

## Correlated projected properties of triaxial mass models

Parijat Thakur and D.K. Chakraborty<sup>1</sup>

*School of Studies in Physics, Pt. Ravishankar Shukla University, Raipur 492010, India*

<sup>1</sup>*IUCAA, Post Bag 4, Ganeshkhind, Pune 411007, India*

**Abstract.** Triaxial mass models producing ellipticity variations and isophote twists are investigated. The spherical mass distributions are made triaxial by adding two more terms, each consisting of a radial function multiplied by a low order spherical harmonic.

Modified Hubble mass model and  $\gamma$ -models of Dehnen are investigated. It is found that the projected properties are correlated when a model with a given set of intrinsic parameters, is viewed in all possible orientations. We propose that these correlation patterns carry the signature of intrinsic shape parameters of the triaxial mass model.

*Key words* : galaxies : elliptical - galaxies : structure

### 1. Introduction

Following a classical scheme set by Schwarzschild (1979) for producing a triaxial mass model, deZeeuw and Carollo (1996), (hereafter, ZC96) studied the projected properties of a triaxial generalisation of the spherical  $\gamma$  - models of Dehnen (1993). Adopting a similar scheme, we (Chakraborty and Thakur, 2000, hereafter, CT2K) studied the projected properties of a triaxial generalisation of a modified Hubble mass model. It is found that the triaxial mass models of these kinds exhibit ellipticity variations and isophote twists in their projections, which can be compared with observations.

We investigated the models of CT2K and ZC96 further. We found strong correlations between the axis ratios of the elliptical isophotes, evaluated at small and at large radii when a model with a given set of intrinsic parameters is viewed in all possible orientations.

We propose that the correlation patterns can be utilised to set constraints on the intrinsic shape of a triaxial mass model independent of the viewing angles, using the methods of Bayesian statistics (Statler 1994a, b; Statler and Fry, 1994).

---

e-mail : [ircrsu@bom6.vsnl.net.in](mailto:ircrsu@bom6.vsnl.net.in)

In section 2, we present the triaxial mass models and their correlated projected properties. Section 3 is devoted for results and discussion.

## 2. Triaxial mass models and their correlated projected properties

The density distribution  $\rho(r, \theta, \phi)$  in usual spherical coordinates  $(r, \theta, \phi)$  is taken as

$$\rho(r, \theta, \phi) = f(r) - g(r)Y_2^0(\theta) + h(r)Y_2^2(\theta, \phi) \quad (1)$$

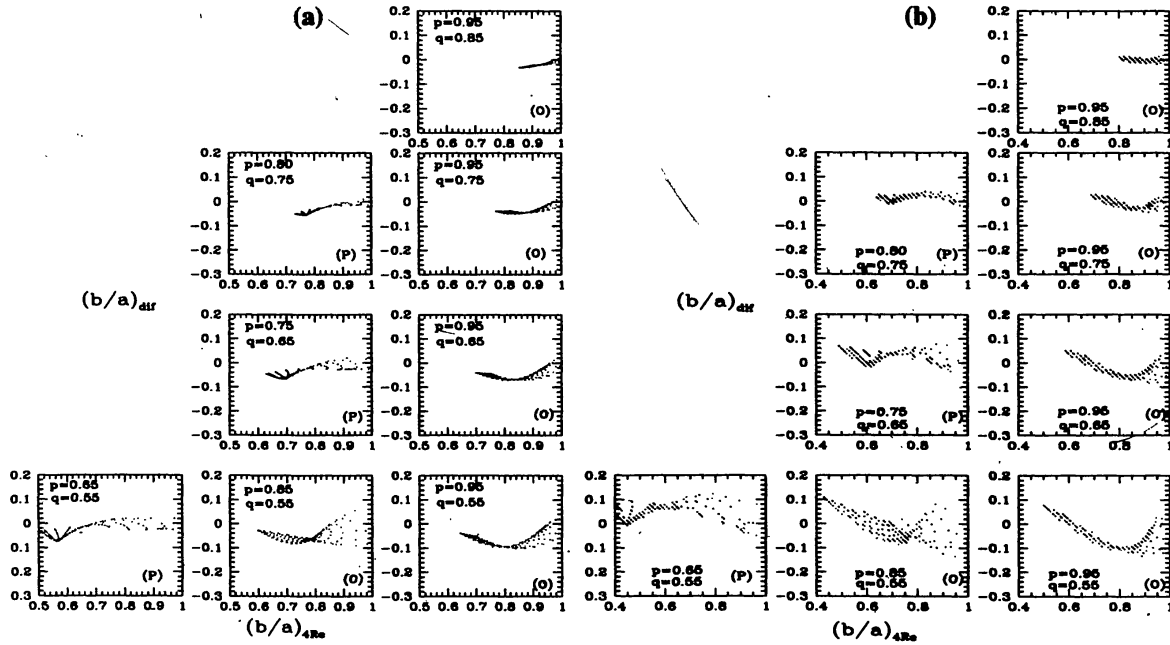
where  $f(r)$ ,  $g(r)$  and  $h(r)$  are the same as adopted in CT2K and in ZC96.  $Y_2^0 = \sqrt{3/2} \cos^2 \theta - 1/2$  and  $Y_2^2 = 3 \sin^2 \theta \cos 2\phi$  are the usual spherical harmonics.

The projected surface density  $\Sigma(R, \Theta)$ , where  $(R, \Theta)$  are the plane polar coordinates in a plane perpendicular to the line of sight along the viewing angles  $(\theta', \phi')$ , can be calculated by evaluating certain  $r$ -integrals of the radial functions  $f$ ,  $g$  and  $h$  (cf : ZC96, CT2K). This allows one to calculate the axis ratio  $\frac{b}{a}$  and the position angle  $\Theta_*$  of the major axis of the approximate elliptical constant surface density contours quite easily and often, analytically.

Besides exhibiting  $\frac{b}{a}$  and  $\Theta_*$  variations along  $R$ , these models show very interesting properties of correlations between the axis ratios at a small and at a large  $R$ , and also, between the position angles at a small and at a large  $R$ . Figure 1 (a, b) show the correlations between  $\left(\frac{b}{a}\right)_{4Re}$  and  $\left(\frac{b}{a}\right)_{dif} = \left(\frac{b}{a}\right)_{.25Re} - \left(\frac{b}{a}\right)_{4Re}$  for CT2K, and ZC96 models with  $\gamma = 0$  (core version of ZC96), respectively. From the figure 1 (a, b), we find that the points are strongly correlated, and further, the patterns are very much different between a prolate triaxial and an oblate triaxial, occupying almost non-overlapping distinct regions in the parameter space of  $\left(\frac{b}{a}\right)_{4Re}$  and  $\left(\frac{b}{a}\right)_{dif}$  when a CT2K model or a ZC96 model, with a particular choice of  $p = p_o = p_\infty$  and  $q = q_o = q_\infty$ , as indicated in the plots, is projected in varying viewing angles. We repeated the same for a ZC96 model with  $\gamma = 1, 2$  and found that the essential features of the correlations are still maintained. Since ZC96 has worked with small changes between  $(p_o, p_\infty)$  and between  $(q_o, q_\infty)$ , we studied the correlations by allowing a non-zero, but small values of either  $\Delta p = |p_o - p_\infty| \sim 0.05$  or  $\Delta q = |q_o - q_\infty| \sim 0.05$ . We found that the correlation patterns are almost similar to those presented in figures 1 (a, b).

## 3. Results and discussion

Observable parameters  $\left(\frac{b}{a}\right)_{4Re}$  and  $\left(\frac{b}{a}\right)_{dif}$  exhibit correlations when a model with a particular choice of a set of intrinsic parameters, is projected in varying viewing angles  $(\theta', \phi')$ . Further, a comparison of figures 1a and 1b suggest that the ‘‘correlation patterns’’ depend on the radial density profile and the asymptotic axis ratios and are insensitive to the details in between (the  $g(r)$  and  $h(r)$  functions). These correlation patterns are maintained even if we change  $p_o, q_o$  slightly from  $p_\infty, p_\infty$  respectively. One can, therefore, exploit these correlations, shown by CT2K and ZC96 models, to estimate the intrinsic shape of an elliptical galaxy, independent of the choice of the models, following the procedure led by Statler and Fry 1994.



**Figure 1.** (a) Axis ratio  $(\frac{b}{a})_{4Re}$  and  $(\frac{b}{a})_{dif}$  for given  $p$  and  $q$  as indicated in each plot when a C12K model is projected at various viewing angles. (b) Axis ratio  $(\frac{b}{a})_{4Re}$  and  $(\frac{b}{a})_{dif}$  for given  $p$  and  $q$  as indicated in each plot when a ZC96 model with  $\gamma = 0$  is projected at various viewing angles. Symbols (P) and (O) indicate the prolate and oblate triaxials respectively.

### Acknowledgements

The authors express their sincere thanks to CSIR, New Delhi India for providing support through the project grant No. 03(0807)/97/EMR-II. We are also thankful to IUCAA, Pune, India for providing support and facilities which made this study possible.

### References

- Chakraborty D.K., Thakur Parijat, 2000, MNRAS, 318, 1273
- Dehnen W., 1993, MNRAS, 265, 250
- De Zeeuw P.T., Carollo C.M., 1996, MNRAS, 281, 1333
- Schwarzschild M., 1979, Ap.J., 232, 236
- Statler T.S., 1994a, Ap.J., 425, 458
- Statler T.S., 1994b, Ap.J., 425, 500
- Statler T.S., Fry A.M., 1994, Ap.J., 425, 481