SOME CONSEQUENCES OF QUADRATIC GRAVITY FOR THE EARLY UNIVERSE

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Abstract. The hypothesis that gravity at very high energies and, hence, around the Planck epoch in the early universe is described by an action quadratic in the curvature, asymptotically free in the coupling is explored. It is shown that the flatness and horizon problems can be resolved in this framework without invoking inflation.

1. Introduction

It is well known that despite several successes of the standard Big-Bang model such as the presence of the microwave background and the prediction of the observed helium abundance there are severe theoretical problems at the earliest epochs.

For example, we have so-called *flatness and horizon problems*. The flatness problem arises from the extrapolation that for the Universe to be within an order of magnitude to the closure density (ρ_c) at the present epoch as implied observationally it ought to have been fine tuned to the closure density to one part in 10^{60} at the Planck epoch, i.e., $(\rho - \rho_c)/\rho$ the relative density difference is a function of the epoch, scaling as T^{-2} (T is the temperature), i.e., proportional to the time t, implying that for $T \approx 10^{19}$ GeV (i.e., at the Planck epoch) this ratio is $< 10^{-60}$. Or in other words, the kinetic energy term $(\dot{R}/R)^2$ and the potential energy term $8\pi G\rho/3$ in the equation $(\dot{R}/R)^2 = 8\pi G\rho/3$ for the expanding scale factor R in the early universe must have been equal and opposite to one part in 10^{60} at the earliest Planck phase, i.e., they must have balanced to an accuracy of some 60 or more decimal places at that epoch.

At $T \approx 1$ s (when helium was being formed) the two terms should have been equal and opposite to about one part in 10^{18}; i.e., to have a spontaneous Big Bang of this very *precise* magnitude the Universe should have started out with a total energy of exactly zero (!) which also implies a density equal to closure density at all epochs.

Another problem is the horizon problem. As is evident from the microwave background the Universe on the largest scales is extremely homogeneous and isotropic (to better than one part in 10^4). However, as is known, standard cosmology has particle horizons. When matter and radiation last interacted vigorously (at $t \approx 10^{13}$ s, $T \approx 0.3$ eV), what was to become the presently observable universe was comprised of $\approx 10^6$ causally distinct regions. The particle horizon at decoupling only subtends an angle of about 0.5 deg on the sky today, so why such uniformity on angular scales $\gg 0.5$ deg.

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The problem arises because the Universe expanded at earliest epochs as $R \sim t^{1/2}$, whereas the horizon expands with light velocity as $\sim t^{1/2}/ct \to \infty$ as $t \to 0$.

For, e.g., at $t \sim 1$ s after the Big Bang there would be $\sim 10^{27}$ causally distinct regions. Still the present universe is isotropic all over. The so-called inflationary universe paradigm was invented (Guth, 1981) to take care of the above problems confronting Big-Bang cosmology at its earliest epochs. This invokes a vacuum-dominated exponential expansion of the Universe at an early phase with a *Hubble constant* $H_{\rm Pl} \approx (8\pi V(0)/3M_{\rm Pl}^2)^{1/2} \approx M_G^2/M_{\rm Pl}$. ($V(0) \approx M_G^4$, M_G is mass scale of scalar field) driving expansion.

A single causally connected region can expand exponentially to give rise to the observed universe thus taking care of the horizon problem. As the curvature term becomes vanishingly small after inflation, we have an explanation of $\Omega = 1$, i.e., the flatness problem.

Of course one could have alternatives to the conventional inflationary scenarios requiring massive scalar fields with very flat potential wells. One such alternative would be the modification of general relativity at Planck scales. We shall consider this in the next section.

2. Gravity at High Energies

One such possibility of modification is to consider the Weyl-type Lagrangian (Equation (1)) for high-energy gravity, i.e., quadratic in curvatures with dimensionless coupling constant, appropriate for a renormalizable theory of gravity in contrast to Einstein's non-renormalizable gravity with dimensional Newtonian constant. The appropriate Lagrangian was considered by Sivaram (1985, 1986a, b, 1990) as

$$I_Q \simeq \alpha_G \int d^4x (C^2 + \beta R^2) , \qquad (1)$$

(α_G, β are dimensionless constants. This was treated as the gravity analogue of the QCD action quadratic in the Yang–Mills field. At the appropriate high-energy scale they describe gravity and strong interactions. At the very earliest phases of the Universe one would expect gravity to be described by such an action which is also scale-invariant. Once the scale invariance is broken at around the Planck mass, the Hilbert term is induced (as discussed in the above works) with a dimensional constant $1/G_N$ and linear in curvature. Note that this is analogous to the emergence of an effective low-energy strong interaction theory of pions from the underlying high-energy gauge theory of QCD, the strong interactions having a global chiral SU(2) × SU(2) symmetry analogous to general coordinates transformations for gravity. The low-energy effective action retains this symmetry, only the scale invariance being broken. In QCD we have analogously the scale $\Lambda_{\rm QCD} \approx 1$ GeV and an action quadratic in the Yang–Mills field. They possess some remarkable properties in common. In QCD, the strong colour interactions between quarks are linear, i.e., the potential $V \propto r$, only systems with zero total colour have finite energy, i.e., leading to confinement of quarks. For the scale-invariant quadratic action, i.e., Equation (1), the potential also grows linearly with distance as the corresponding Poisson equation is

$$\alpha \nabla^4 \phi = km\delta^3(r)\,, \tag{2}$$

with the solution

$$\nabla^{-4} m\delta^3(r) \sim mr \tag{3}$$

(for a point mass m, k being a constant and for the usual general relativistic case $\nabla^2\phi \sim m\delta^3(r)$ giving $\phi - m/r$). Here the field equations are of fourth order. This implies that for scale invariant quadratic gravity only systems with zero total energy have finite energy, i.e., energy is confined analogous to colour in QCD. In fact, it can be shown rigorously that all exact classical solutions of the field equations following from the above action have zero total energy for $\alpha_G \beta > 0$ (Sivaram, 1986a, and references given therein).

3. Some Consequences for the Early Universe

This aspect of the total energy being zero for actions such as Equation (1) would have interesting consequences for the earliest phases of the Universe when gravity would have been described by such equations. Thus the initial state of the Universe would in this picture be a zero-energy configuration thus naturally accounting for the flatness problem (i.e., the equality between kinetic and potential energy terms to $O(10^{-60})$ at the Planck epoch). Thus a flat universe ($\Omega = 1$) is in this picture dictated as initial condition arising from gravity at energies $\approx E_{\rm Pl}$ being described by an action like Equation (1). Configurations with $K \neq 0$ would not be energetically favoured.

As far as the horizon problem is concerned, we note that if during an early epoch (say at $t \approx t_{\rm Pl} \approx 10^{-43}$ s), the scale factor R increased as rapidly or more rapidly than the time t (for, e.g., as $t^{1/2}$ or faster) then the horizon distance $d_H \to \infty$, thus eliminating the horizon problem. The horizon problem thus arises if one persists with the usual GR Lagrangian with a potential $\sim r^{-1}$ at very earliest epochs.

This would imply for the scale factor, a relation $R = at^{1/2}$ (a constant), giving rise to the horizon problem as the horizon expands as $R = ct$.

For the case of the quadratic theory given by Equation (1) with the solution suggested by Equation (3), the equation for the scale factor is modified as

$$\dot{R}^2 = bR \tag{4}$$

(b a constant) giving the corresponding solution for the scale factor of the form

$$R = bt^2 \tag{5}$$

rather than the usual $R \propto t^{1/2}$ solution.

As Equation (5) indicates, R now increases much faster than t so that the horizon problem is eliminated. Thus both the horizon and flatness problems are naturally resolved if one accepts the hypothesis that gravity around the Planck epoch is described by the action (1) rather than the usual Hilbert one of GR.

It may be pointed out that many authors starting from Starobinsky (1980) have discussed inflation in the context of actions of the type

$$I = R + \beta R^2 , \tag{6}$$

i.e., a combination of Hilbert and quadratic terms. In general, these models are equivalent to those invoking GR plus massive scalar fields, there being a general transformation due to Whitt (1984), linking these theories. In the present case, we did not consider the R term, but solely the quadratic term, not in the *ad hoc* sense, but as the appropriate action for gravity at energies $\sim E_{\mathrm{Pl}}$ at the very earliest phases.

As elaborated above this has the consequence that the flatness and horizon problems are taken care of automatically without invoking inflation. Once the Hilbert term is induced as a result of breaking of the scale invariance the Universe expands in the usual Robertson–Walker manner. It is possible to picture an intermediate stage in which both the terms are present (as in Equation (6)) giving rise to a phase giving rise to density perturbations $\sim \beta \sim 10^{-4}$. This would be explored in a subsequent work.

Also we can note that actions of the form $\sim R^n$, with n integer have power-law solutions with scale factor increasing faster than t ($\sim t^n$), thus resolving the horizon problem. But only for the quadratic case, as in Equation (1) we have uniquely the zero-energy solution also accounting for the flatness problem.

For arguments of domination of quadratic over higher-order terms, see Sivaram (1985, 1986a, 1991).

References

Guth, A.: 1981, *Phys. Rev.* **D23**, 347.
Sivaram, C.: 1985, *Bull. Astron. Soc.* **13**, 339.
Sivaram, C.: 1986a, *Astrophys. Space Sci.* **125**, 189.
Sivaram, C.: 1986b, *Int. J. Theor. Phys.* **26**, 1125.
Sivaram, C.: 1988, *Astrophys. Space Sci.* **137**, 603.
Sivaram, C.: 1991, *Int. J. Theor. Phys.* (in press).
Starobinsky, A.: 1980, *Soviet Astron. Letters* **9**, 302.
Whitt, A.: 1984, *Phys. Letters* **145B**, 176.