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Compton cross sections in highly dense plasmas

Vinod Krishan

Indian Institute of Astrophysics, Bangalore 560034, India

Abstract

The properties of the electromagnetic radiation undergo a fundamental change in the presence of a medium in general and a plasma in particular. Specially the dispersion relation is modified in a way that in a plasma, the square of the four-momentum of a photon is not zero. In an unmagnetized plasma, it is equal to the square of the plasma frequency. Further, the electron-photon coupling vertex along with the energy momentum conservation laws are also modified if the Compton processes occur in a plasma. As a result the cross sections of the three Compton processes, viz., the Compton scattering, the electron-positron pair annihilation and production become functions of the plasma density. The cross sections are found to change significantly in highly dense plasmas.

1. Introduction

The processes of the Compton scattering, the electron-positron pair annihilation and production play an important role in the functioning of the high energy astrophysical sources such as pulsars, supernovae, γ−ray bursts and the early universe (Wolfgang, Fabian & Giovannelli 1990; Rose 1973). The cross sections of these processes are used, almost always, either in the classical Thomson limit or at best including the Klein-Nishina cross sections for high energy photons and electrons. The effect of the plasma medium on the cross section has not been investigated. In this paper, it is shown that the cross sections undergo a significant modification when the Compton processes take place in a dense plasma. In section two, the Compton processes are described in a plasma. In section three, the astrophysical objects and sites, where the effect of dense plasma could be substantial, are mentioned.

2. Compton processes in a plasma

There are three changes that one has to incorporate in order to calculate the Compton cross sections in a plasma. These are:
(i) the square of the four vector of a photon in a plasma is given by (Krishan 1999):

$$K^2 = \vec{K}^2 c^2 - \omega^2 = -\omega_p^2 \tag{1}$$

where $\omega_p^2 = 4\pi n e^2/m$ is the square of the plasma frequency and n is the electron density.
(ii) The change in the electron-photon coupling vertex is related to the change in the energy density of an electromagnetic field by the well known Von-Laue factor. The electric energy density $\frac{E^2}{8\pi}$ in vacuum becomes

$$\frac{E^2}{8\pi} \cdot \frac{\partial}{\partial \omega}(\omega \epsilon)$$

in a medium of dielectric constant ϵ. Similarly the magnetic energy density B^2 is related to the electric energy density as $B^2 = \epsilon E^2$. Both these effects change the normalization $(2\pi\hbar c^2/\omega)^{1/2}$ of the vector potential of the electromagnetic field in vacuum to (Harris 1975a, Landau & Lifshitz 1960)

$$\left[\frac{2\pi\hbar c^2}{\omega} \frac{1}{1/2\frac{\partial}{\partial \omega}(\omega^2 \epsilon)}\right]^{\frac{1}{2}}$$

in a medium.
(iii) the energy-momentum conservation law (equation (2)) is modified to

$$\gamma' = \frac{\gamma + p\gamma^2}{1 + \gamma\left[1 - \sqrt{1-p}\sqrt{1-p'}\cos\theta\right]} \tag{2}$$

where $p = \omega_p^2/\omega^2$ and $p' = \omega_p^2/\omega'^2$.

After incorporating all these changes, the differential cross section for the Compton scattering averaged over photon polarizations is found to be:

$$\begin{aligned}\frac{d\bar{\sigma}_c}{d\Omega_{k'}}\frac{1}{r_0^2} = \Bigg[&\frac{\gamma'^2}{\gamma^2}\frac{(1+\cos^2\theta)}{2\sqrt{AB}} + \frac{\gamma'^2}{4\gamma^2}\Bigg[\frac{\gamma'}{\gamma A} + \frac{\gamma}{\gamma' B} - \frac{2}{\sqrt{AB}} \\ &+ \left(1 - \frac{\sqrt{A}}{\sqrt{B}}\right)\frac{\gamma}{A}(1-p)\sin^2\theta - \left(1 - \frac{\sqrt{B}}{\sqrt{A}}\right)\frac{\gamma'}{B}(1-p')\sin^2\theta \\ &- \frac{p\gamma}{AB}\left\{\frac{2\gamma}{\gamma'}\sqrt{AB} + A\left(\frac{\gamma^2}{\gamma'^2} + \frac{\gamma}{\gamma'}\right) - B\left(\frac{\gamma'}{\gamma} + 1\right) - 2\sqrt{AB}\right\}\Bigg]\Bigg] G\end{aligned} \tag{3}$$

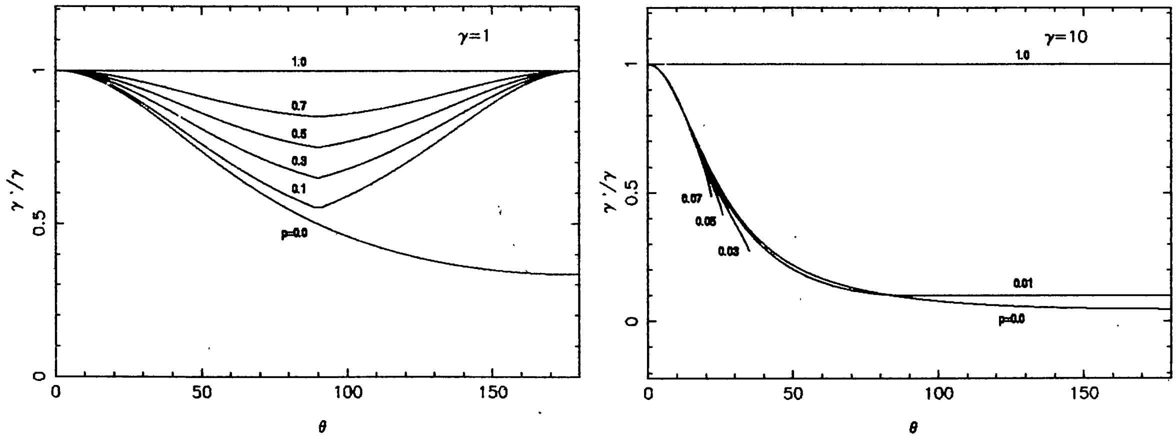

Figure 1. Variation of the first root (γ'/γ) of equation (2) vs the scattering angle θ for different values of the plasma density parameter p and initial photon energy γ.

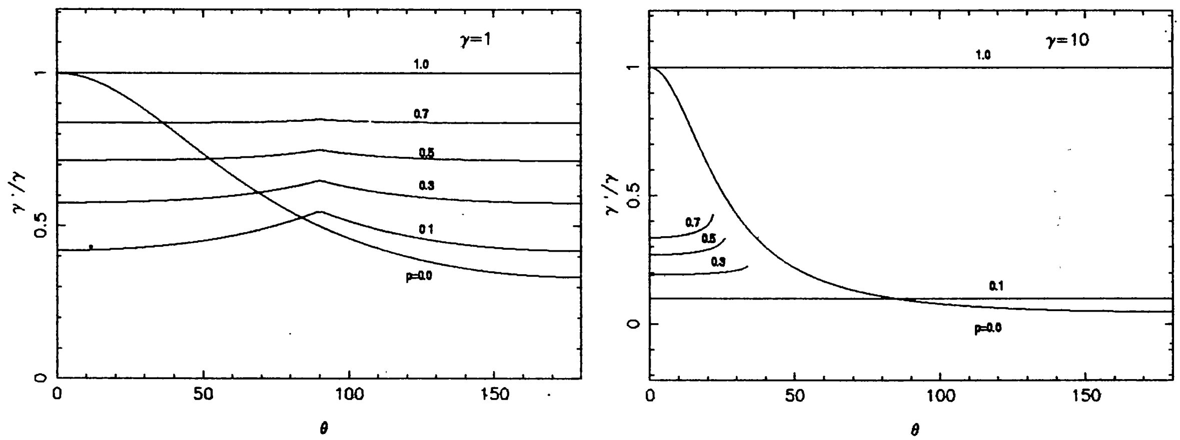

Figure 2. Variation of the second root (γ'/γ) of equation (2) vs the scattering angle θ for different values of the plasma density parameter p and initial photon energy γ.

where $\gamma = \hbar\omega/mc^2$ and $\gamma' = \hbar\omega'/mc^2$ are the initial and the final photon energies related by the energy-momentum conservation law (Eq. (2)),

$$G = \left[1 + p\gamma + p'\gamma'\sqrt{1-p}\cos\theta/\sqrt{1-p'}\right]^{-1}\sqrt{1-p'}$$

Here θ is the scattering angle, $A = (1 + p\gamma/2)^2, B = (1 - p'\gamma'/2)^2$, $p = \omega_p^2/\omega^2$, $p' = \omega_p^2/\omega'^2$ and $r_0^2 = (e^2/mc^2)^2$. One can easily identify the Thomson, the Klein-Nishina and the additional terms which depend upon the plasma frequency. Unlike in vacuum, the initial and the final photon frequencies are related through a quadratic expression. For every incoming photon, there are two possible scattered photons. The two values of the ratio (γ'/γ) are plotted against the scattering angle θ for various values of p and γ in figures (1) and (2). The first root (for

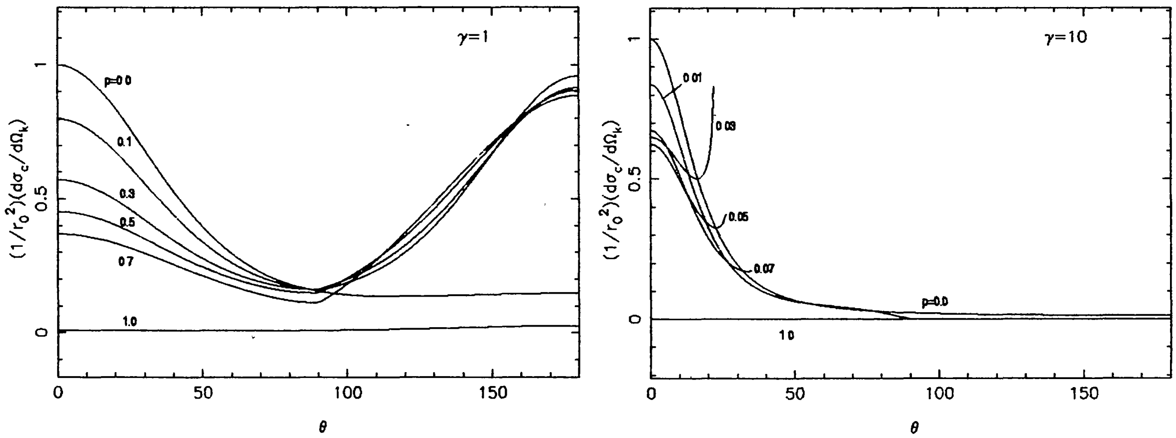

Figure 3. Variation of the differential cross section for the Compton scattering with the scattering angle θ for different values of the plasma density parameter p, the initial photon energy γ and the final photon energy γ' corresponding to the first root of equation (2).

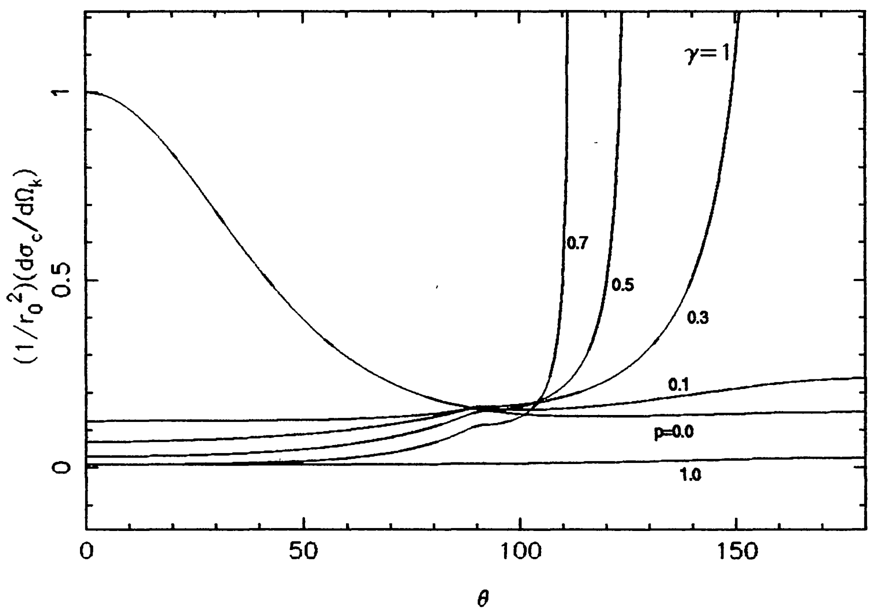

Figure 4. Variation of the differential cross section for the Compton scattering with the scattering angle θ for different values of the plasma density parameter p, the initial photon energy $\gamma = 1$ and the final photon energy γ' corresponding to the second root of equation (2).

plus sign) corresponds to an increase of the frequency of the scattered photon as the plasma density or the parameter p increases for $\gamma = 1$. For $\gamma = 10$, the variations with p are very small. The second root also increases with p but lies below the $p = 0$ line for small values of scattering angle θ. Further the quadratic (2) has real roots only for a limited range of scattering angle for a given value of p. The variation of the two differential cross sections corresponding to the two

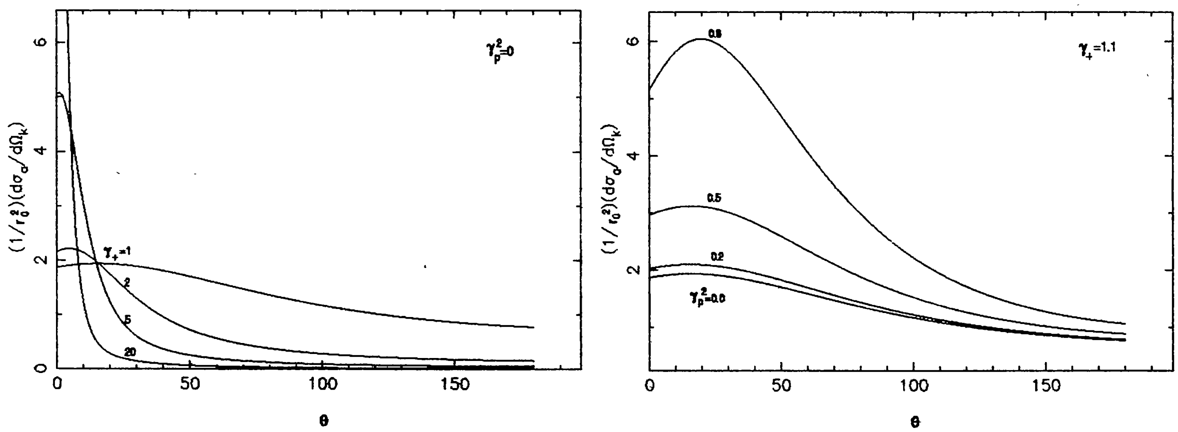

Figure 5. Variation of the differential cross section for pair annihilation with the angle θ between the positron beam and the photon (γ_1) for different positron energies γ_+ and plasma density parameter γ_p^2.

roots with θ for different values of the parameter p and initial photon energy γ is shown in figures (3) and (4). The cross section corresponding to the first root is found to decrease significantly with an increase in p for $\theta < 90°$ and increase for $\theta \geq 90°$. For $\gamma = 10$, the cross section begins to increase for $\theta << 90°$. Further the scattering takes place only for limited values of the scattering angle. The cross section for the second root (Figure 4) shows a similar behaviour but becomes very very large for large scattering angles. This root may not represent the physical situation.

2.1. Pair Annihilation

The cross section for annihilation of an electron with four-momentum p_- and spin S_- and a positron with four-momentum p_+ and spin S_+ into two photons with four-momenta and polarizations k_1, Λ_1 and k_2, Λ_2 is found.

$$\frac{d\bar{\sigma}_a}{d\Omega_{k_1}}\frac{1}{r_0^2} = \frac{(1+\gamma_+)\left[\frac{\gamma_2}{\gamma_1}\frac{1}{A_1} + \frac{\gamma_1}{\gamma_2}\frac{1}{B_2} + D\sin^2\theta_o + E\right]\sqrt{1-p_1}}{2\beta_+\{1+\gamma_+ - (1-p_1)^{-1/2}\beta_+\cos\theta\}\{1+\gamma_+ - (1-p_1)^{1/2}\beta_+\cos\theta\}} \tag{4}$$

where $\gamma_+ mc^2$ is the positron energy, γ_2 and γ_1 are the energies of the two photons, $A_1 = (1 - p_1\gamma_1/2)^2, B_2 = (1 - p_2\gamma_2/2)^2$,

$$D = 1/A_1B_2 - (1 - \sqrt{A_1/B_2})\gamma_1(1-p_1)/2A_1 - (1 - \sqrt{B_2/A_1})\gamma_2(1-p_2)/2B_2$$

$$E = (-p_1\gamma_1/2A_1B_2)\{(-2\gamma_1/\gamma_2)\sqrt{A_1B_2} + A_1(\gamma_1^2/\gamma_2^2 - \gamma_1/\gamma_2) - B_2(1-\gamma_2/\gamma_1) - 2\sqrt{A_1B_2}\}$$

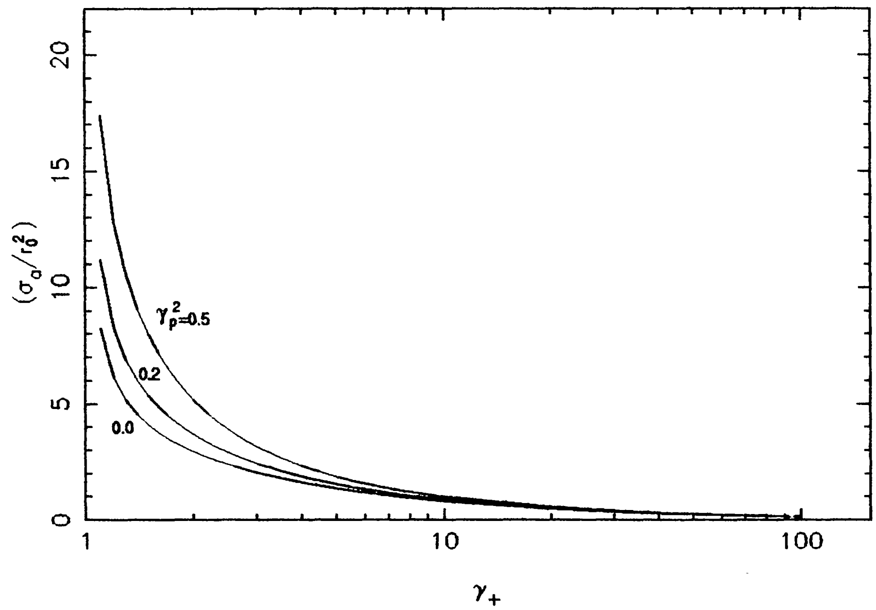

Figure 6. Variation of the total cross section σ_a for pair annihilation with the positron energy γ_+ for different values of the plasma density parameters γ_p^2.

$$\sin^2\theta_o = \frac{\beta_+ \sin\theta}{\gamma_2(1-p_2)^{1/2}},\quad p_1 = \gamma_p^2/\gamma_1^2,\quad p_2 = \gamma_p^2/\gamma_2^2,\quad \gamma_p = \hbar\omega_p/mc^2, \beta_+^2 = \gamma_+^2 - 1,$$

$$\gamma_2 = \gamma_+ + 1 - \gamma_1,\quad \gamma_1 = \frac{1+\gamma_+}{1+\gamma_+ - (\gamma_+ - t)\sqrt{1-p_1}},\quad \cos\theta = \frac{\gamma_+ - t}{\beta_+}$$

The variation of the differential cross section for pair annihilation with θ, the angle between the positron beam and the photon (γ_1), for different values of the positron energy γ_+, plasma density parameter γ_p^2 is shown in figures 5 and 6. Integrating over the solid angle $d\Omega_k$ the total cross section σ_a for pair annihilation can be determined. The variation of σ_a with γ_+ for different values of γ_ρ^2 is shown in figure (7). It is seen that the cross section for pair annihilation increases with an increase in the plasma density.

2.2. Pair Production

It is known that the total cross section σ_p for pair production is related to σ_a (Reinhardt 1992, 1994) as: $\sigma_p = 2(\gamma_+^2 - 1)\sigma_a/\gamma_+^2$. The variation of σ_p with γ_+ for different values of γ_p^2 is shown in figure (8). One can see the threshold behaviour of the pair production cross section as well as its increase with an increase of the plasma density.

The enhancement of the pair annihilation and production cross sections and the decrement of the Compton scattering cross section (for small θ) in a plasma

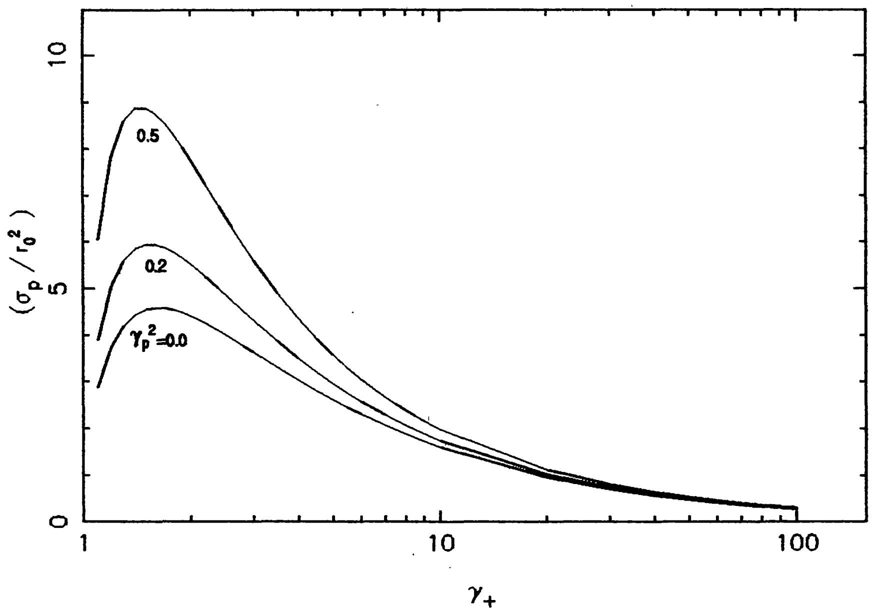

Figure 7. Variation of the pair production cross section σ_p with positron energy γ_+ for different values of the plasma density parameters γ_p^2.

are the consequences of the finite mass that a photon acquires in a plasma. Typical plasma densities at which the effects are appreciable are of the order of 10^{30} cm^{-3}.

3. Whence such effects

The importance of the plasma effect can be quantified in terms of the parameter $p = \omega_p^2/\omega^2$. We find $p = 3 \times 10^{-27}$n for ω at 1KeV. Thus, it is evident that the Compton cross sections will undergo substantial changes in compact objects such as the electron-positron supernovae. Also in the early universe when the electron density is proportional to the cube of the temperature, extremely high density plasma and radiation provide the right conditions for the plasma affected Compton processes. Other likely objects could be the $\gamma-$ray bursts. In their so called fireball model, the initial density before it undergoes substantial expansion could be extremely large (Mitra 1998). The full import of the plasma effects on the Compton processes in astrophysics awaits to be appreciated.

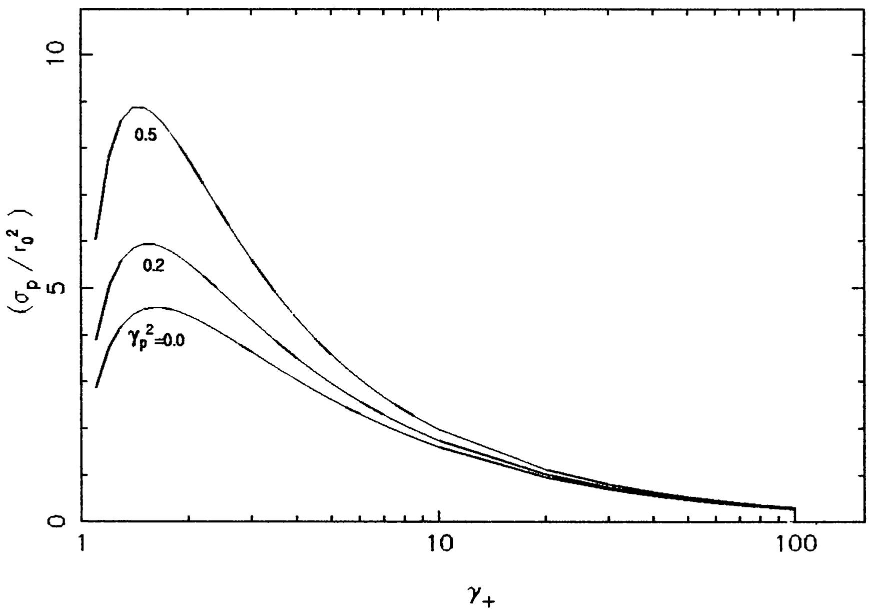

Figure 8. Variation of the pair production cross section σ_p with positron energy γ_+ for different values of the plasma density parameters γ_p^2.

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