (see [4]), there was an initial nonlinear rise. This was followed by a linear portion, for collision times greater than the free fall time.

For a clumpy protogalaxy, typically $N \sim 10^4$, whether we consider the top-down or the bottom-up scenario for galaxy formation [2]. From our result we see that collapse times are more than doubled for collision times in the rage 2–5. This implies $f \sim 10^{-2} - 10^{-3}$, which interesting, compares well with that obtained by [7] through a totally different approach. The collapse times we obtain, compare well with the dissipation time obtained for virial theorem random motions, in simulations by [8] (see also [4]), and in the analysis by [3].

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