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Estimation of Vital Shower Parameters in Wavefront Sampling Technique

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Abstract. Wavefront sampling experiments record arrival times of Čerenkov photons with high precision at various locations in Čerenkov pool using a distributed array of telescopes. It was shown earlier that this photon front can be fitted with a spherical surface traveling at a speed of light and originating from a single point on the shower axis. Radius of curvature of the spherical shower front (R) is approximately equal to the height of shower maximum from observation level. For a given primary species, it is also found that R varies with the primary energy (E) and this provides a method of estimating the primary energy. In general, one can estimate the arrival times at each telescope using the radius of curvature, arrival direction of the primary and the core location. This, when compared with the data enables us to estimate the above parameters for each shower. This method of obtaining the arrival direction alleviates the difficulty in the form of systematics arising out of the plane wavefront approximation for the Čerenkov front. Another outstanding problem in the field of atmospheric Čerenkov technique is the difficulty in locating the shower core. This method seems to solve both these problems and provides an elegant method to determine the arrival direction as well as the core location from timing information alone. In addition, using the Čerenkov photon density information and the core position we can estimate the energy of the primary if the nature of the primary is known. Combining these two independent estimates of the primary energy, the energy resolution can be further improved. Application of this methodology to simulated data and the results will be presented. The intrinsic uncertainties on the various estimated parameters also will be discussed.

Key words: VHE $\gamma-$ray astronomy, Air shower simulations

1. Čerenkov shower front fitting for vertical showers

In a typical wavefront sampling experiment arrival time of Čerenkov shower front is recorded at several locations in Čerenkov pool with high precision. This information can be used to reconstruct the shower direction. It is shown earlier that for vertically incident showers initiated by γ−rays and cosmic ray primaries, the relative arrival time delay $[t(r)]$ of Čerenkov shower front at a core distance r can be approximated by

$$t(r) = \frac{\sqrt{(R^2 + r^2)}}{c} - \frac{R}{c}$$

where R is the radius of curvature of the spherical front (Battistoni et al. (1998), Chitnis and Bhat (1999)).

In effect, Čerenkov shower front can be approximated with a wavefront moving at a speed of light, originating from a single point on shower axis. Figure 1 shows the variation of mean arrival time of Čerenkov shower front as a function of core distance for γ−rays, protons and Fe nuclei of various energies incident vertically at the top of the atmosphere. These showers were simulated using a package known as CORSIKA (Heck et al., 1998). For each case, arrival time is averaged over typically 10 showers. Best fit spherical wavefront is also indicated by smooth curve. Radius of the fitted wavefront (R) approximately corresponds to the height of shower maximum (h_e) from observation level. This is expected since large fraction of emission arises from the region in the vicinity of shower maximum.

2. Algorithm for spherical wavefront fitting for inclined showers

Here we extend this approach to the showers generated by primaries incident at an angle with respect to the vertical. Consider a distributed of telescopes at observation level. Shower is incident at a zenith angle θ and azimuth angle ϕ. Here it is assumed that the shower axis passes through the centre of the array. Coordinates of the telescopes in the array are given by (x_i,y_i), with respect to the centre of the array.

The arrival time of the shower front at a telescope situated at (x_i,y_i), with respect to that at the footprint of the front, is given by

$$t(r) = \frac{1}{c}\left[\sqrt{\frac{R^2}{cos^2\theta} - 2Rtan\theta\,(x_i cos\phi + y_i sin\phi) + x_i^2 + y_i^2} - R\right]$$

where R is the vertical height of the emission point on shower axis from the observation level.

Using this algorithm one can fit the measured arrival time and estimate the arrival direction of shower front (θ and ϕ) as well as the radius of curvature of the shower front (R').

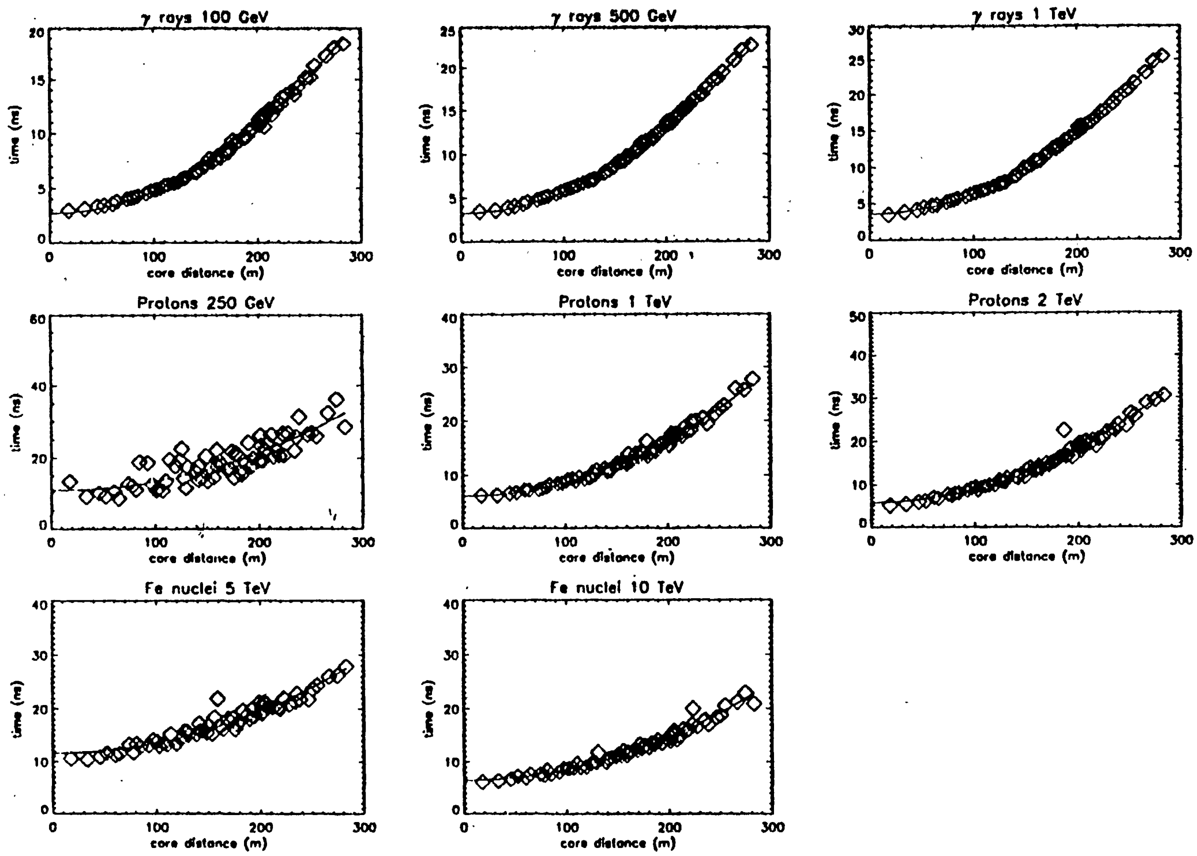

Figure 1. Variation of mean arrival time of Čerenkov shower front as a function of core distance for γ–rays, protons and Fe nuclei of various energies incident vertically at the top of the atmosphere. For each case, arrival time is averaged over typically 10 showers. The smooth curve corresponds to the best fit spherical wavefront.

3. Spherical wavefront fitting of simulated data

Efficacy of this algorithm is tested using simulated data. A large number of showers initiated by protons incident at an average angle of 15° with respect to vertical are simulated using CORSIKA (Heck et al., 1998). Energies of these proton primaries are chosen randomly in the range 250 GeV - 20 TeV with the spectral index of -2.65. Zenith angles for showers are chosen randomly within 2° around the average value. Whereas azimuthal angles are randomly selected in the range 0-360°. Axes of all the showers pass through the central telescope of the array. Arrival times of Čerenkov photons are recorded at 357 telescopes spread over an area of 400 m × 400 m at an altitude of 1 km.

For each shower, arrival times at various core distances are fitted with a spherical wavefront. Angles θ and ϕ and vertical height of emission point from observation level R are estimated using Marquardt routine. This exercise is carried out for a sample of

50 showers. Figure 2(a) shows the distribution of fitted radii of curvature (R) of shower front. For comparison, distribution of the shower maxima h_e from observation level is also shown in the same figure. Distribution of difference in R and h_e is shown in Figure 2(b). Mean value of this difference distribution is 3.3 km with an RMS spread of 2.8 km, indicating that effective point of emission is above the shower maximum (also refer to Chitnis and Bhat, 2001). Larger difference between R and h_e for inclined showers could be intrinsic. Reduction in the relative contribution of the curvature to the arrival time differences at various telescopes compared to those due to propagation delay could result in a larger error in fitted radius as well. This problem can perhaps be solved by correcting the arrival times for differences arising out of the spatial separation of detectors.

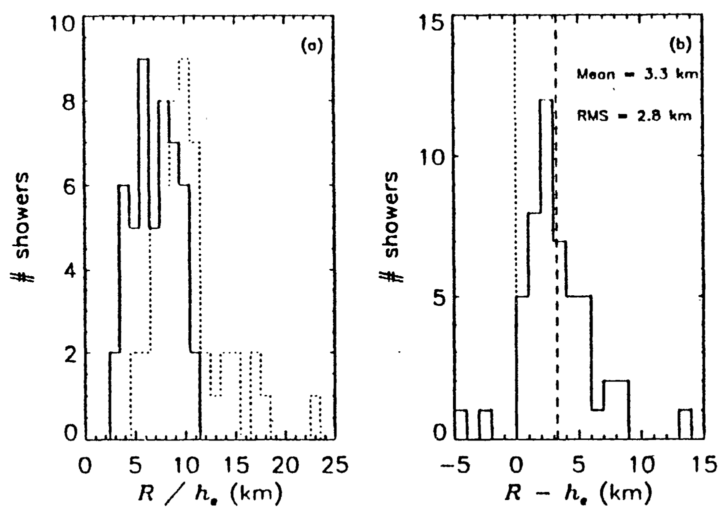

Figure 2. (a) Distribution of the fitted radii of shower front R (continuous line) and distribution of the actual shower maxima h_e from observation level (shown by dotted line). (b) Difference distribution of R and h_e. Mean of the distribution and the expected value are marked by dashed and dotted lines. Mean and RMS of the difference distribution are indicated.

Difference distribution of fitted azimuthal angle (ϕ) and the actual azimuthal angle is shown in Figure 3a. Similarly difference distribution of fitted zenith angle (θ) and actual zenith angle is shown in Figure 3b.

So far we have carried out spherical wavefront fitting assuming shower core to be exactly at the centre of the array, which is also the origin of our coordinate system. However, in an actual experiment shower core may lie anywhere in the telescope array and at times even outside the array. By introducing the core location as another variable, same algorithm can be further extended to get location of the shower core. In the present estimate, the known core position is re-fitted by the routine. Hence uncertainties in the core location are under-estimated.

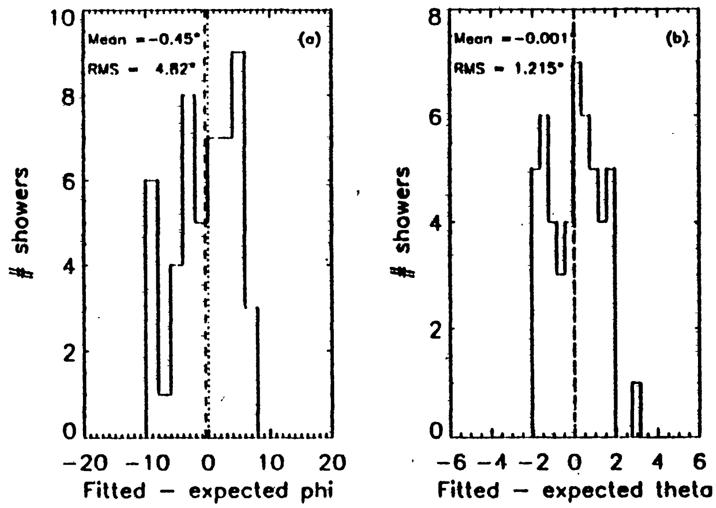

Figure 3. (a) Difference distribution of fitted and actual ϕ. Mean of the distribution and the expected value are marked by dashed and dotted lines. Mean and RMS of the difference distribution are indicated. (b) Difference distribution of fitted and actual θ. Mean of the distribution and the expected value are marked by dashed and dotted lines. Mean and RMS of the difference distribution are indicated.

4. Conclusions

Čerenkov shower front from protons showers incident at an angle of 15° is fitted with a spherical wavefront. Fitted radius R is found to be less than the height of shower maximum h_e by about 3 kms. For vertical showers R is found to be about 1 km lower than h_e (Chitnis and Bhat, 2001). Larger difference between R and h_e for inclined showers could be intrinsic. Reduction in the relative contribution of the curvature to the arrival time differences at various telescopes compared to those due to propagation delay could result in a larger error in fitted radius as well. Using this method azimuthal and zenith angles of the shower axis are estimated and are expected to give better estimate than those estimated using plane front approximation. Using this method location of the shower core can be estimated. This information will be particularly useful to derive the primary energy and the nature of the primary using photon density distributions. Using R, core location as well as photon density information at the observation level, it will be possible to improve the energy resolution of the experiment.

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