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Analytical study of standing shock around black hole

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Abstract. Shocks form around black holes as the matter slows down by centrifugal barrier. At the shocks, kinetic energy is converted into thermal energy and hard-X rays, outflows etc. are produced which are observed. We compute the locations of these shocks analytically by using a set of approximations and found that they match very well with those obtained by numerical techniques. From the shock properties we compute QPO oscillation frequencies and outflow rate in the jets.

Key words: accretion, accretion disks — black hole physics — hydrodynamics — shock waves

1. Introduction

It is well accepted that standing or oscillating shocks are essential ingredients in the accretion disk around compact objects. The spectral properties of black holes (Chakrabarti & Titarchuk, 1995) and frequencies of quasi-periodic oscillations seem to be explicitly related to the location x_s of the shock (Molteni, Sponholz & Chakrabarti, 1996). Comparing with ordinary stars, region $x < x_s$ is often known as CENBOL (centrifugal pressure supported boundary layer). Because of their importance, it would be interesting to obtain the shock location x_s using fundamental parameters of the flow.

2. Model Equations

We consider thin, inviscid and steady accretion around a Schwarzschild black hole. The is assumed to be in equilibrium in a direction transverse to the flow and the potential is describe by the Paczyński-Wiita pseudo-Newtonian potential $\phi = \frac{GM_{BH}}{x-r_g}$ (Paczyński & Witta, 1980), where $r_g = 2GM_{BH}/c^2$ is the radius of a black hole of mass M_{BH}, G is the gravitational constant and c is the velocity of light respectively. r_g, r_g/c and

M_{BH} are used as units of length, time and mass respectively. The dimensionless energy conservation equation (Chakrabarti, 1989, hereafter C89) is given by,

$$\mathcal{E} = \frac{1}{2}\vartheta^2 + na^2 + \frac{\lambda^2}{2x^2} - \frac{1}{2(x-1)}, \tag{1}$$

where, $n(= \frac{1}{\gamma-1})$ is the polytropic constant and γ is the adiabatic index. $\mathcal{E}$ and λ are the specific energy and angular momentum respectively. Here x, ϑ and a are all dimensionless. The mass flux conservation equation (C89) in terms of ϑ and $a = \sqrt{\gamma P/\rho}$ in given by,

$$\dot{\mathcal{M}} = \vartheta a^q x^{3/2}(x-1), \tag{2}$$

where, $q = \frac{\gamma+1}{\gamma-1}$, P is the isotropic pressure and ρ is the local density of the flow.

Using sonic point analysis (C89) point conditions in the usual way one obtains the fourth order equation (Das et al. 2001),

$$\mathcal{N}x_c^4 - \mathcal{O}x_c^3 + \mathcal{P}x_c^2 - \mathcal{Q}x_c + \mathcal{R} = 0. \tag{3}$$

where, $\mathcal{N} = 10\mathcal{E}$; $\mathcal{O} = 16\mathcal{E}+2n-3$; $\mathcal{P} = 6\mathcal{E}+\lambda^2(4n-1)-3$; $\mathcal{Q} = 8n\lambda^2$; and $\mathcal{R} = (1+4n)\lambda^2$. Equation (3) is a quartic equation and among the four root all of them may be complex or two complex and two real or all four are real (Abramowitz and Stegan, 1970). For shock, all the four roots must be real. One solution is found to be inside the horizon and others would be outside and out of them two would be X-type and one in between must be O-type.

3. Shock Location Analysis

For the formation of standing shock Rankine-Hugoniot shock conditions are to be satisfied (see, Landau and Lifshitz 1959 and Chakrabarti 1989). We use this conditions in the Eq. (1) and Eq. (2) and get the shock invariant quantity in terms of pre- and post-shock Mach numbers (M_- and M_+) of the flow:

$$C = \frac{[M_+(3\gamma-1) + (2/M_+)]^2}{2+(\gamma-1)M_+^2} = \frac{[M_-(3\gamma-1) + (2/M_-)]^2}{2+(\gamma-1)M_-^2}. \tag{4}$$

where, subscripts "-" and "+" refer, respectively, to quantities before and after shock. Simplifying the shock invariant relation (Eq. 4) for relativistic flow with $\gamma = 4/3$, i.e. $n = 3$, we obtain,

$$2(M_+^2 + M_-^2) - 21M_+^2M_-^2 + 12 = 0. \tag{5}$$

We expand pre- and post-shocks Mach number in quadratic so that Eq. (5) becomes:

$$\mathcal{A}x_s^4 + \mathcal{B}x_s^3 + \mathcal{C}x_s^2 + \mathcal{D}x_s + \mathcal{F} = 0, \tag{7}$$

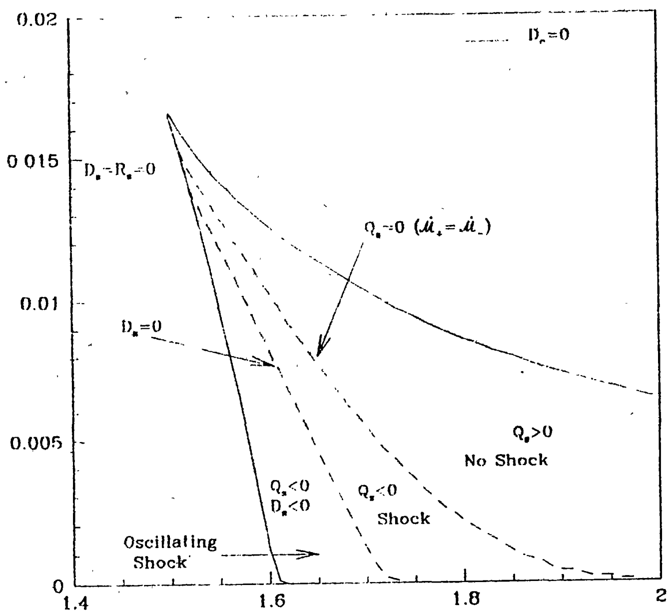

Figure 1: Division of parameter space for the formation of shock as spanned by the initial parameter $\mathcal{E}, \lambda$. Solid curve represents the multiple sonic point regions. Dashed curve ($D_s = 0$) surrounds the region with shocks in accretion. When $D_s < 0$ and there are three sonic points, shocks are oscillatory, giving rise to quasi-periodically varying hard X-rays.

quartic (Das et al. 2001) and is solvable analytically. Here, $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$, and $\mathcal{F}$ are the functions of the initial parameters $\mathcal{E}$ and λ (see, Das et al. 2001).

In Figure 1, we draw the parameter space for the shock locations. The solid line boundary separates the single and multiple sonic point region. The boundary of the parameter space for shock is obtained by using the approximate analytical method and it shows a little mismatch at the region of the cusp ($D_s = R_s = 0$). In the region ($Q_s < 0$ and $D_s < 0$) the shock location is imaginary and shocks oscillates back and forth causing Quasi-Periodic Oscillation (QPO). Below the no-shock region flow has only one sonic point and obviously there is no shock. Comparison of the boundary of the parameter space obtained from numerical integration result suggests very little difference.

In Figure 2, we draw shock location x_s as a function of specific energy and specific angular momentum (λ). ($= 1.51$ for rightmost and increasing by 0.01 leftward). For higher λ, the shock is generally located farther from the black hole. Results are similar to numerically obtained values (C89).

Once shock location is known, the compression ratio at the shock can be computed analytically (Das et al. 2001b). Assuming the outflow is formed from the CENBOL, and it is isothermal till the sonic point, the outflow rate could also be computed (Das et. al. 2001b). Figure 3, we present the variation of the ratio $R_{\dot{m}} = outflow\ rate/inflow\ rate$ with compression ratio for different angular momentum (marked).

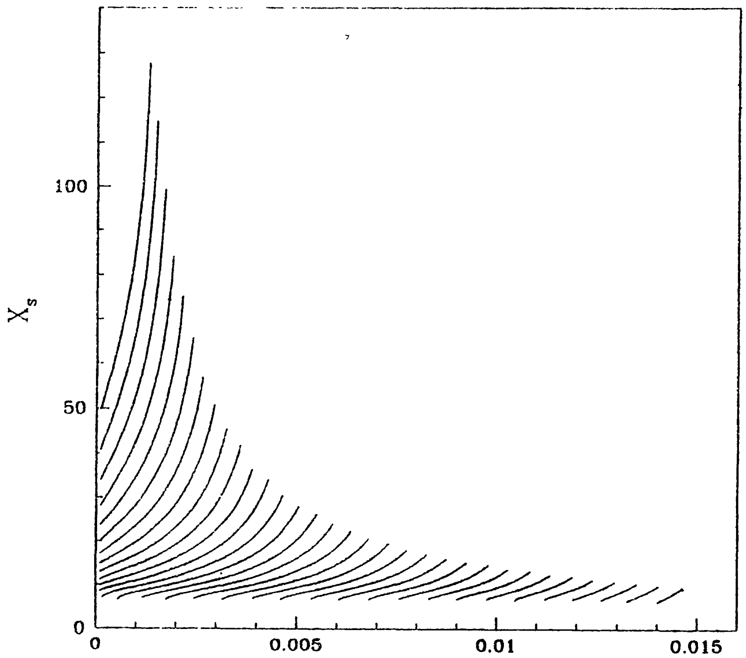

Figure 2: Variation of shock location (x_s along y-axis) with specific energy ($\mathcal{E}$ along x-axis) of the flow. Each curve is drawn for a specific angular momentum λ. From right to left curved are drawn for $\lambda = 1.51,\ 1.52,\ 1.53, ...$ till 1.84 respectively. For a given specific energy $\mathcal{E}$, shock location increases with increasing centrifugal force (through λ). Similarly, for a given λ, shock location increases with energy.

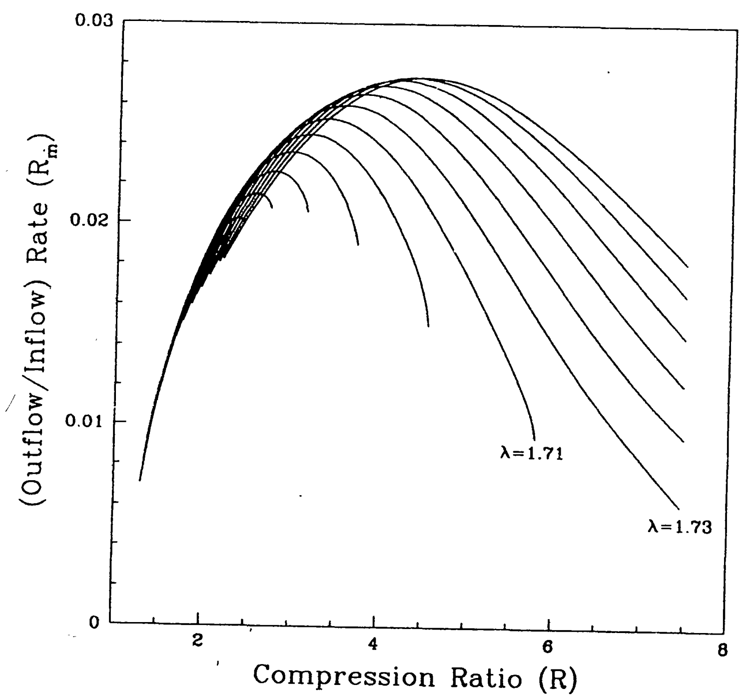

Figure 3: Variation of ratio of outflow to inflow rates $R_{\dot{m}}$ as a function of compression ratio for various specific angular momentum. $\lambda = 1.57$ (inner most) to 1.83 (outer most). Curves are drawn at intervals of $d\lambda = 0.02$. Outflow rate is maximum at some intermediate compression ratio.

4. Conclusion

In this paper, we demonstrated that the shock location and the associated properties could be studied completely analytically. We note that the location of the shock moves away from the black hole as the angular momentum increases, indicating that shocks are mainly centrifugal supported. At the shock, flow kinetic energy is mainly converted to thermal energy thereby heating the flow. The hot post-shock flow intercepts soft photon from pre-shock region and produces hard photons by inverse Comptonization processes. So the spectral states and time dependent behavior of the hard X-rays are directly related to the properties of the shock location. From the location of the shock one could also get the information of QPO frequency.

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