Bull. Astr. Soc. India (1997) **25**, 485-488

Thermodynamic inflation

B.G. Sidharth

B. M. Birla Science Centre, Adarsh Nagar, Hyderabad 500 063, India

Received 3 March 1997; accepted 28 May 1997

Abstract. Using the well known fact that at very high temperatures, Bosons and Fermions both satisfy the same energy, pressure and volume relation, we deduce an inflationary scenario for the early universe.

It is well known that at high temperatures the Fermi-Dirac statistics and Bose-Einstein statistics converge to the Maxwell-Boltzmann statistics [1]. In-fact we have for a Bose gas (Huang, 1975)

$$\frac{P}{kT} = \frac{4\pi}{h^3} \int_0^\infty dp p^2 log\,(1 - ze^{-\beta p^2/2m}) - \frac{1}{V} log\,(1 - z) \tag{1}$$

The last term in (1) corresponds to the momentum $\vec{p} = 0$. The average occupation number of this momentum is given by

$$< n_0 > = \frac{z}{1 - z} \tag{2}$$

and as $z \to 1$ corresponding to Bose condensation, the last term in (1) tends to ∞.

However if there is a negligible fraction of Bosons corresponding to $\vec{p} = 0$, that is, as can be seen from (2) if $z << 1$ then the last term in (1) can be dropped and as is well known (Huang, 1975), we get

$$U = \frac{3}{2} PV \tag{3}$$

Equation (3) also holds good for a Fermi gas.

There is another way of looking at this situation. The occupation number for photons of momentum $\vec{k}$ irrespective of polarization is :

$$< n_k > = \frac{2}{e^{\beta \hbar \omega} - 1}, \beta \equiv \frac{1}{kT}, \tag{4}$$

If all the photons are of very high frequency that is

$$\hbar \omega \geq kT \tag{5}$$

then we have from (4),

$$< n_k > \sim 1 \text{ or } 0 \tag{6}$$

exactly as for Fermions.

Infact from both points of view, the fact that $z << 1$ and $T \to \infty$ implies that Bose-Einstein statistics and Fermi-Dirac statistics coincide as pointed out earlier.

While the well known fact that equation (3) holds good for Bosons also under the conditions stated is all that is required in the sequel, let us now examine this circumstance from the point of view of Quantum Field Theory. For simplicity we consider the Klein-Gordon field. The Hamiltonian obtained from a suitable variational principle is (Bjorken & Drell 1965):

$$H = \frac{1}{2} \int d^3 k w_k \, [a_k a_k^+ + a_k^+ ak] \tag{7}$$

where $a^+ (k)$ and $a(k)$ are the creation and destruction operators. It is at this stage that the commutation relations between these operators are invoked to obtain the various harmonic oscillator levels of the boson field. It is also known (reference 2) that if instead, we invoke anti commutators for $a^+(k)$ and $a(k)$ then H in (7) becomes an infinite constant, as can be seen by substitution.

However in a high energy situation where the temperature $T \to \infty$, or for practical purposes $T >> 1$ where condition (5) holds good for all the participating Bosons, the occupation numbers for all but high energies vanish, and the infinite constant poses no problem in the sense that H is extremely large. This means that we can use anti commutators so that we have the usual Fermionic occupation number,

$$a^+ a = N = 1 \text{ or } 0,$$

which agrees with (6). That is, in this situation, the Bosons actually behave like Fermions.

One way of looking at this is that as all the Bosons have high energy, the uncertainty in their energy is also high and it is no longer meaningful to talk about Bosons occupying well defined energy levels, as we can in normal circumstances.

A testing ground for the Fermionic behaviour of Bosons would be in the very early universe, or indeed any other collection of Bosons at a sufficiently high temperature. Because of the Fermionic behaviour, the energy would be effectively greater. Indeed in this case we have to use equation (3) for the pressure instead of the usual relation,

$$U = \frac{1}{3} PV \tag{8}$$

Thus we should have an inflation type scenario (Guth 1981). To examine this further, using (3) instead of (8) in the usual cosmological equations (Misner et al. 1973), we get for the radiation dominated phase of the early universe,

$$\dot{\rho}'_r + \rho'_r \dot{V}' + \frac{2}{3} \rho'_r \dot{V}' = 0$$

so that,

$$\rho'_r V'^{5/3} = \text{constant}$$

or

$$\rho'_r a'^5 = \text{constant} \tag{9}$$

where ρ and a stand for the density of the radiation and scale factor of the universe respectively and primes denote that we are in the "effective" scenario of Fermionic behaviour.

In the usual picture we get (cf. Misner et al. 1973)

$$\rho_r a^4 = \text{constant} \tag{10}$$

Also substitution of (9) in the field equations gives

$$\left(\frac{\dot{a}'}{a'}\right)^2 = \frac{8\pi}{3} a'^{-5} \quad (a' << 1)$$

or

$$a' \propto t^{2/5} \tag{11}$$

as against

$$a \propto t^{1/2} \tag{12}$$

which we get using (10), in the usual formulation.

Effectively the scale factor of the early universe is a' given by (11) and not a given by (12) as we would normally expect. Comparing (12) and (11) we get

$$a' = Za, Z = t^{-\frac{1}{10}} \tag{13}$$

We can see immediately that the factor Z becomes arbitrarily large as $t \to 0$. Effectively at a given time, the volume of the early universe is increased by a factor Z^3, while the entropy which is of the order of the total number of particles at that time remains constant. Equivalently the entropy is effectively reduced by the factor Z^{-3}, the volume or scale factor being treated as the same in both the scenarios. In any case the expansion is effectively non adiabatic unlike in the standard formulation. This is exactly the starting point for the inflationary scenario (Guth 1981).

If as is required, Z has to be of the order 10^{27}, this requires that

$$t \sim 10^{-270} \text{ seconds.}$$

While this is well within the Planck epoch as infact is desirable [Abbott & Pi 1986), the effects are pushed back very much closer to $t = 0$!

In any case we have an inflation type scenario without invoking GUTS or SUSY, but we have treated Bosons and Fermions on the same footing via equation (3).

References

Abbott L.F., Pi, S – Y., (Ed.) 1986, Inflationary Cosmology, World Scientific, Singapore.

Bjorken J.D., Drell S.D., 1965, "Relativisitic Quantum Fields", McGraw-Hill Book Co., New York.

Guth A.H., 1981, Phys. Rev. D23, 347.

Huang K., 1975, Statistical Mechanics, Wiley Eastern Ltd., New Delhi.

Misner C.W., Thorne K. S., Wheeler J.A., 1973, Gravitation, W.H. Freeman, San Francisco.