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The effect of aberrations on image quality – A study for the 1.2 m Gurusikhar Infrared Telescope

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Abstract. The achromatic aberrations, which in general affect image quality in telescopes, have been studied in context of the 1.2 m Gurusikhar Infrared Telescope. The analysis gives the magnitudes of aberrations for different fields of view at the Cassegrain and prime focus thereby indicating the optimal field diameter that may be used for observations. The range of acceptable back - focal distances have also been worked out, thereby setting constraints on the location of the focal plane in back - end instruments to be used on the GIRT. This study should be especially useful while designing instruments for the 1.2 m telescope, in planning observations or in subsequent data analysis.

Key words : aberrations, Cassegrain focus, prime focus

1. Introduction and Aim

The 1.2m Gurusikhar Infrared Telescope (GIRT) of the Physical Research Laboratory, Ahmedabad has been operational since the last few years (Deshpande 1995) and its optical performance, following the refiguring of the primary mirror by the Sinden Optical Works, U.K. in 1994, has been good. Achromatic aberrations, however, affect telescope performance in general. In this study, we have quantified the aberrations for the 1.2 m reflector at the Cassegrain and prime focus, with emphasis on the former. The aim of this analysis is manifold. Several back - end instruments, both of the home institute and other institutions, are presently being used or are being planned for the 1.2m telescope. In instances, where imaging applications are pursued, it is necessary to know the optimum field-of-view within which the image quality is acceptable. This can be quantified by examining the aberrations at the edge of the field. Further, off-axis guiding generally selects stars well-removed from the centre of the field and the image quality of the guiding star is therefore likely to be affected by optical aberrations.

Quantifying these can help in ensuring accurate guiding. A third important point pertains to fixing an optimal range for the back focal distance i.e. the distance from the primary mirror surface to the final Cassegrain focus. This parameter essentially decides, for a given back-end instrument, the location of the instrument's focal plane from the Cassegrain plate. As will be shown later, a restricted range is permissible beyond which the image quality begins to deteriorate due to spherical aberration. This study will therefore help in fixing the focal plane position while designing instruments for the GIRT. This is specially relevant in the case of instruments designed by other institutions and intended to be used on the GIRT (some are already being used), where the designers will not have easy access to the GIRT optical lay-out details. The current study therefore, has been motivated by all these points outlined above, and principally aims at providing an analysis which should be beneficial to all users of the GIRT.

2. Optical configuration and lay-out

We first present some of the optical parameters of the GIRT (Technical Report on GIRT optics; Sinden Optical Company correspondence), which are required for subsequent calculations, and also give the symbols.

R = Axial radius of curvature of primary (paraboloid) = 7242 mm

f_1 = focal length of primary mirror = $R / 2$

D_1 = Diameter of primary mirror = 1210 mm

b_1 = Conic constant of primary = - 1

f_2 = focal length of secondary (convex hyperboloid) = 1047 mm

D_2 = Diameter of secondary mirror = 301 mm

b_2 = Conic constant of secondary = - 2.56

Movement of secondary from nominal position = 20 mm up, 60 mm down

The conic constant, in general, is defined as $b = -(eccenticity)^2$. In figure 1, the optical configuration of the telescope is shown. The relevant quantities here are :

p = distance from secondary mirror to primary focus

p' = distance from secondary mirror to final focus

d = $f_1 - p'$ = distance between the primary and secondary mirror

A = secondary amplifying ratio ($A = p' / p$)

f = effective focal length ($f = Af_1$)

l = $p' + p - f_1$ = back focal distance or distance from primary mirror surface to final focus

The values of p and p' are determined by the criterion that the Cassegrain focus should be free of spherical aberration. For this, the secondary should have a conic constant defined by the condition

$$b_2 = -\left(\frac{A+1}{A-1}\right)^2 \tag{1}$$

Since $b_2 = -2.56$, the secondary magnification ratio $A = 4.333$. Hence $p' = 4.333p$. Also, from the Gaussian formula we have, $1/p' + 1/p = 1/f_2$. Therefore, solving for p and p' give $p = 805.4$ mm and $p' = 3490$ mm. These values of p and p' give the optimum configuration of the telescope for the best focus at the Cassegrain end.

3. Aberrations at the Cassegrain focus

The theory of aberrations in telescopes has been well studied and is presented in several places (eg. Wilson 1996; Schroeder 1987; Lucas 1978). We have followed the treatment of Lucas, which is based on the pioneering work of the German astronomer, K. Schwarzchild. The equations used here are based on the third - order theory of aberrations. It may be pointed out, that there were a few errors in the equations of Lucas (missed symbols, typographical errors etc.) which the authors have taken care off.

We first study aberrations at the Cassegrain focal plane. The aberrations (measured in seconds of arc) at the edge of a field of diameter α (measured in degrees) are given by (Lucas, 1978)

$$\text{Coma (t)} = 1351\alpha\, D_1^2\, F \tag{2}$$

$$\text{Radial Astigmatism} = 15.7(2C + D)\, \alpha^2\, D_1 \tag{3}$$

$$\text{Tangential Astigmatism} = 15.7\, \alpha^2\, D_1 D \tag{4}$$

$$\text{Distortion} = -\, 0.138\alpha^3\, E \tag{5}$$

and the field curvature parameters are given by :

$$\frac{1}{g_t} = 2(2C + D) \tag{6}$$

$$\frac{1}{g} = 2(C + D) \tag{7}$$

$$\frac{1}{g_s} = 2D \tag{8}$$

In the above equations, the coefficients C, D, E, F given by :

$$C = \frac{Ad}{2p'f} \tag{9}$$

$$D = \frac{(A2 - 1)\, d}{2p'f} \tag{10}$$

$$E = -\frac{(A - 1)\, df}{4p'2} \tag{11}$$

$$F = -\frac{1}{4f^2} \tag{12}$$

It may be noted that the distortion in equation 5 is of the pin - cushion type and the coma referred to (equation 2) is the tangential coma. The field curvature parameters g_t and g_s give the radius of the curved surfaces on which the tangential and sagittal astigmatic images are formed. The best focus images are however formed on the curved surface having radius g. On any other flat surface, separate from these three curved surfaces, the image has an elliptical shape whose dimensions are given by equations 3 and 4 (Lucas, 1978).

Substituting the values of f, p, p', A, d and D_1 from section 2 in equations 9 to 12, and then using equations 2 to 5, the magnitude of the aberrations for different fields of view can be calculated. These values are shown in table 1, for the telescope in the optimal condition i.e. $A = 4.333$ and there is no spherical aberration in the Cassegrain image.

Table 1 . Aberrations at Cassegrain focus at optimum position .

Field diameter (α) (arc min)	Coma (arc sec)	Radial astigmatism (arc sec)	Tangential astigmatism (arc sec)	Distortion (arc sec)
5.0	0.17	0.09	0.06	0.0
10.0	0.33	0.36	0.24	0.0
15.0	0.50	0.81	0.54	.01
20.0	0.67	1.44	0.96	.02
30.0	1.00	3.23	2.17	.05
40.0	1.34	5.74	3.86	.12
50.0	1.67	8.97	6.03	.24
60.0	2.01	12.92	8.68	.42

$g_t = 735.4$ mm $g_s = 1094$ mm $g = 879.6$ mm Sph. Ab. = 0

As may be seen the coma and distortion are relatively unimportant, even for a large diameter of the field. However the radial and tangential astigmatism start becoming severe beyond the edge of a 10 arc minute field. For astronomical purposes, a reasonable limit on an acceptable field diameter can be set by considering the point where the seeing and the magnitude of the aberrations become comparable. At Gurusikhar, the full width to half maximum of the seeing profile is typically between 1.0 to 1.5 arc seconds. Under good seeing conditions, it may thus be said, *that a field of 10 to 12 arc minute diameter is optimal for observations.*

3.1 Movement of secondary mirror and spherical aberration at the Cassegrain focus :

When the secondary mirror is moved up or down, the distances p and p' are changed (refer figure 1). As a consequence, the secondary amplifying ratio $A = p'/p$ also changes. Hence, equation 1 is no longer satisfied and spherical aberration will affect the image at the Cassegrain focus. The magnitude of this tranverse spherical aberration (TSA) is given by (Schroeder, 1987)

$$TSA = \left(\frac{f D_1^3}{8R^3}\right)\left[b_1 + 1 - \frac{k^4}{\rho^3}\left(b_2 + \left[\frac{A+1}{A-1}\right]^2\right)\right] \tag{13}$$

where $k = D_2 / D_1$ and $\rho = Ak/(A-1)$.

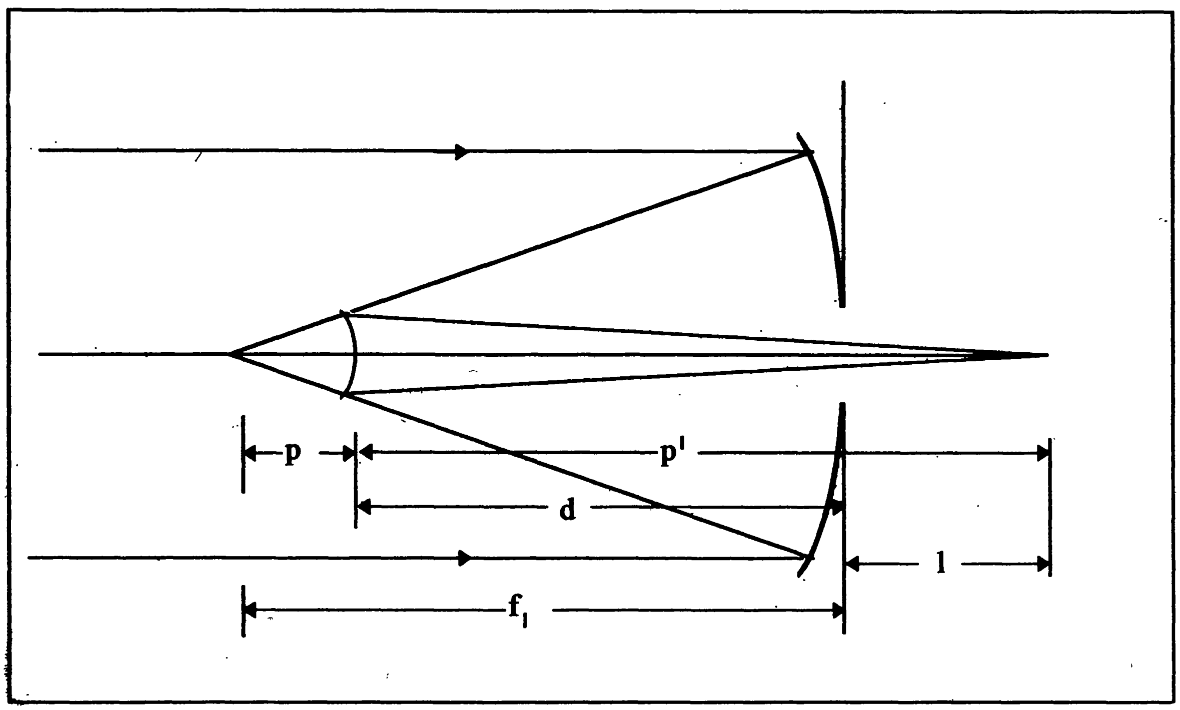

Figure 1. The optical configuration of the telescope showing the relevant parameters.

It may be seen, that the condition for zero spherical aberration expressed in equation 1 is inbuilt in the above equation. This is because $b_1 = -1$ for a paraboloid, and for no spherical aberration (TSA = 0), it can be seen from equation 13 that the condition of equation 1 must hold. Further, it may also be noted from equation 13, that the two variables on which the TSA effectively depends are the magnification factor A and the effective focal length f, which in turn depend on the position of the secondary. Since all quantities in the right hand side of equation 13 are known, the TSA for different positions of the secondary may be calculated. This data is presented in table 2. In it we have calculated TSA/2, instead of TSA, since the former represents the diameter of the circle of least confusion (Schroeder, 1987), and gives a good visualization of the spread of the image at best focus.

Table 2. Spherical Aberration at Cassegrain focus with movement of secondary mirror

Secondary movement (UP – ; Down +) (mm)	p' (mm)	Secondary amplifying ratio (A)	Effective focal length (f) (mm)	Back focal distance (l) (mm)	TSA/2 (arc sec)
– 10	3310	4.161	15068	483.9	.61
– 8	3345	4.194	15188	520.8	.49
–6	3380	4.228	15311	558.3	.37
–4	3416	4.263	15436	596.4	.25
–2	3453	4.298	15563	635.0	.13
0	3490	4.333	15691	674.3	.00
2	3528	4.370	15822	714.1	.13
4	3566	4.406	15956	754.6	.25
6	3606	4.444	16091	795.8	.38
8	3645	4.482	16229	837.6	.51
10	3686	4.520	16369	880.1	.65

In column 1 of table 2, the secondary mirror movement has been given with reference to the optimum position at which $A = 4.333$ (i.e. $p = 805$ mm, $p' = 3490$ mm). Further, the back focal distance has been shown only for a limited range of the secondary mirror position, much lesser than the –20 mm up and 60 mm down movement possible. In fact, the maximum and minimum values of the back focal distance are 307 mm and 2233 mm corresponding to the secondary position being 20 mm upwards or 60 mm downwards of the optimal position. However this entire range of back focal distance cannot be used because the image is affected by spherical aberration. As may be seen, *the image at the Cassegrain focus will be characterised by a circle of least confusion with diameter more than 0.5 arc seconds if the focus is moved more than approximately 150 mm from the optimal position of 674 mm. The exact range of the back focal length is 519 to 833 mm.* The total dimensions of all elements between the primary mirror surface and the Cassegrain plate, where back-end instruments are coupled, is 316 mm for the GIRT. Therefore the focal - plane within the back-end instrument should preferably be within the range of 203 to 517 mm from the Cassegrain plate with the nominal position at 358 mm. It may be mentioned, that the other aberrations considered here (coma, astigmatism etc.), change very marginally at the Cassegrain focus from the values given in Table 1, for other positions of the secondary mirror and are hence not tabled here.

4. Aberrations at the prime focus

Aberrations at the prime focus arise only from the primary mirror. Although at present, no observations are being done at the prime focus, it is envisaged that in the future, optical - fibres may be employed there to feed the signal to a grating spectrograph system or other instruments. Thus the aberrations at the prime focus are also presented. From Lucas (1978), the aberrations are again given by equations 2 to 5 but the coefficients B, C, D, E, F are now given by :

$$B = \frac{(b_1 + 1)}{8 f_1^3} \tag{14}$$

$$C = \frac{1}{2f_1} \tag{15}$$

$$D = E = 0 \tag{16}$$

$$F = -\frac{1}{4 f_1^2} \tag{17}$$

The additional coefficient B introduced here is needed because the spherical aberration at the prime focus is given by :

$$\text{Sp. Aberration} = 51{,}566\, D_1^3 B \tag{18}$$

The magnitudes of the aberrations calculated from equations 2 to 5, equations 14 to 17 and equation 18 are presented in table 3.

Table 3. Aberrations at prime focus

Field dia (α) in arc minn.	Coma in arc sec.	Radial astigmatism in arc sec.	Other aberrations
1.0	0.63	.00	Sph. Ab. = 0
2.0	1.26	.01	Tang. Astig = 0
3.0	1.89	.01	Distortion = 0
4.0	2.51	.02	
5.0	3.14	.04	
6.0	3.77	.05	
7.0	4.40	.07	
8.0	5.03	.09	
9.0	5.66	.12	
10.0	6.29	.15	

As may be seen from the table 3, spherical aberration, tangential astigmatism and distortion are zero at the prime focus and radial astigmatism is relatively insignificant. The limit on the usable field is set by the coma which is rather severe even for a moderate field of view. Hence, if observations over an extended field are to be done then it is essential to have a corrector at the prime focus (e.g. a Wynne triplet or a equivalent optical component).

To conclude this work, the results presented here are briefly summarized in the next section.

5. Summary

The third - order theory of aberrations have been used to quantify the aberrations at the Cassegrain and prime focus of the 1.2m Mount Abu reflector. At the Cassegrain focus, under best focus conditions, it is found that optimum image quality will be obtained in a 10 to 12 arc minute field of view. Instruments for Cassegrain use, should have the focal-plane located within a range of 203 to 517 mm from the Cassegrain plate, and nominally at 358 mm, to minimize the effects of spherical aberration deteriorating image quality. The magnitudes of the aberrations for different fields of view are given in the three tables presented here.

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