GEOMETRICAL OPTICS AND DIFFRACTION VIS-À-VIS MIE THEORY OF SCATTERING OF ELECTROMAGNETIC RADIATION BY A SPHERE

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Abstract. The usefulness of the classical Geometrical Optics and Diffraction (GOD) has been illustrated for scattering of electromagnetic radiation by very large dielectric and absorbing spheres. Various scattering parameters such as extinction efficiency, asymmetry parameter, radiation pressure, etc., have been calculated on the basis of GOD and compared with the equivalent results obtained as per the Mie theory. The spheres are assumed to be composed of pure and impure silicate-like or polystyrene material in the visual wavelengths. The representative indices of refraction $m = m' - im''$ are chosen to be $m' = 1.6$ and $m'' = 0.00, 0.05, 0.10, 0.30, 1.00, 2.00$, and 4.00. It is shown that the asymptotic values of a given scattering parameter obtained from the Mie theory calculations agree reasonably well with the corresponding result based on GOD. It is thus possible to estimate the minimum value ($x_{\min}$) of the size-to-wavelength parameter x ($= 2\pi a/\lambda$; a, the radius of the sphere; and λ, the wavelength of the incident radiation), such that, for $x > x_{\min}$, GOD holds good for certain specified accuracy.

1. Introduction

The problem of scattering of electromagnetic radiation by a homogeneous, isotropic and smooth sphere with arbitrary size and index of refraction can be studied rigorously by the Mie theory (see, for example, Mie, 1908; Debye, 1909; Shifrin, 1951; van de Hulst, 1957; Shifrin and Zelmanovich, 1964; Kerker, 1969; Liou, 1980). However, in many applications, such as, for instance, rainbow, halo, glory, biological cells, scattering by interstellar, circumstellar, and interplanetary grains, radar/lidar experiments and X-ray scattering, very large size-to-wavelength parameters,

$$x = \frac{2\pi a}{\lambda} \gg 1$$

occur, where a is the radius of the scattering spherical particle and λ the wavelength of the incident radiation. The Mie theory calculations in such cases may become cumbersome, time consuming and expensive if not difficult. Therefore, one may seek recourse to some short-cut method.

For very large particles ($x \gg 1$), it is possible to use the classical theories of geometrical optics and/plus diffraction (hereafter referred to as GOD). For sufficiently large x, the exact calculations as per the Mie theory show that some of the scattering parameters, in the limit of very large x, tend to approach asymptotically the values obtained on the basis of God. However, it is imperative to know the minimum value of the size-to-wavelength parameter, say, $x_{\min}$, such that for $x \geq x_{\min}$, GOD holds good for a given index

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of refraction and certain specified accuracy. In what follows we propose to study GOD vis-à-vis the Mie theory and delineate the approximate values of $x_{\min}$ for various scattering parameters in the case of dielectric as well as absorbing spheres.

2. Theoretical Consideration

The basic technique of geometrical optics consists of tracing the ray path through intervening media. Here one is guided by the elementary laws of reflection and refraction at an interface between two media with different indices of refraction. A part of the incident radiation energy can also be absorbed within the sphere. The Fresnel reflection and transmission coefficients as well as the absorption coefficient are the most important physical quantities involved in dealing with the problem of scattering by a very large sphere treated according to geometrical optics. In addition, one has to consider the contribution due to Fraunhofer diffraction around the edge of the sphere. An appealing feature of GOD (geometrical optics and/plus diffraction) is that, conceptually in line with the familiar laws of physics at undergraduate level, one can keep track of various individual contributions due to the phenomena of reflection, refraction, transmission, absorption, and diffraction.

The symbols and notation have the following meanings.

The independent variables are: a, the size of the sphere; λ, the wavelength of the incident radiation; $m = m' - im''$, the index of refraction of the material of the sphere; $x = 2\pi a/\lambda$, the size-to-wavelength parameter (X); and θ, the scattering angle.

The derived quantities are: Q_{ext}, the extinction efficiency (QEXT); Q_{sca}, the scattering efficiency (QSCA); $Q_{\text{abs}} = Q_{\text{ext}} - Q_{\text{sca}}$, the absorption efficiency (QABS); $\langle \cos\theta \rangle$, the asymmetry parameter or factor (ASYM); Q_{pr}, the efficiency for radiation pressure (QPR); Q_{back}, the back-scattering efficiency (QBACK); Albedo $= Q_{\text{sca}}/Q_{\text{ext}}\Xi$ (ALBEDO); NN, the number of the Mie coefficients considered from 1 to NN.

The symbols used in the tables for the Mie theory as well as GOD are shown within the brackets. The scattering cross sections corresponding to each efficiency can be defined by the product of particular efficiency and the geometrical cross section of the target sphere. For example, the extinction cross section $C_{\text{ext}} = \pi a^2 Q_{\text{ext}}$. The functional dependence of the derived quantities has the form $Q = Q(a, \lambda, m)$ or $Q = Q(x, m)$ where Q represents any of the derived quantities. We shall omit all the relevant equations because they are well documented in the literature.

The essential conditions for GOD to be valid in the case of scattering by a sphere can be described in nutshell as follows: (i) the size-to-wavelength parameter $x \gg 1$ and (ii) the phase shift parameter along a diameter of the sphere; viz., $2x\,|m - 1| \gg 1$. Some approximate procedures for calculating the scattering parameters under these conditions of GOD are available in the literature (see, for example, van de Hulst, 1957; Irvine, 1963, 1965; Shah, 1991a, b). However, they need to be assessed in the context of quantitative specification of the size-to-wavelength parameter of the sphere. This aspect can be studied by comparison between the results obtained according to GOD and the corresponding calculations based on the Mie theory.

For very large dielectric sphere ($x \gg 1$, $m'' = 0$) we have followed the procedure described by van de Hulst (1957), Irvine (1963), and Shah (1991a). For very large absorbing sphere ($x \gg 1$ and $m'' > 0$) we have used the expressions given by van de Hulst (1946) and Irvine (1965). The latter also includes the procedure of Born and Wolf (1959) suitable for computing the reflectances. For very large and sufficiently absorbing sphere ($a\alpha \gg 1$, α is the absorption coefficient), the refracted light will be completely absorbed within the sphere and so there will be no transmitted light. Therefore, one needs to consider only the contributions due to external reflection and diffraction.

3. Results of Calculations and Discussion

The Mie theory calculations have been performed throughout with the help of the FORTRAN Program MIEHISS (Shah, 1977) operated on Mighty Frame II computer. The size-to-wavelength parameter is varied between $x = 10^{-5}$ to 10^5 at selected intervals. The primary objective here is to search for x_{min}, the minimum value of the size-to-wavelength parameter such that, for $x \geq x_{min}$, the Mie theory and GOD both give nearly the same results for a given scattering parameter subject to certain specified accuracy.

The representative results of the exact calculations of various scattering parameters according to the Mie theory are listed in Table I for a dielectric sphere with $m = 1.6 - i0.0$ and in Tables II and III for an absorbing sphere with $m = 16 - i0.1$. These indices of refraction correspond to pure and impure (absorbing) silicate-like or polystyrene material. The equivalent results based on GOD have been entered in the bottom lines of Tables I, II, and III.

These tables show that various scattering parameters such as extinction efficiency, asymmetry parameter, etc., approach asymptotically the results given by GOD albeit at different values of x_{min}.

Consider first the dielectric sphere in Table I for $m = 1.6 - i0.0$. Suppose that the tolerance of accuracy is restricted to three significant digits. The Q_{ext}(GOD) = 2.00. The same value of Q_{ext} according to the Mie theory begins to occur at $x_{min} \simeq 15\,000$ in Table I. Thus, the approximation based on GOD hold good for $x > 15\,000$ in the case of the extinction efficiency correct to 3 significant digits. The case of absorption efficiency or albedo is trivial. It is difficult to estimate x_{min} in the case of the asymmetry factor ($\langle \cos\theta \rangle$), the radiation pressure efficiency (Q_{pr}), and the back-scattering efficiency (Q_{back}) because of the oscillations attributed to the major resonances, the ripple structure, and the surface wave phenomena (see, for example, van de Hulst, 1957; Kerker, 1969). In particular, the back-scattering efficiency (Q_{back}) for a very large dielectric sphere may oscillate with large amplitudes depending not only on the size-to-wavelength parameter ($x \gg 1$) but on the index of refraction also. The functional dependence of Q_{back} on m is such that initially Q_{back} increases as m increases beyond $m = 1$. The maximum of Q_{back} occurs at $m \simeq 1.8$ (Kerker, 1969). For $m > 1.8$, the trend would reverse, finally to conform to the case of very large perfectly reflecting/conducting sphere. Thus $Q_{back} = 1$ when $x \gg 1$ and m tends to infinity. The recent calculations

TABLE I

Extinction efficiency (QEXT), asymmetry parameter (ASYM), efficiency for radiation pressure (PQR), and back-scattering efficiency (QBACK) of a dielectric sphere as function of size-to-wavelength parameter (X) as per the Mie theory. NN = the number of Mie coefficients considered from 1 to NN. The bottom line gives the corresponding results based on geometrical optics plus diffraction (GOD).

Index of refraction: real part = 1.6, Imaginary part = 0.0					
X	QEXT	ASYM	QPR	QBACK	NN
0.10000E − 04	0.31209E − 20	0.20816E − 10	0.31209E − 20	0.46814E − 20	2
0.10000E − 02	0.31210E − 12	0.20816E − 06	0.31210E − 12	0.46814E − 12	2
0.10000E + 00	0.31255E − 04	0.20797E − 02	0.31190E − 04	0.46653E − 04	3
0.10000E + 01	0.30681E + 00	0.20965E + 00	0.24248E + 00	0.25819E + 00	5
0.10000E + 02	0.25628E + 01	0.74828E + 00	0.64510E + 00	0.10546E + 02	17
0.20000E + 02	0.26119E + 01	0.73470E + 00	0.69295E + 00	0.12224E + 02	29
0.40000E + 02	0.22595E + 01	0.77994E + 00	0.49769E + 00	0.12917E + 01	50
0.60000E + 02	0.21177E + 01	0.76962E + 00	0.48789E + 00	0.29693E + 01	73
0.80000E + 02	0.20599E + 01	0.76379E + 00	0.48656E + 00	0.81517E + 01	94
0.10000E + 03	0.20472E + 01	0.78235E + 00	0.44557E + 00	0.60074E + 01	115
0.20000E + 03	0.20460E + 01	0.78574E + 00	0.43838E + 00	0.85334E + 01	219
0.40000E + 03	0.20294E + 01	0.79351E + 00	0.41906E + 00	0.14041E + 02	421
0.60000E + 03	0.20305E + 01	0.80104E + 00	0.40398E + 00	0.22124E + 02	626
0.80000E + 03	0.20316E + 01	0.79530E + 00	0.41586E + 00	0.38612E + 02	827
0.10000E + 04	0.20213E + 01	0.80231E + 00	0.39960E + 00	0.29705E + 02	1029
0.11000E + 04	0.20189E + 01	0.79930E + 00	0.40520E + 00	0.22682E + 02	1130
0.12000E + 04	0.20114E + 01	0.79727E + 00	0.40777E + 00	0.11619E + 02	1231
0.13000E + 04	0.20099E + 01	0.80130E + 00	0.39937E + 00	0.27014E + 02	1328
0.14000E + 04	0.20154E + 01	0.80176E + 00	0.39953E + 00	0.29563E + 02	1429
0.15000E + 04	0.20165E + 01	0.79947E + 00	0.40437E + 00	0.43073E + 02	1530
0.16000E + 04	0.20121E + 01	0.80019E + 00	0.40203E + 00	0.27172E + 02	1631
0.17000E + 04	0.20159E + 01	0.80243E + 00	0.39828E + 00	0.34228E + 02	1732
0.18000E + 04	0.20179E + 01	0.80183E + 00	0.39988E + 00	0.24894E + 02	1833
0.19000E + 04	0.20166E + 01	0.79910E + 00	0.40513E + 00	0.46852E + 02	1933
0.20000E + 04	0.20100E + 01	0.80085E + 00	0.40028E + 00	0.36482E + 02	2034
0.30000E + 04	0.20109E + 01	0.79954E + 00	0.40312E + 00	0.84867E + 02	3039
0.40000E + 04	0.20092E + 01	0.80250E + 00	0.39681E + 00	0.76017E + 02	4041
0.50000E + 04	0.20064E + 01	0.80215E + 00	0.39696E + 00	0.12904E + 03	5046
0.60000E + 04	0.20056E + 01	0.79972E + 00	0.40169E + 00	0.94478E + 02	6044
0.70000E + 04	0.20067E + 01	0.80209E + 00	0.39715E + 00	0.15669E + 03	7049
0.80000E + 04	0.20054E + 01	0.80241E + 00	0.39625E + 00	0.14235E + 03	8051
0.90000E + 04	0.20038E + 01	0.80120E + 00	0.39837E + 00	0.16610E + 03	9053
0.10000E + 05	0.20055E + 01	0.80077E + 00	0.39955E + 00	0.17440E + 03	10054
0.20000E + 05	0.20032E + 01	0.80004E + 00	0.40056E + 00	0.45753E + 03	20067
0.25000E + 05	0.20028E + 01	0.80226E + 00	0.39602E + 00	0.61049E + 03	25072
0.30000E + 05	0.20019E + 01	0.80111E + 00	0.39816E + 00	0.60675E + 03	30073
0.40000E + 05	0.20016E + 01	0.80182E + 00	0.39668E + 00	0.71606E + 03	40083
0.50000E + 05	0.20051E + 01	0.80207E + 00	0.39614E + 00	0.12161E + 04	50089
0.60000E + 05	0.20012E + 01	0.80208E + 00	0.39608E + 00	0.98623E + 03	60095
0.70000E + 05	0.20011E + 01	0.80185E + 00	0.39652E + 00	0.15264E + 04	70097
0.80000E + 05	0.20010E + 01	0.80120E + 00	0.39779E + 00	0.17344E + 04	80103
0.90000E + 05	0.20010E + 01	0.80111E + 00	0.39797E + 00	0.14954E + 04	90098
0.10000E + 06	0.20011E + 01	0.80144E + 00	0.39734E + 00	0.24552E + 04	100104
$x \gg 1$	0.20000E + 01	0.80153E + 00	0.39695E + 00		

TABLE II

Extinction efficiency (QEXT), absorption efficiency (QABS), and Albedo (ALBEDO) of an absorbing sphere according to the Mie theory. X, NN, and the bottom line have the same meanings as in Table I.

Index of refraction: real part = 1.6, Imaginary part = − 0.1

X	QEXT	QABS	ALBEDO	NN
0.10000E − 04	0.18457E − 05	0.18457E − 05	0.17395E − 14	2
0.10000E − 02	0.18457E − 03	0.18457E − 03	0.17395E − 08	3
0.10000E + 00	0.18612E − 01	0.18579E − 01	0.17275E − 02	5
0.10000E + 01	0.57024E + 00	0.27560E + 00	0.51670E + 00	8
0.10000E + 02	0.24569E + 01	0.11962E + 01	0.51313E + 00	21
0.20000E + 02	0.22648E + 01	0.10844E + 01	0.52119E + 00	34
0.40000E + 02	0.21648E + 01	0.10015E + 01	0.53734E + 00	58
0.60000E + 02	0.21262E + 01	0.97008E + 00	0.54376E + 00	80
0.80000E + 02	0.21045E + 01	0.95324E + 00	0.54706E + 00	102
0.10000E + 03	0.20903E + 01	0.94264E + 00	0.54905E + 00	124
0.20000E + 03	0.20573E + 01	0.91980E + 00	0.55290E + 00	229
0.40000E + 03	0.20363E + 01	0.90706E + 00	0.55455E + 00	436
0.60000E + 03	0.20277E + 01	0.90247E + 00	0.55494E + 00	640
0.80000E + 03	0.20229E + 01	0.90007E + 00	0.55507E + 00	843
0.10000E + 04	0.20198E + 01	0.89858E + 00	0.55511E + 00	1046
0.11000E + 04	0.20186E + 01	0.89803E + 00	0.55511E + 00	1148
0.12000E + 04	0.20175E + 01	0.89757E + 00	0.55511E + 00	1249
0.13000E + 04	0.20166E + 01	0.89718E + 00	0.55511E + 00	1350
0.14000E + 04	0.20158E + 01	0.89683E + 00	0.55510E + 00	1451
0.15000E + 04	0.20151E + 01	0.89654E + 00	0.55509E + 00	1552
0.16000E + 04	0.20145E + 01	0.89627E + 00	0.55508E + 00	1653
0.17000E + 04	0.20139E + 01	0.89604E + 00	0.55507E + 00	1754
0.18000E + 04	0.20134E + 01	0.89583E + 00	0.55506E + 00	1855
0.19000E + 04	0.20129E + 01	0.89565E + 00	0.55505E + 00	1946
0.20000E + 04	0.20125E + 01	0.89548E + 00	0.55504E + 00	2057
0.25000E + 04	0.20108E + 01	0.89483E + 00	0.55498E + 00	2560
0.30000E + 04	0.20095E + 01	0.89438E + 00	0.55493E + 00	3063
0.35000E + 04	0.20086E + 01	0.89406E + 00	0.55488E + 00	3566
0.40000E + 04	0.20079E + 01	0.89382E + 00	0.55484E + 00	4069
0.45000E + 04	0.20073E + 01	0.89363E + 00	0.55481E + 00	4571
0.50000E + 04	0.20068E + 01	0.89348E + 00	0.55477E + 00	5073
0.60000E + 04	0.20060E + 01	0.89324E + 00	0.55472E + 00	6077
0.70000E + 04	0.20054E + 01	0.89308E + 00	0.55467E + 00	7080
0.80000E + 04	0.20050E + 01	0.89295E + 00	0.55463E + 00	8084
0.90000E + 04	0.20046E + 01	0.89285E + 00	0.55460E + 00	9086
0.10000E + 05	0.20043E + 01	0.89277E + 00	0.55457E + 00	10089
0.15000E + 05	0.20033E + 01	0.89252E + 00	0.55447E + 00	15099
0.20000E + 05	0.20027E + 01	0.89240E + 00	0.55440E + 00	20107
0.25000E + 05	0.20023E + 01	0.89232E + 00	0.55436E + 00	25114
0.30000E + 05	0.20021E + 01	0.89227E + 00	0.55432E + 00	30120
0.40000E + 05	0.20017E + 01	0.89221E + 00	0.55428E + 00	40130
0.50000E + 05	0.20015E + 01	0.89217E + 00	0.55424E + 00	50138
0.60000E + 05	0.20013E + 01	0.89214E + 00	0.55422E + 00	60144
0.70000E + 05	0.20012E + 01	0.89212E + 00	0.55420E + 00	70150
0.80000E + 05	0.20011E + 01	0.89211E + 00	0.55419E + 00	80156
0.90000E + 05	0.20010E + 01	0.89210E + 00	0.55417E + 00	90161
0.10000E + 06	0.20009E + 01	0.89209E + 00	0.55416E + 00	100165
$x \gg 1$	0.20000E + 01	0.89199E + 00	0.55401E + 00	

TABLE III

Asymmetry parameter (ASYM), efficiency for radiation pressure (QPR) and back-scattering efficiency (QBACK) of an absorbing sphere as per the Mie theory. X, NN, and the bottom line have the same meanings as in Table I.

Index of refraction: real part = 1.6, Imaginary part = − 0.1				
X	ASYM	QPR	QBACK	NN
0.10000E − 04	0.20780E − 10	0.18457E − 05	0.48160E − 20	2
0.10000E − 02	0.20780E − 06	0.18457E − 03	0.48160E − 12	3
0.10000E + 00	0.20761E − 02	0.18612E − 01	0.47993E − 04	5
0.10000E + 01	0.21674E + 00	0.50638E + 00	0.24183E + 00	8
0.10000E + 02	0.91553E + 00	0.13030E + 01	0.15248E + 00	21
0.20000E + 02	0.93190E + 00	0.11648E + 01	0.48130E − 01	34
0.40000E + 02	0.93682E + 00	0.10750E + 01	0.54684E − 01	58
0.60000E + 02	0.93816E + 00	0.10416E + 01	0.54658E − 01	80
0.80000E + 02	0.93874E + 00	0.10238E + 01	0.54656E − 01	102
0.10000E + 03	0.93903E + 00	0.10126E + 01	0.54655E − 01	124
0.20000E + 03	0.93944E + 00	0.98869E + 00	0.54653E − 01	229
0.40000E + 03	0.93945E + 00	0.97544E + 00	0.54653E − 01	436
0.60000E + 03	0.93938E + 00	0.97068E + 00	0.54653E − 01	640
0.80000E + 03	0.93932E + 00	0.96820E + 00	0.54653E − 01	843
0.10000E + 04	0.93927E + 00	0.96667E + 00	0.54653E − 01	1046
0.11000E + 04	0.93925E + 00	0.96611E + 00	0.54653E − 01	1148
0.12000E + 04	0.93923E + 00	0.96563E + 00	0.54653E − 01	1249
0.13000E + 04	0.93921E + 00	0.96522E + 00	0.54653E − 01	1350
0.14000E + 04	0.93919E + 00	0.96487E + 00	0.54653E − 01	1451
0.15000E + 04	0.93918E + 00	0.96457E + 00	0.54653E − 01	1552
0.16000E + 04	0.93917E + 00	0.96430E + 00	0.54653E − 01	1653
0.17000E + 04	0.93915E + 00	0.96406E + 00	0.54653E − 01	1754
0.18000E + 04	0.93914E + 00	0.96385E + 00	0.54653E − 01	1855
0.19000E + 04	0.93913E + 00	0.96366E + 00	0.54653E − 01	1956
0.20000E + 04	0.93912E + 00	0.96348E + 00	0.54653E − 01	2057
0.25000E + 04	0.93907E + 00	0.96281E + 00	0.54653E − 01	2560
0.30000E + 04	0.93904E + 00	0.96236E + 00	0.54653E − 01	3063
0.35000E + 04	0.93901E + 00	0.96204E + 00	0.54653E − 01	3566
0.40000E + 04	0.93899E + 00	0.96179E + 00	0.54653E − 01	4069
0.45000E + 04	0.93897E + 00	0.96159E + 00	0.54653E − 01	4571
0.50000E + 04	0.93896E + 00	0.96144E + 00	0.54653E − 01	5073
0.60000E + 04	0.93893E + 00	0.96120E + 00	0.54653E − 01	6077
0.70000E + 04	0.93891E + 00	0.96103E + 00	0.54653E − 01	7080
0.80000E + 04	0.93890E + 00	0.96090E + 00	0.54653E − 01	8084
0.90000E + 04	0.93888E + 00	0.96079E + 00	0.54653E − 01	9086
0.10000E + 05	0.93887E + 00	0.96071E + 00	0.54653E − 01	10089
0.15000E + 05	0.93883E + 00	0.96046E + 00	0.54653E − 01	15099
0.20000E + 05	0.93881E + 00	0.96034E + 00	0.54653E − 01	20107
0.25000E + 05	0.93879E + 00	0.96026E + 00	0.54653E − 01	25114
0.30000E + 05	0.93878E + 00	0.96021E + 00	0.54653E − 01	30120
0.40000E + 05	0.93877E + 00	0.96014E + 00	0.54653E − 01	40130
0.50000E + 05	0.93876E + 00	0.96010E + 00	0.54653E − 01	50138
0.60000E + 05	0.93875E + 00	0.96008E + 00	0.54653E − 01	60144
0.70000E + 05	0.93874E + 00	0.96006E + 00	0.54653E − 01	70150
0.80000E + 05	0.93874E + 00	0.96004E + 00	0.54653E − 01	80156
0.90000E + 05	0.93873E + 00	0.96003E + 00	0.54653E − 01	90161
0.10000E + 06	0.93873E + 00	0.96002E + 00	0.54653E − 01	100165
$x \gg 1$	0.93869E + 00	0.95993E + 00	0.54653E − 01	

based on GOD (Shah, 1991a) have revealed a new feature of shallow resonance in $\langle \cos\theta \rangle$ and Q_{pr} both as function of m for dielectric sphere. The asymmetry parameter for a very large dielectric sphere has the minimum value $\langle \cos\theta \rangle_{min} = 0.476792$ and the corresponding efficiency for radiation pressure has the maximum value $\{Q_{pr}\}_{max} = 1.04642$, in the common interval of index of refraction defined by $11.201 \leq m' \leq 11.203$ and $m'' = 0$.

Next we consider a very large absorbing sphere with the representative index of refraction $m = 1.6 - i0.1$. The scattering parameters calculated according to the Mie theory are presented in Tables II and III. The corresponding results for GOD are also included in the bottom line. In order that the asymptotic results of the scattering parameters in Tables II and III may match with the corresponding results based on GOD at the level of three significant digits, the minimum values of the size-to-wavelength parameter turn out to be approximately $x_{min} = 7000$, 15000, 15000, 400, 15000, and 200 for Q_{ext}, Q_{abs}, albedo $\langle \cos\theta \rangle$, Q_{pr}, and Q_{back}, respectively. Note that the oscillations, which were prominent in the case of the dielectric sphere with $m = 1.6 - i0.0$ in Table I, are now considerably damped out with the addition of absorptivity. In particular the asymptotic limit for Q_{back} is attained at much smaller value of x_{min} as compared to other scattering parameters in Tables II and III and all the scattering parameters in Table I. This holds good for $\langle \cos\theta \rangle$ also to certain extent provided $0 < m'' \ll 1$.

It is instructive to study the variation of x_{min} with change in the imaginary part of the index of refraction. The results of detailed calculations are summarized in Table IV. Here Mie and GOD indicate the exact values as per the Mie theory and the approximate values based on GOD, respectively. The tolerance of accuracy is assumed to be three significant digits for each of the scattering parameters listed in Table IV. Some interesting features may be noted. GOD holds good for $x > x_{min}$. The asymmetry factor $\langle \cos\theta \rangle$ and the efficiency for radiation pressure Q_{pr} are almost independent of absorptivity provided m'' is sufficiently small but not zero. The same is true for Q_{ext}, Q_{pr}, albedo, and Q_{back} too. Such a feature of near constancy of $\langle \cos\theta \rangle$ and Q_{pr} has been described by Irvine (1965) for a complex index of refraction satisfying the condition $0 < |m''/(m' - 1)| \leq 1$. Again x_{min}, in the case of back-scattering efficiency (Q_{back}) for an absorbing sphere, is unusually small as compared to the cases of other scattering parameters.

Figure 1 illustrates the manner in which the absorption efficiency as per the Mie theory approaches the asymptotic value for a variety of complex indices of refraction. The dashed line in each plot corresponds to the result obtained according to GOD. As x increases, the difference between Q_{abs}(Mie) and Q_{abs}(GOD) decreases monotonically in each plot. The question is: how large the size-to-wavelength parameter should be in order for GOD to be valid asymptote to the Mie theory? The answer depends among other things on the tolerance of accuracy that one would desire. For instance, Table IV gives the minimum values x_{min} for various scattering parameters by limiting the accuracy to three significant digits.

The effects of varying the real as well as the imaginary parts of the index of refraction

TABLE IV

The variation of x_{min} with absorptivity and/or the scattering parameters. Mie and GOD refer to the calculations based on the Mie theory and geometrical optics plus diffraction, respectively.

Index of refraction $m = m' - im''$		Extinction efficiency Q_{ext}	Absorption efficiency Q_{abs}	Albedo	Asymmetry parameter $\langle \cos\theta \rangle$	Radiation pressure efficiency Q_{pr}	Back-scattering efficiency Q_{back}
$1.6 - i0.0$	Mie	2.0037	0.00000	1.0000	0.80144	0.39734	2455.2
	x_{min}	15000	Arbitrary	Arbitrary	*	*	*
	GOD	2.00	0.00000	1.0000	0.80152	0.39696	?
$1.6 - i0.05$	Mie	2.0054	0.89343	0.55389	0.93967	0.96049	0.053605
	x_{min}	7000	70000	400	80	20000	80
	GOD	2.0000	0.89330	0.55335	0.93966	0.96008	0.053604
$1.6 - i0.10$	Mie	2.0054	0.89252	0.55447	0.93945	0.96046	0.054653
	x_{min}	7000	15000	15000	400	15000	200
	GOD	2.0000	0.89199	0.55401	0.93869	0.95993	0.054653
$1.6 - i0.30$	Mie	2.0054	0.87939	0.56145	0.92943	0.95940	0.065725
	x_{min}	7000	10000	5000	800	10000	40
	GOD	2.0000	0.87864	0.56068	0.92866	0.95864	0.065693
$1.6 - i1.00$	Mie	2.0051	0.76847	0.61649	0.84542	0.95945	0.17536
	x_{min}	8000	9000	20000	3500	40000	40
	GOD	2.0000	0.76771	0.61615	0.84456	0.95926	0.17526
$1.6 - i2.00$	Mie	2.0054	0.56743	0.71654	0.71340	0.97939	0.40536
	x_{min}	8000	30000	60000	15000	10000	40
	GOD	2.0000	0.56718	0.71641	0.71281	0.97867	0.40520
$1.6 - i4.00$	Mie	2.0054	0.29141	0.85351	0.58329	1.0054	0.71889
	x_{min}	9000	30000	1700	15000	10000	200
	GOD	2.0000	0.29114	0.85443	0.58251	1.0046	0.71880

* Indicates that the oscillations in the scattering parameter persist even up to $x = 10^5$. Therefore, x_{min} cannot be specified or else $x_{min} > 10^5$.

on the asymptotic behaviour of the asymmetry parameter and the efficiency for radiation pressure have been set out in Figure 2. For real m, the oscillations in $\langle \cos\theta \rangle$ and Q_{pr} become more pronounced with increasing x or m. However, with the addition of a small imaginary part to the index of refraction, the oscillations fade out resulting in smooth monotonic variations of $\langle \cos\theta \rangle$ and Q_{pr}.

The relative error in Q(GOD) with respect to Q(Mie) can be defined by

$$\frac{\Delta Q}{Q} = \left\{ \frac{Q(\mathrm{GOD}) - Q(\mathrm{Mie})}{Q(\mathrm{Mie})} \right\}, \tag{1}$$

where Q stands for one of the scattering parameters. Q(GOD) and Q(Mie) are the values of the scattering parameter calculated according to GOD and the Mie theory, respectively. Table V illustrates the percent errors in extinction and radiation pressure efficiencies for a dielectric ($m = 1.6 - i0.0$) and an absorbing ($m = 1.6 - i0.0$) sphere.

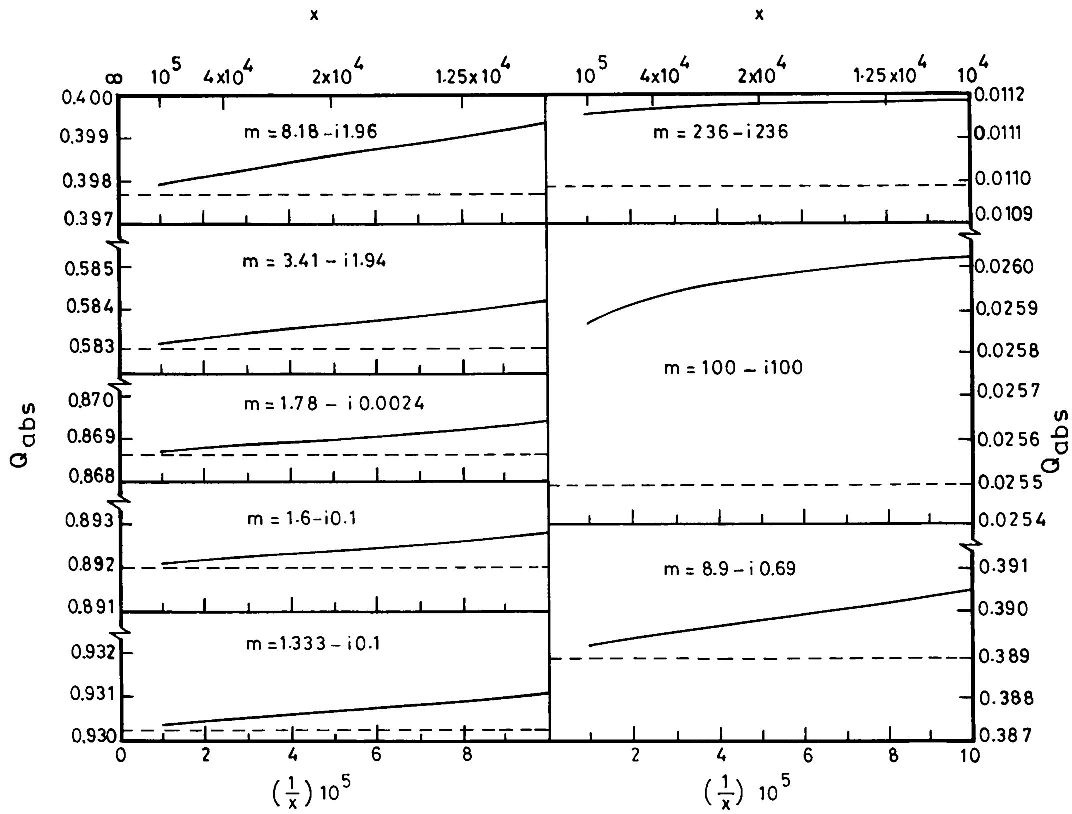

Fig. 1. Absorption efficiency (Q_{abs}) versus $x^{-1} \times 10^5$ for very large absorbing spheres with a variety of complex indices of refraction ($m = m' - im''$). The solid line (——) curves represent the exact calculations according to the Mie theory and the dashed line (----) curves correspond to GOD (geometrical optics and diffraction). The abscissae in all plots correspond to 10^5 times the reciprocal of the size-to-wavelength parameter (x). The ordinate scales for the plots on the right- (left-)hand side are given on the extreme right (left).

For moderate values of the size-to-wavelength parameter in the range $1 \ll x < 50$, the absolute values of the errors in Q_{ext}(GOD) are more in the case of the real $m = 1.6 - i0.0$ compared to the complex $m = 1.6 - i0.1$. Similar trend is observed in the case of Q_{pr} for $1 \ll x < 500$. The errors at a level of $\leq \pm 1\%$ in Q_{ext}(GOD) as well as Q_{pr}(GOD) show up beginning with $x \approx 1000$. In order to match the asymptotic values as per the Mie theory with the equivalent results from GOD within an error of $\leq 0.1\%$, the size-to-wavelength parameter should be enhanced to $x > 30000$ and $x > 40000$ for Q_{ext} and Q_{pr}, respectively, in the case of $m = 1.6 - i0.0$. Similarly, $x > 30000$ and $x > 10000$ for Q_{ext} and Q_{pr}, respectively, in the case of $m = 1.6 - i0.1$.

Table V shows that, for dielectric spheres, there is no monotonic trend in the errors as x increases. This can happen due to the major maxima and minima, the ripple structure, and the effects of surface waves (see, for example, van de Hulst, 1957; Irvine, 1965). On the other hand, for absorbing spheres, the errors in Q_{ext} and Q_{pr} decrease montonically as x goes on increasing. These aspects have been brought out succinctly in Figures 1 and 2 for a variety of indices of refraction.

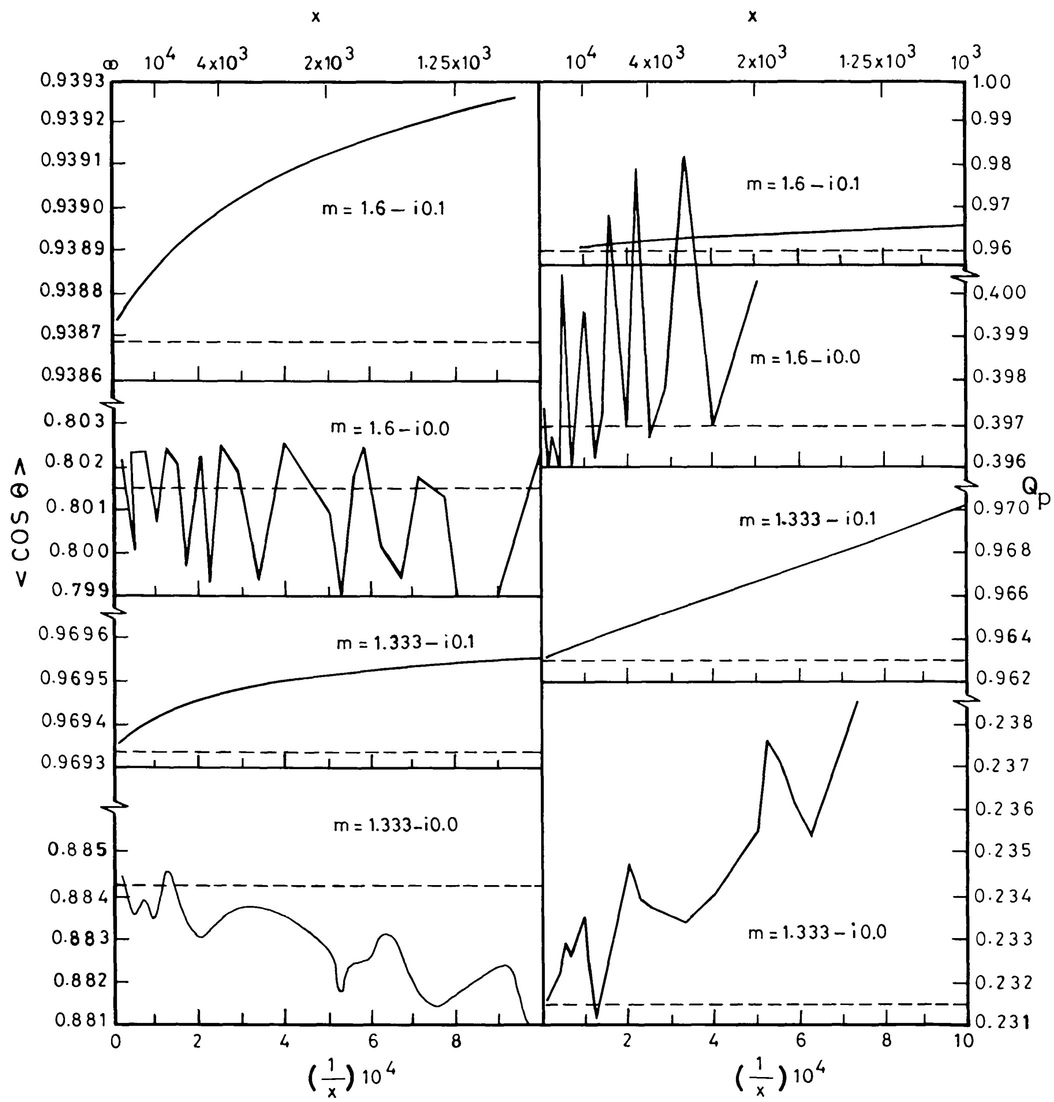

Fig. 2. The asymmetry parameter ($\langle\cos\theta\rangle$, the set on the left) and the efficiency for radiation pressure (Q_{pr}, the set on the right) for real and complex indices of refraction ($m = m' - im''$) representative of the dielectric and absorbing spheres, respectively. The continuous curves are for the Mie theory and the dashed line curves are for GOD. The abscissae in all plots correspnd to 10000 times the reciprocal of the size-to-wavelength parameter (x). The ordinate scales for $\langle\cos\theta\rangle$ are on the left and those for Q_{pr} are on the right. Note that the dashed line itself is the abscissa in the case of Q_{pr} for $m = 1.6 - i0.1$.

In general, Q_{ext} versus x curve for dielectric sphere ($m'' \approx 0.0$) shows up ripple structure supercomposed on the maxima and minima due to major resonances. With increasing absorptivity ($m'' > 0$) this ripple structure begins to fade out, first at larger values of x. Next at sufficiently higher values of m'', even the major maxima and minima are damped out; again this behaviour begins prominently first at larger values of x (see, for example, Bohren and Huffmann, 1983, pp. 104, 300, 306). For high absorptivity such that $(m''/(m' - 1)) > 1$, minor oscillations possibly due to surface waves may reappear in Q_{ext}, Q_{sca}, $\langle\cos\theta\rangle$, and Q_{pr} for certain range of size-to-wavelength parameter; the

TABLE V

Percent errors in extinction and radiation pressure efficiencies according to Equation (1)

x	Dielectric sphere $m = 1.6 - i0.0$		Absorbing sphere $m = 1.6 - i0.1$	
	$\left(\frac{\Delta Q_{ext}}{Q_{ext}}\right) \times 100$	$\left(\frac{\Delta Q_{pr}}{Q_{pr}}\right) \times 100$	$\left(\frac{\Delta Q_{ext}}{Q_{ext}}\right) \times 100$	$\left(\frac{\Delta Q_{pr}}{Q_{pr}}\right) \times 100$
10	− 22.0	− 38.5	− 18.6	− 26.3
20	− 23.4	− 42.7	− 11.7	− 17.6
40	− 11.5	− 20.2	− 7.6	− 10.7
60	− 5.6	− 18.6	− 5.9	− 7.8
80	− 2.9	− 18.4	− 5.0	− 6.2
100	− 2.3	− 10.9	− 4.3	− 5.2
500	− 1.2	− 3.2	− 1.5	− 1.3
1000	− 1.0	− 0.66	− 0.98	− 0.70
2000	− 0.50	− 0.83	− 0.62	− 0.37
5000	− 0.32	− 0.0025	− 0.34	− 0.16
10000	− 0.27	− 0.65	− 0.21	− 0.084
20000	− 0.16	− 0.90	− 0.14	− 0.046
30000	− 0.095	− 0.30	− 0.10	− 0.032
40000	− 0.080	+ 0.068	− 0.085	+ 0.025
50000	− 0.075	+ 0.20	− 0.075	− 0.021
60000	− 0.060	+ 0.22	− 0.065	− 0.019
70000	− 0.055	+ 0.11	− 0.060	− 0.017
80000	− 0.050	− 0.21	− 0.055	− 0.015
90000	− 0.050	− 0.26	− 0.050	− 0.013
10^5	− 0.055	− 0.098	− 0.050	− 0.012

case of Q_{ext} has been illustrated in Figure 8 of Irvine (1965). However, for very large values of x, it is unlikely that an absorbing sphere may exhibit such oscillations.

In spite of the oscillations in Q_{ext}, $\langle \cos\theta \rangle$, and Q_{pr} for dielectric spheres, the asymptotic value at a level of at least two significant digits are possible for certain x_{min} within the range $1 \ll x_{min} \ll 10^5$. But in the case of Q_{back} for dielectric spheres, there is no sign of asymptotic value even up to $x = 10^5$. The asymptotic values of all the scattering parameters including Q_{back} for absorbing spheres are possible with the accuracy of 3 or more significant digits and x_{min} in the range $1 \ll x_{min} \ll 10^5$.

4. Conclusions

The theories of geometrical optics and diffraction (GOD) can be useful for calculating the electromagnetic scattering parameters of sufficiently large homogeneous, isotropic, and smooth spheres. In order for GOD to be valid, the size-to-wavelength parameter (x) should be larger than certain minimum value (x_{min}) which depends among other things upon the index of refraction and the tolerance of accuracy. The latter should be restricted to a moderate level of ≤ 3 significant digits. It may be noted that x_{min} may be different for different scattering parameters. In practice, for routine application, $x_{\min}$ for each scattering parameter can be estimated by comparison of sample results based on GOD and the Mie theory.

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