Bull. Astr. Soc. India (1997) 25, 401–416

Averaging problem in general relativity, macroscopic gravity and using Einstein's equations in cosmology*

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Abstract. The averaging problem in general relativity is briefly discussed. A new setting of the problem as that of macroscopic description of gravitation is proposed. A covariant space-time averaging procedure is described. The structure of the geometry of macroscopic space-time, which follows from averaging Cartan's structure equations, is described and the correlation tensors present in the theory are discussed. The macroscopic field equations (averaged Einstein's equations) derived in the framework of the approach are presented and their structure is analysed. The correspondence principle for macroscopic gravity is formulated and a definition of the stress-energy tensor for the macroscopic gravitational field is proposed. It is shown that the physical meaning of using Einstein's equations with a hydrodynamic stress-energy tensor in looking for cosmological models means neglecting all gravitational field correlations. The system of macroscopic gravity equations to be solved when the correlations are taken into consideration is given and described.

1. The averaging problem in General Relativity

The usual practice of using Einstein's equations in looking for cosmological solutions is to assume that the real complex lumpy universe with a discrete matter distribution (stars, galaxies, clusters of galaxies, etc.) can be adequately approximated by a "smoothed", or hydrodynamic, stress-energy tensor usually taken to be representable by a perfect fluid. Such an approximation assumes an effective averaging of the discrete matter distribution. It is tacitly assumed at the same time, and it is the essence of employing the field equations of general relativity in modern comology, that Einstein's equations remain unchanged in the structure of their field operator under such an averaging. Apart from the question of whether or not a hydrodynamic picture is satisfactory for the matter distribution in the universe, such a procedure of employing Einstein's equations in cosmology raises questions of principle which

* Received Honourable Mention at the 1996 competition of the Gravity Research Foundation, Massachusetts, USA.

constitute the so-called averaging problem in general relativity [1] - [5] (see [3], [6] for a review and references). Indeed, a correct statement of the problem requires the averaging out of Einstein equations in both sides, in the matter source and in the field operator. And it is the averaged equations which should then be solved to find the relevant cosmological models[1]. The results of averaging are expected to be far from trivial, since Einstein's equations are highly non-linear, and the avaraging is likely to change their structure. Solving the averaged equations may therefore be expected to bring about a new view on cosmology, which could in turn alter our understanding and modify the predictions, which are heavily based, in modern cosmology, on the solutions of Einstein's equations.

This essay is aimed to give an overview of results on the averaging problem within the macroscopic gravity approach proposed recently [4], [8] - [13], and to reveal, within its context, the physical meaning and the range of validity of usign Einstein's equations with an averaged, hydrodynamic stress-energy tensor in studies of cosmological problems.

2. A setting of the problem of macroscopic gravity

The averaging problem in general relativity can be reformulated in a broader context as the problem of macroscopic description of gravitation [4], [8]-[11]. The idea of macroscopic gravity can be considered as an extension of Lorentz's idea, formulated first for electrodynamics [14], regarding the existence of two levels, microscopic and macroscopic, of understanding classical physical phenomena. The microscopic and macroscopic, of understanding classical physical phenomena. The microscopic description deals with the matter structured by discrete constituents, while the macroscopic description represents matter from a hydrodynamic point of view. However, unlike electrodynamics where the field operator is linear in the fields and it can be easily averaged out by applying either space, time, or statistical averagings (or a combination of them), and the remaining problem is the construction of models of continuous electromagnetic media which relates to the structure of averaged (macroscopic) current (see [15] for discussion and details), the problem of derivation of a macroscopic theory of gravity does require one to overcome severe difficulties, even on the first step of averaging the Einstein field operator. These are clearly connected with the non-flat geometry underlying the general relativity theory and the nonlinear character of the gravitational field, resulting in the need for considering the field correlations. In such a setting Einstein's equations are to be considered as the microscopic ones with a microscopic stress-energy tensor $t_{\beta}^{\alpha(\mathrm{micro})}$, which may be considered as well-grounded, for it is these equations that are believed to provide us with an exact solution for the gravitational field of an isolated point mass (Schwarzchild's solution)[2].

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1 The averaging problem in general relativity applies also in cases of using Einstein's equations inside gravitating extended bodies where a hydrodynamic matter model is being used again. Similar arguments as in cosmology question here the applicability of Einstein's equations in such settings, but this is not discussed in this essay (see, however, [7]).

2 There exists, however, another point of view where Einstein's equations are considered to be macroscopic in their nature and they can be used consistently only with a continuous matter model. Such a setting requres one to pose a problem of finding corresponding equations under an appropriate averaging procedure. This problem is similar to that which Lorentz had resolved by discovering a microscopic theory of electromagnetism and showing Maxwell's theory to be its macroscopic version.

It should be pointed out also that macroscopic description has a direct observational status since a (space-time) averaging procedure is a model of a measurement procedure. Physically, a macroscopic theory has a direct observational status and answers the questions of principle of which objects can be observed in classical measurement of fields and matter (as induction in electrodynamics, for example).

The overwhelming majority of approaches to the averaging problem [6] has three main features : (I) they are *perturbative* (an averaging of perturbed Einstein's equations is carried out); (II) they follow the generally adopted way, evoked by Lorentz's approach to electrodynamics, of an averaging of *Einstein's equations* to arrive at the averaged field equations and to understand thereby a structure of averaged gravity, and (III) no proposal has been made about the *correlation functions* which should inevitably emerge in an averaging of a non-linear theory. The results derived failed to give and, cannot in fact give, a satisfactory solution to the problem since (I') any perturbation analysis cannot give information about an averaged geometry; (II') an averaging of Einstein's equations themselves can be easily demonstrated to be insufficient because (III') they themselves cannot be used as a definition of the correlation functions if one wishes to have them as *the field equations*. Indeed, consider the Einstein equations in the mixed form

$$g^{\alpha\epsilon} r_{\epsilon\beta} - \frac{1}{2}\,\delta^{\alpha}_{\beta} g^{\mu\nu} r_{\mu\nu} = -\kappa t^{\alpha(\mathrm{micro})}_{\beta}, \tag{2.1}$$

which looks preferable for one must deal with only products of metric by curvature, as otherwise, in their other forms, one faces triple products of metric by metric by curvature. Suppose now that a space-time averaging procedure $\langle\cdot\rangle$ for tensors on space-time has been defined. Then the averaging of (2.1) brings

$$\langle g^{\alpha\epsilon} r_{\epsilon\beta}\rangle - \frac{1}{2}\,\delta^{\alpha}_{\beta}\langle g^{\mu\nu} r_{\mu\nu}\rangle = -\kappa\langle t^{\alpha(\mathrm{micro})}_{\beta}\rangle, \tag{2.2}$$

which can be rewritten as

$$\langle g^{\alpha\epsilon}\rangle\,\langle r_{\epsilon\beta}\rangle - \frac{1}{2}\,\delta^{\alpha}_{\beta}\langle g^{\mu\nu}\rangle\,\langle r_{\mu\nu}\rangle + C^{\alpha}_{\beta} = -\kappa\langle t^{\alpha(\mathrm{micro})}_{\beta}\rangle, \tag{2.3}$$

where $\langle g^{\alpha\epsilon}\rangle$ and $\langle r_{\epsilon\beta}\rangle$ denote the averaged inverse metric and the Ricci tensors, and C^{α}_{β} stands for a correlation function, which is just the difference between (2.2) and (2.3). The averaged Einstein equations (2.3) become now a definition of the correlation function. In order to bring back their status of the field equations one should define the object and find the properties and equations for C^{α}_{β} by using some information outside the Einstein equations (2.1).

To resolve all these problems it has been proposed in the macroscopic gravity approach that in order to derive the form of the averaged Einstein operator one should study, first of all, the problem of how *to average out a (pseudo-) Riemannian space-time* itself, i.e. Cartan's structure equations [16], [17] describing the structure of a (pseudo)-Riemannian geometry. While doing this it is necessary to understand which averaged geometrical object - metric, connection, or curvature - can characterize an averaged space-time. Another necessary part of such an approach is the splitting of the averages of products of the objects, being found in averaging out Cartan's equations. This is the problem of *introducing the correlation functions*. Upon deriving the structure equations for the averaged manifold, the Einstein equations which are known to be additional conditions to Cartan's equations, can successfully be averaged out. Such an approach to formulate a macroscopic theory of gravity is essentially *non-perturbative* and provides us with both *the geometry* underlying the macroscopic gravitational phenomena and the *macroscopic (averaged) field equations*.

3. A space-time averaging procedure

Let us remind the definition of the space-time averages adopted in macroscopic gravity [4], [8]. The space-time averaging procedure is a generalization of the space-time averaging procedure used in electrodynamics (see, for example, [18]-[21])[3] and it is based on the concept of Lie-dragging of averaging regions[4], which makes it valid for any differentiable manifolds with a volume n-form. Chosen a compact region $\Sigma \subset \mathcal{M}$ of a differentiable space-time manifold $(\mathcal{M}, g_{\alpha\beta})$ and a supporting point $x \in \Sigma$ to which the average value will be prescribed, the average value of an object (tensor, geometric objects, etc.) $p^{\alpha}_{\beta}(x)$, $x \in \mathcal{M}$, over a region Σ at the supporting point $x \in \Sigma$ is defined as

$$\bar{p}^{\alpha}_{\beta}(x) = \frac{1}{V_\Sigma} \int_\Sigma \mathbf{p}^{\alpha}_{\beta}(x,x') \sqrt{-g'} d^4x' \equiv \langle \mathbf{p}^{\alpha}_{\beta} \rangle, \tag{3.1}$$

3 Averaging procedures applied in the derivation of macroscopic electrodynamics have incorporated one or more combinations of three types of averaging procedures, viz., spatial, time, ensemble averagings. The discussion [22]-[25] seems to lead to a conclusion that a space averaging is always necessary and unavoidable in all macroscopic settings. Further application of either a time or statistical averaging depends on the problem of interest, or even could be unnecessary if there is no time periodicity in the problem under study. In generalization to a space-time formulation of macroscopic theories it is reasonable and mathematically preferable, in our opinion, to consider a space-time averaging procedure as fundamental one, with spatial averages taken as a projection of the space-time averages on hypersurfaces if the microscopic physics possesses no regularities and periodicity along a time-like direction. It is also known that it is space-time averages of physical fields that have the physical meaning [26], [27]. An alternative approach to that adopted here may be development of a space-time ensemble averaging procedure, though space-time procedures have its own advantages.

4 A definition of Lie-dragging (or, dragging) of a manifold's region of along a vector field (congruence), that is a mapping of the region into itself along the vector field (congruence), can be found in any standard textbook on differential geometry (see, for example, [17], [28]). Hereafter Lie-draggings of regions are supposed to be diffeomorphisms of the same differentiability class as that of the space-time manifold $(\mathcal{M}, g_{\alpha\beta})$.

where V_Σ is the 4-volume of the region Σ,

$$V_{\Sigma} = \int_{\Sigma} \sqrt{-g} d^4x \,, \tag{3.2}$$

with the averaged object $\bar{p}^{\alpha}_{\beta}$ keeping the same tensorial character as p^{α}_{β}. Here the integration is carried out over all points $x' \in \Sigma$, $g' = \det(g_{\alpha\beta}(x'))$, and the boldface object $\mathbf{p}^{\alpha}_{\beta}(x, x')$ in the integrand of (3.1) is a bilocal extension of the object $p^{\alpha}_{\beta}(x)$, $\mathbf{p}^{\alpha}_{\beta}(x, x') = \mathcal{A}^{\alpha}_{\mu'}(x, x')\, p^{\mu'}_{\nu'}(x')\, \mathcal{A}^{\nu'}_{\beta}(x', x)$, by means of bilocal averaging operators $\mathcal{A}^{\alpha}_{\beta'}(x, x')$ and $\mathcal{A}^{\alpha'}_{\beta}(x', x)$. The averaging scheme is covariant and linear by its structure with corresponding algebraic properties. The operator $\mathcal{A}^{\alpha}_{\beta'}$, is supposed to exist on $\mathcal{U} \subset M$, $x, x' \in \mathcal{U}$, and to have the following properties[5]: (i) idempotency, $\mathcal{A}^{\alpha}_{\beta'}\,, \mathcal{A}^{\beta'}_{\gamma''} = \mathcal{A}^{\alpha}_{\gamma''}$, that results in the idempotency of the averages, $\bar{\bar{p}}^{\alpha}_{\beta} = \bar{p}^{\alpha}_{\beta}$; (ii) the coincidence limit $\lim_{x' \to x} \mathcal{A}^{\alpha}_{\beta'}(x, x') = \delta^{\alpha}_{\beta}$, with both properties defining $\mathcal{A}^{\alpha'}_{\beta}$ as an inverse operator to $\mathcal{A}^{\alpha}_{\beta'}\,, \mathcal{A}^{\alpha}_{\beta'}\,, \mathcal{A}^{\beta'}_{\gamma} = \delta^{\alpha}_{\gamma}$ and $\mathcal{A}^{\alpha}_{\beta'}\,, \mathcal{A}^{\gamma'}_{\alpha} = \delta^{\gamma'}_{\beta'}$. By assigning an averaging region Σ_x near each point x of M and by defining a law of the correspondence between the neighbouring averaging regions as Lie-dragging of regions by means of another bilocal operator $\mathcal{W}^{\alpha'}_{\beta}(x', x)$, one can define directional, partial and covariant derivatives of average fields. Requirements that (*a*) all averaging regions have the same value of volume (3.2), $V_{\Sigma_x} = \text{const}$, for all $x \in M$ and (*b*) a region remains the same after its Lie-dragging along a circuit constructed from vectors with vanishing Lie brackets, bring about special conditions on the two bivectors, taken for simplicity as $\mathcal{W}^{\alpha'}_{\beta} = \mathcal{A}^{\alpha'}_{\beta}$,

$$\mathcal{W}^{\alpha'}_{\beta;\alpha'} = 0 \tag{3.3}$$

where the semicolon denotes a covariant derivative with respect to connection coefficients $\gamma^{\alpha}_{\beta\delta}$, and

$$\mathcal{W}^{\alpha'}_{[\beta,\gamma]} + \mathcal{W}^{\alpha'}_{[\beta,\delta'}\,, \mathcal{W}^{\delta'}_{\gamma]} = 0. \tag{3.4}$$

The first requirement (3.3) means that the averaging regions' volume is a free parameter of the procedure and this assumes physically that measurements carried out on a differentiable manifold by an observer with a given measurement system can be characterized by an invariant number $V_{\Sigma_x} = \text{const}$ which is to be fixed, or chosen, for a situation under consideration. The second requirement (3.4) means that the covering of a manifold by averaging regions is defined uniquely, which results in a non-trivial analytical property of the averages (3.1) as single-valued *local* functions of the supporting point, $\bar{p}^{\alpha}_{\beta,[\mu\nu]} = 0$ and the standard calculus is therefore applicable to deal with them. These requirements are particular differential conditions on the bivectors, which means geometrically that the bivectors act on M as volume-preserving diffeomorphisms holonomic in a defined bilocal sense (biholonomic, see [8]). The bivectors satisfying (3.3), (3.4) together with the algebraic properties (i) and (ii) above have been shown

5 They are formalization of the properties of the space-time averages in macroscopic electrodynamics in the language of bilocal kernels.

[4], [8] to exist on an arbitrary differentiable manifold with a volume n-form and solutions of the system of algebraic and partial differential equations have been found for the bivectors in a factorized form. These solutions are bilocal products of a vector basis e^{α}_{i} with constant anholonomicity coefficients, taken at one point, by its dual basis e^{k}_{μ} in another point, $\mathcal{W}^{\alpha'}_{\beta}(x', x) = e^{\alpha'}_{i}(x')e^{i}_{\beta}(x)$. It should be noted here that solutions of such structure are in fact the general solution for equation (3.4), for a bilocal operator satisfying the algebraic properties (i) and (ii) can be shown to be idempotent if it is factorized into the bilocal product [29]. In the simplest case when e^{α}_{i} is a coordinate basis, $e^{\alpha}_{i} = \partial x^{\alpha} / \partial \phi^{i}$ where $\phi^{i}(x)$ are four arbitrary scalar functions, and the bivector takes the form

$$\mathcal{W}^{\alpha'}_{\beta}(x', x) = \frac{\partial x^{\alpha'}}{\partial \phi^{i}} \frac{\partial \phi^{i}}{\partial x^{\beta}}, \tag{3.5}$$

a proper coordinate system formed by ϕ^{i}'s is a volume-preserving coordinate system, in which $\mathcal{W}^{\alpha'}_{\beta}(x', x) = \delta^{\alpha'}_{\beta}$ and $\det(g_{\alpha\beta}) = \text{const}$ and all definitions and relations within the averaging scheme acquire especially simple form. The coordinate system is an analogue of Cartesian coordinates in the flat space-time case.

The basic commutation formula for the averaging and exterior derivative, obtained in the framework of the formalism, has the following form [8] :

$$d\bar{p}^{\alpha}_{\beta}(x)\, \langle \text{đ}\mathbf{p}^{\alpha}_{\beta} \rangle, \tag{3.6}$$

where đ is a bilocal exterior derivative.

It should be pointed out here that such pretendents to be averaging kernels as either the parallel transportation bivector, or the one constructed by taking derivatives of Synge's world function in two different points [27], [30] *do not satisfy* the conditions (3.3) and (3.4), with the latter not satisfying also the idempotency condition (*i*). Those objects are well-defined as single-valued functions of their arguments only for Whitehead's normal convex neighbourhoods (where there exists one and only one geodesic between each pair of points), which makes problematic their use for averaging the gravitational field over the space-time regions with matter. Such cases are of the most interest from the physical point of view and a possibility to be able to treat them is an ultimate goal in the framework of the macroscopic description of classical fields. Just to make use of such bivectors within the formalism makes many things very complicated [29] (for example, the commutation formula (3.6) loses its transparent meaning and simple form). If, nevertheless, one decides to utilize the parallel transportation bivector, $\mathcal{W}^{\alpha'}_{\beta}(x', x) = g^{\alpha'}_{\beta}(x', x)$, and to be restricted to the normal convex neighbourhoods, it is necessary to require the parallelly transported bases to have the constant anholonomicity coefficients to fulfill the condition (3.4). The additional requirement for the operator $g^{\alpha'}_{\beta}(x', x)$ to be volume-preserving (3.3) leads [29] to the class of D'Arti spaces (see, for example [31] for definitions and relevant results), which is a special class of (pseudo)-Riemannian spaces with particular restrictions of their curvature. On the contrary, the volume-preserving bases $e^{\alpha}_{i}(x)$ with constant anholonomicity coefficients defining the averaging and coordination operators in the space-time averaging scheme adopted in macroscopic gravity, exist always, at least, locally[6], on any (pseudo)-Riemannian space-times without any restrictions on the curvature[7]. This averaging scheme, therefore, in addition to the possibility to define averaged fields (3.1) with reasonable algebraic and differential properties, is applicable (locally, as discussed above) on any space-time manifold. This is an essential advantage of the scheme, which allows one to consider it, as well as the results of its application for the space-time averaging of (pseudo)-Riemannian geometry and general relativity, as being generic from both geometrical and physical points of view[8].

The averages (3.1) of products and corresponding correlation functions are taken to be one-point functions of supporting point x, which means that the approach developed is related to the equilibrium macroscopic gravitational processes (for a non-equlibrium theory [32] it is necessary to introduce many-points version of averaged products and correlation functions).

4. The geometry of macroscopic space-time

Resulting from the averaging out of Cartan's structure equations, the geometry of the averaged (macroscopic) space-time has the following structure [4], [8]. The average $\overline{\mathcal{F}}^{\alpha}{}_{\beta\gamma} = \langle \mathcal{F}^{\alpha}{}_{\beta\gamma} \rangle$ of the microscopic Levi-Civita connection[9] $\gamma^{\alpha}{}_{\beta\gamma}$, is supposed to be Levi-Civita's connection of the averaged space-time. A metric tensor $G_{\alpha\beta}$ always exists locally due to Frobenius'theorem with given $\overline{\mathcal{F}}^{\alpha}{}_{\beta\gamma}$ [17], and $G_{\alpha\beta}$ is considered to be the macroscopic metric tensor. There are two

6 The space-time averaging scheme being considered here is essentially of local character in the sense that the average values are defined by (3.1) over local regions Σ of a microscopic manifold M, and thereby the average fields are defined locally on $\mathcal{U}$ with the topological and differentiable structure of M remaining unchanged. Such local character of the macroscopic picture is dictated, first of all, from the physical point of view, by our experience and observations which show that physical quantities are represented by local functions determined by means of measurements which are themselves fundamentally of local character (i.e., a measurement of a physical quantity is carried out always during a finite time period over a finite space region, to be small compared with the characteristic extension of the system under interest and its time of existence). Thus, from the mathematical point of view, to describe such objects adequately, it is sufficient to formulate a calculus of the averages on a differentiable manifold. If such a calculus is formulated, a definition of an average field globally can be done in the same way as one constructs a global field on a manifold if it possesses a non-trivial topology.

7 It follows from the well-known fact that on any (pseudo)-Riemannian space there always exists a coordinate system in which the connection coefficients $g^{\alpha}_{\beta\delta}$ have $g^{\alpha}_{\beta\alpha} = 0$, or, what is the same, det $(g_{\alpha\beta})$ = const.

8 This analysis gives an indication, in our opinion, that using such bivectors as, for instance, the parallel transportation bivector, for the averaging of general relativity implies another set of basic assumptions about the nature of averaged gravity and the character of space-time measurements [32]. It also may be related to a quantum regime of gravitation as a physical setting where such averaging operators are more adequate (see, for example, [27]). For formulation of a classical macroscopic theory of gravity, the proposed space-time avaraging procedure based on a Lie-dragging model of space-time measurements is relevant and it is a simplest generalization of the flat space-time procedure adopted in hydrodynamics and macroscopic electrodynamics.

9 Here $\mathcal{F}^{\alpha}{}_{\beta\gamma}$ is a bilocal extension of the connection coefficients $\gamma^{\alpha}{}_{\beta\gamma}$ [4].

curvature tensors, $M^{\alpha}{}_{\beta\gamma\delta}$ and $R^{\alpha}{}_{\beta\gamma\delta}$ Riemannian and non-Riemannian, respectively. The Riemannian curvature is assumed to correspond to the average curvature tensor $\bar{r}^{\alpha}{}_{\beta\gamma\delta} = R^{\alpha}{}_{\beta\gamma\delta}$ for another symmetric connection $\Pi^{\alpha}{}_{\beta\gamma}$ which is non-metric (i.e. the connection is incompatible with the metric tensor $G_{\alpha\beta}$), and the average curvature is non-Riemannian in that sense. There is a remarkable relation between the two curvature tensors, which results from averaging the second Cartan equation[10] $r^{\alpha}{}_{\beta\gamma\delta} = 2\gamma^{\alpha}{}_{\beta[\delta,\gamma]} + 2\gamma^{\alpha}{}_{\epsilon[\gamma}\gamma^{\epsilon}{}_{\underline{\beta}\delta]}$

$$\mathrm{R}^{\alpha}{}_{\beta\rho\sigma} = \mathrm{M}^{\alpha}{}_{\beta\rho\sigma} + \mathrm{Q}^{\alpha}{}_{\beta\rho\sigma} \, . \tag{4.1}$$

This relation is of the form of a constitutive relation between the induction, $M^{\alpha}{}_{\beta\rho\sigma}$, and average field, $R^{\alpha}{}_{\beta\rho\sigma}$, with $Q^{\alpha}{}_{\beta\rho\sigma}$ standing for the polarization tensor defined below in Eq. (4.3). The origin of this *geometric relation* lies in the simple, but non-trivial, geometric fact of the non-linear definition of the affine curvature in terms of connection, which results in the curvature determined by the average connection not being equal to the average curvature. The curvature tensors satisfy the corresponding Bianchi identities, the Bianchi identities for $\mathrm{M}^{\alpha}{}_{\beta\rho\sigma}$ resulting from averaging out the microscopic ones [4], [8].

There is an affine deformation tensor $A^{\alpha}{}_{\beta\gamma} = \bar{\mathcal{F}}^{\alpha}{}_{\beta\gamma} - \Pi^{\alpha}{}_{\beta\gamma}$ in this geometry, which plays the role of the polarization potential. It satisfies the partial differential equation [4], [8][11].

$$A^{\alpha}{}_{\beta[\sigma\|\rho]} - A^{\alpha}{}_{\epsilon[\rho}A^{\epsilon}{}_{\underline{\beta}\sigma]} = -\frac{1}{2}\, Q^{\alpha}{}_{\beta\rho\sigma} \, , \tag{4.2}$$

which is always integrable with necessity on an arbitrary averaged manifold. The tensor $\mathrm{A}^{\alpha}{}_{\beta\gamma}$ therefore exists and the theory is not empty.

In addition to these objects, in a 4-dimensional space-time there are three correlation tensors. The correlation 2-form $Z^{\alpha}{}_{\beta[\gamma}{}^{\nu}{}_{\underline{\nu}\sigma]}(x)$ is defined as

$$Z^{\alpha}{}_{\beta[\gamma}{}^{\mu}{}_{\underline{\nu}\sigma]} = \langle \mathcal{F}^{\alpha}{}_{\beta[\gamma}\mathcal{F}^{\mu}{}_{\underline{\nu}\sigma]}\rangle - \bar{\mathcal{F}}^{\alpha}{}_{\beta[\gamma}\bar{\mathcal{F}}^{\mu}{}_{\underline{\nu}\sigma]} \, , \tag{4.3}$$

with $Q^{\alpha}{}_{\beta\gamma\lambda} = -2Z^{\delta}{}_{\beta[\gamma}{}^{\alpha}{}_{\underline{\delta}\lambda]}$ (note the change in the sign in the definition of $Q^{\alpha}{}_{\beta\gamma\lambda}$ here and in [10] – [13] as compared with [4], [8], [9]), and the correlation 3-form $Y^{\alpha}{}_{\beta[\gamma}{}^{\mu}{}_{\underline{\nu}\sigma}{}^{\theta}{}_{\underline{\kappa}\pi]}(x)$ and 4-form $X^{\alpha}{}_{\beta[\gamma}{}^{\mu}{}_{\underline{\nu}\sigma}{}^{\theta}{}_{\underline{\kappa}\pi}{}^{\tau}{}_{\underline{\varrho}\psi]}(x)$ which are defined analogously to (4.3) as triple and quadruple connection correlation tensors. These tensors are constructed from the connection coefficients and the number of them is *finite* since the dimension of space-time is finite.

In their geometrical sense the correlation functions are non-trivial generalizations of the concept of the affine curvature tensor. The geometric meaning of the trace part $\mathrm{Q}^{\alpha}{}_{\beta\gamma\lambda}$ of

[10] Note that underlined indices are not affected by antisymmetization.

[11] The covariant derivatives with respect to the connections $\Pi^{\alpha}{}_{\beta\gamma}$ and $\bar{\mathcal{F}}^{\alpha}{}_{\beta\gamma}$ are denoted as | and ||, respectively.

$Z^{\alpha}{}_{\beta[\gamma}{}^{\mu}{}_{\underline{\nu}\sigma]}$ can be shown to be the difference between the defect δv^{α} for a vector v^{α} after its parallel transportation along a circuit with square $\Delta\sigma^{\mu\nu}$ in a microscopic manifold with the curvature $r^{\alpha}{}_{\beta\gamma\delta}$ of the connection $\gamma^{\alpha}{}_{\beta\gamma}$ with its subsequent averaging, and the defect $\delta\bar{v}^{\alpha}$ for the averaged vector $\bar{v}^{\alpha}$ after its parallel transportation along the same circuit in a macroscopic manifold with the induction curvature $M^{\alpha}{}_{\beta\gamma\delta}$ of the averaged connection $\overline{\mathcal{F}}^{\alpha}{}_{\beta\gamma}$

$$\langle\delta \mathbf{v}^{\alpha}\rangle - d\bar{v}^{\alpha} = Q^{\alpha}{}_{\beta\mu\nu}\, \bar{v}^{\beta}\, \Delta\sigma^{\mu\nu}. \tag{4.4}$$

The geometrical meaning of the tensor $Z^{\alpha}{}_{\beta[\gamma}{}^{\mu}{}_{\underline{\nu}\sigma}$ itself and the tensors $Y^{\alpha}{}_{\beta[\gamma}{}^{\mu}{}_{\underline{\nu}\sigma}{}^{\theta}{}_{\kappa\pi}$ and $X^{\alpha}{}_{\beta[\gamma}{}^{\mu}{}_{\underline{\nu}\sigma}{}^{\theta}{}_{\underline{\kappa}\pi}{}^{\gamma}{}_{\varphi\psi}$ can be understood as generalization of (4.4) in the fibre bundle picture [32] of the space-time averaging scheme. There are the structure equations for the correlation tensors. In the simplest case when the other correlations tensors are put to zero, the structure equation for the tensor $Z^{\alpha}{}_{\beta[\gamma}{}^{\mu}{}_{\underline{\nu}\sigma]}$ reads

$$Z^{\alpha}{}_{\beta[\gamma}{}^{\mu}{}_{\underline{\nu}\sigma||\lambda]} = 0 \tag{4.5}$$

which is an analogue of the Bianchi identities, with the corresponding integrability conditions [4], [8].

The macroscopic metric tensor $G_{\alpha\beta}$ of the averaged space-time is compatible with the Levi-Civita connection $\overline{\mathcal{F}}^{\alpha}{}_{\beta\gamma}$, $G_{\alpha\beta||\gamma} = 0$. At the same time the metric $G_{\alpha\beta}$ is incompatible with the connection $\Pi^{\alpha}{}_{\beta\gamma}$, $G_{\alpha\beta|\gamma} = N_{\alpha\beta\gamma}$ where $N_{\alpha\beta\gamma}$ is the non-metricity object. The averaged metric tensors $\bar{g}_{\alpha\beta} \neq G_{\alpha\beta}$ and $\bar{g}^{\alpha\beta} \neq G^{\alpha\beta}$ in general and they are not metric tensors any more, i.e. $\bar{g}^{\alpha\beta}\, \bar{g}_{\beta\gamma} \neq \delta^{\alpha}_{\gamma}$, although the averaging has been shown to keep them covariantly constant, $\bar{g}_{\alpha\beta||\gamma} = 0$ and $\bar{g}^{\alpha\beta}{}_{||\gamma} = 0$, as a consequence of the splitting rule

$$\langle \mathbf{c}^{\mu\ldots}_{\upsilon\ldots}\, \mathcal{F}^{\alpha}{}_{\beta\gamma}\rangle = \bar{c}^{\mu\ldots}_{\upsilon\ldots}\, \overline{\mathcal{F}}^{\alpha}{}_{\beta\gamma}\,, \tag{4.6}$$

which is assumed to hold for any covariantly constant tensors, Killing vectors and tensors, and similar objects denoted as $\bar{c}^{\mu\ldots}_{\upsilon\ldots}$. It means geometrically that the averaging preserves the symmetries of microscopic space-time. Due to the structure of the relations between $\bar{g}_{\alpha\beta}$, $G_{\alpha\beta}$ and $M^{\alpha}{}_{\beta\gamma\delta}$ one can always put, in addition,

$$\bar{g}_{\alpha\beta} = G_{\alpha\beta}\,. \tag{4.7}$$

The covariantly constant symmetric tensor $\bar{g}^{\alpha\beta}$ is then an object of the theory and it is convenient to define a tensor $U^{\alpha\beta} = \bar{g}^{\alpha\beta} - G^{\alpha\beta}$ which is covariantly constant,

$$U^{\alpha\beta}{}_{||\gamma} = 0. \tag{4.8}$$

The meaning of the tensor $U^{\alpha\beta}$ is that it describes the algebraic metric correlations (all other metric correlations are contained in the correlation tensors $Z^{\alpha}{}_{\beta[\gamma}{}^{\mu}{}_{\underline{\nu}\sigma]}$, $Y^{\alpha}{}_{\beta[\gamma}{}^{\mu}{}_{\underline{\nu}\sigma}{}^{\theta}{}_{\underline{\kappa}\pi]}$ and

$X^{\alpha}{}_{\beta[\gamma}{}^{\mu}{}_{\underline{\nu}\sigma}{}^{\theta}{}_{\underline{\kappa}\pi}{}^{\tau}{}_{\underline{\phi}\psi]}$) in this geometry[12]. The covariantly constant correlation tensor $\Delta^{\alpha}_{\beta}(x)$, defined by the relation

$$\delta^{\alpha}_{\beta} = \langle \mathbf{g}^{\alpha\epsilon}\mathbf{g}_{\epsilon\beta}\rangle = \bar{g}^{\alpha\epsilon}\bar{g}_{\epsilon\beta} + \Delta^{\alpha}_{\beta} \ , \tag{4.9}$$

can be shown to have the following structure when taking (4.7) into account :

$$\Delta^{\alpha}_{\beta} = -U^{\alpha\epsilon}G_{\epsilon\beta} \ . \tag{4.10}$$

The presence of the tensor $\bar{g}^{\alpha\beta}$, or $U^{\alpha\beta}$, has been proved to make the macroscopic space-time reducible due to a classification theorem [4], [8] of all possible macroscopic space-times according to Petrov's types of the induction tensor and kinds of the macroscopic metric tensor reducibility. This theorem determines also the algebraic structure of the tensor $U^{\alpha\beta}$ in terms of covariantly constant vectors and symmetric idempotent tensors.

The geometry of averaged space-time is a new geometry being a non-trivial generalization of the metric affine connection one and it is this geometry that underlies the macroscopic theory of gravity.

5. The field equations of macroscopic gravity

The splitting rule for the average of the product of *metric times curvature* derived in the theory

$$\langle \mathbf{r}^{\alpha}{}_{\beta\gamma\lambda}\mathbf{g}^{\epsilon\rho}\rangle - R^{\alpha}{}_{\beta\gamma\lambda}\bar{g}^{\epsilon\rho} = -2Z^{\alpha}{}_{\beta[\gamma}{}^{\epsilon}{}_{\underline{\delta}\lambda]}\bar{g}^{\delta\rho} - 2Z^{\alpha}{}_{\beta[\gamma}{}^{\rho}{}_{\underline{\delta}\lambda]}\bar{g}^{\epsilon\delta} \tag{5.1}$$

plays the most important role in macroscopic gravity, for it is the only rule needed for the Einstein equations (2.1) to be averaged out. The result is the macroscopic field equations [4]

$$\bar{g}^{\alpha\epsilon}M_{\epsilon\beta} - \frac{1}{2}\delta^{\alpha}_{\beta}\bar{g}^{\mu\nu}M_{\mu\nu} = -\kappa\langle \mathbf{t}^{\alpha(\mathrm{micro})}_{\beta}\rangle + (Z^{\alpha}{}_{\mu\nu\beta} - \frac{1}{2}\delta^{\alpha}_{\beta}Q_{\mu\nu})\bar{g}^{\mu\nu} \tag{5.2}$$

which are not Riemannian in their geometrical meaning, though $\bar{g}^{\mu\nu}$ is covariantly constant, $M_{\mu\upsilon}$ is the Ricci tensor of the Riemannian curvature $M^{\alpha}{}_{\beta\gamma\delta}$, and the divergence of the left-hand side of (5.2) vanishes [4], yielding the equations of motion, or the conservation law for averaged matter together with correlation terms,

$$\kappa\langle \mathbf{t}^{\alpha(\mathrm{micro})}_{\beta}\rangle_{||\alpha} = (Z^{\alpha}{}_{\mu\nu\beta||\alpha} - \frac{1}{2}Q_{\mu\nu||\beta})\bar{g}^{\mu\nu}. \tag{5.3}$$

Here $Z^{\alpha}{}_{\mu\nu\beta} = 2Z^{\alpha}{}_{\mu[\epsilon}{}^{\epsilon}{}_{\underline{\nu}\beta]}$ is a Ricci-tensor like object for the correlation tensor $Z^{\alpha}{}_{\beta[\gamma}{}^{\mu}{}_{\underline{\nu}\lambda]}$, $Q_{\mu\nu} = Q^{\epsilon}{}_{\mu\nu\epsilon}$, and $\langle \mathbf{t}^{\alpha(\mathrm{micro})}_{\beta}\rangle$ stands for an averaged stress-energy tensor. The problem of calculation, or

[12] It should be noted that one can use both tensors $\bar{g}_{\alpha\beta}$ and $\bar{g}^{\alpha\beta}$ without fixiing the ansatz (4.7) and defining $U^{\alpha\beta}$, but it simplifies the formalism without loss of generality and restriction of the geometric content of the metric part of this geometry.

construction, of $\langle \mathbf{t}^{\alpha(\text{micro})}_{\beta} \rangle$ from a given microscopic stress-energy tensor $t^{\alpha(\text{micro})}_{\beta}$ constitutes the problem of construction of macroscopic gravitating media. This is still an open problem in general relativity[13] (see an attempt in [33] and also relevant approaches [34], [35]). The usual practice in cosmology is to assume phenomenologically one or another form of smoothed, hydrodynamic stress-energy tensors on the basis of obervational data, and the results of theoretical analysis are then compared with the data again to conclude about the assumptions made.

Similar to macroscopic electrodynamics, the problem of construction of models of gravitating media in macroscopic gravity is to be considered [32] after finding the macroscopic space-time geometry (see Section IV) and the form of averaged Einstein's operator (5.2).

On using the tensor $U^{\alpha\beta}$ one can write the macroscopic field equations in a remarkable form

$$G^{\alpha\epsilon} M_{\epsilon\beta} - \frac{1}{2}\,\delta^{\alpha}_{\beta}\, G^{\mu\nu} M_{\mu\nu} = -\kappa T^{\alpha(\text{micro})}_{\beta} \tag{5.4}$$

where the macroscopic stress-energy tensor $T^{\alpha(\text{micro})}_{\beta}$ is defined to be

$$\kappa T^{\alpha(\text{macro})}_{\beta} = \kappa\langle T^{\alpha(\text{micro})}_{\beta}\rangle - (Z^{\alpha}{}_{\mu\nu\beta} - \frac{1}{2}\,\delta^{\alpha}_{\beta} G_{\mu\nu})\bar{g}^{\mu\nu} + G^{\alpha\epsilon} M_{\epsilon\beta} - \frac{1}{2}\,\delta^{\alpha}_{\beta}\, U^{\mu\nu} M_{\mu\nu}\,, \tag{5.5}$$

and the stress-energy tensor can be shown to satisfy the conservation law

$$T^{\;\alpha\;(\text{macro})}_{\beta\|\alpha} = 0. \tag{5.6}$$

Though somewhat unexpected, the result seems to be natural : a space-time averaging out of the Einstein equations brings the field equations which can be written *in the form* of the Einstein equations for the induction Ricci tensor defined through the Riemannian macroscopic metric $G_{\alpha\beta}$. The macroscopic stress-energy tensor (5.5) includes, in addition to the averaged matter, the correlation tensor terms with $Z^{\alpha}{}_{\mu\nu\beta}$, $Q_{\mu\nu}$ and $U^{\alpha\epsilon}$ for geometric correction of the averaged matter. The correlation terms reveal the structure of the correlation tensor C^{α}_{β} in (2.3), but now all the correlation objects have geometrical origin and definitions with corresponding differential equations for them (see (4.5) and (4.8)). In their geometrical meaning, the equations (5.4) however *are not Riemannian*, as the Einstein equations (2.1) of general relativity are, which is due to a different underlying geometry for macroscopic gravitation.

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[13] Unlike electrodynamics where there is the whole industry of modelling electromagnetic continuous media, though there are still a number of unsolved problems of principle regarding the derivation of material relations from the microscopic equations of motion (see, for example [22]).

The macrovacuum equations of the theory, following from the averaging of the vacuum Einstein equations $r_{\alpha\beta} = 0$, or from (5.2) directly, read

$$M_{\alpha\beta} = -Q_{\alpha\beta}, \quad Z^{\alpha}{}_{\mu\nu\beta}\bar{g}^{\mu\nu} = -\bar{g}^{\alpha\epsilon}Q_{\beta\epsilon}, \tag{5.7}$$

with $\bar{g}^{\alpha\beta}Q_{\alpha\beta} = 0$ as a consequence. They show the *Ricci non-flat* character of macroscopic gravitation in the absence of averaged matter, in contrast to vacuum microscopic general relativity.

6. The correspondence principle and stress-energy of macroscopic gravitational field

The correspondence principle for the theory of macroscopic gravity [8], [9] states that the macrovacuum equations (5.7) become Issacson's equations [36]

$$M_{\alpha\beta} = -\kappa T^{(\mathrm{GW})}_{\alpha\beta} \tag{6.1}$$

in the high-frequency limit. As a result, the correlation tensor $Q_{\mu\nu} = -2Z^{\delta}{}_{\mu[\nu}{}^{\epsilon}{}_{\underline{\delta}\epsilon]}$, which serves as the macrovacuum source in (5.7), is equal to $\kappa T^{(\mathrm{GW})}_{\alpha\beta}$ in the high-frequency limit and it therefore describes the stress-energy of the macrovacuum gravitational field. This result gives evidence in favour of considering the correlation term on the right-hand side of the macroscopic field equation (5.2) as the stress-energy tensor of macroscopic gravitation

$$(Z^{\alpha}{}_{\mu\nu\beta} - \frac{1}{2}\,\delta^{\alpha}_{\beta}Q_{\mu\nu})\bar{g}^{\mu\nu} = -\,\kappa T^{\alpha(\mathrm{grav})}_{\beta}, \tag{6.2}$$

(which is $Z^{\alpha}{}_{\mu\nu\beta}\bar{g}^{\mu\nu} = -\,\kappa T^{\alpha(\mathrm{grav})}_{\beta}$ in the macrovacuum case (5.7)). Indeed, a simple consideration shows that averaging out over a space-time region makes the gravitational field stress-energy localizable, which brings about the corresponding *tensor* object. This fundamental fact has first been established by Issacson [36] within the high frequency approximation for general relativity. The macriscopic gravity approach provides a general solution to the problem. The correlation tensor $Z^{\alpha}{}_{\mu\nu\beta}$ (and $Q_{\mu\nu} = Z^{\alpha}{}_{\mu\nu\alpha}$ as a consequence of algebraic properties of $Z^{\alpha}{}_{\beta[\gamma}{}^{\mu}{}_{\underline{\nu}\lambda]}$ [4], [8]) is just an *averaged* effect of the "product of connection" which is known in general relativity to be the stress-energy of gravitational field in its physical meaning. Then the conservation law (5.3) tells us that only the *total stress*-energy of the averaged matter and the macroscopic gravitational field is conserved

$$\left(\langle \mathbf{t}^{\alpha(\mathrm{micro})}_{\beta}\rangle + T^{\alpha(\mathrm{micro})}_{\beta}\right)_{\|\alpha} = 0 \tag{6.3}$$

It should be pointed out here that there is a remarkable similarity of the structure of the definition of $T^{\alpha(\mathrm{grav})}_{\beta}$ (6.2) to the structure of Einstein's equations since $Z^{\alpha}{}_{\mu\nu\beta}$ is a Ricci-tensor like object for the correlation tensor $Z^{\alpha}{}_{\beta[\gamma}{}^{\mu}{}_{\underline{\nu}\lambda]}$ and $Q_{\mu\nu} = Z^{\alpha}{}_{\mu\nu\beta}$ is an analogue of the curvature scalar as a trace of $Z^{\alpha}{}_{\mu\nu\beta}$ (or, it is just the Ricci tensor of $Q^{\alpha}{}_{\beta\gamma\delta}$).

An analysis of the structure equations (4.5) for $Z^{\alpha}{}_{\beta[\gamma}{}^{\mu}{}_{\underline{\nu}\lambda]}$ shows that in this simplest case the symmetric part $Z^{\alpha}{}_{(\mu\nu)\beta}$ remains undetermined from the equations and

$$Z^{\alpha}{}_{(\mu\nu)\beta||\alpha} - \frac{1}{2} Q_{\mu\nu||\beta} = 0, \tag{6.4}$$

which means that the stress-energy tensor $T^{\alpha(\mathrm{grav})}_{\beta}$ is conserved separately. Then, due to (6.3), or (5.3), the averaged matter stress-energy tensor $\langle \mathbf{t}^{\alpha(\mathrm{micro})}_{\beta} \rangle$ is also conserved separately. The fact of $Z^{\alpha}{}_{(\mu\upsilon)\beta}$ being undetermined from (4.5) is analogous to the fact that in the Riemannian geometry the Bianchi identities do not determine the Ricci tensor which has to be fixed by an additional hypothesis. In general relativity it is the Einstein equations which relate the Ricci tensor part of the curvature to a matter distribution. Thus, with one non-vanishing correlation tensor $Z^{\alpha}{}_{\beta[\gamma}{}^{\mu}{}_{\underline{\nu}\lambda]}$ the relations (6.2) must be taken as *the field equations* for the tensor $Z^{\alpha}{}_{(\mu\upsilon)\beta}$ with a *given* stress-energy tensor $T^{\alpha(\mathrm{grav})}_{\beta}$, the equations being algebraic.

When the higher correlation tensors, $Y^{\alpha}{}_{\beta[\gamma}{}^{\mu}{}_{\underline{\nu}\sigma}{}^{\theta}{}_{\underline{\kappa}\pi]}$ and $X^{\alpha}{}_{\beta[\gamma}{}^{\mu}{}_{\underline{\nu}\sigma}{}^{\theta}{}_{\underline{\kappa}\pi}{}^{\tau}{}_{\underline{\varphi}\psi]}$, are taken into account a distribution of the stress-energy tensor $T^{\alpha(\mathrm{grav})}_{\beta}$ is to be calculated from (6.2) by solving the generalized analogue of equations (4.5) for $Z^{\alpha}{}_{\beta[\gamma}{}^{\mu}{}_{\underline{\nu}\lambda]}$ [4], [8], and the conservation law (6.3) holds. In such a case one should find some additional hypotheses for the higher correlation tensors [32].

7. Using Einstein's equations in cosmolology

A study of the structure of the field equations of macroscopic gravity enables one to answer also a fundamental question of cosmology about the physical meaning and the range of applicability of the Einstein equations with a continuous (smoothed) matter source. Indeed, if *all correlations functions vanish*, $Z^{\alpha}{}_{\beta[\gamma}{}^{\mu}{}_{\underline{\nu}\sigma]} = 0$ (connection correlations) and $U^{\alpha\beta} = 0$ (metric correlations) the equations (5.4) (as well as (5.7)) become the Einstein equations

$$G^{\alpha\epsilon} M_{\epsilon\beta} - \frac{1}{2} \delta^{\alpha}_{\beta} G^{\mu\nu} M_{\mu\nu} = -\kappa T^{\alpha(\mathrm{macro})}_{\beta} \tag{7.1}$$

for the *macroscopic metric* $G_{\alpha\beta}$ with a stress-energy tensor

$$T^{\alpha(\mathrm{macro})}_{\beta} + \langle \mathbf{t}^{\alpha(\mathrm{micro})}_{\beta} \rangle \tag{7.2}$$

on the right-hand side of (7.1), which is usually taken as a perfect fluid tensor while looking for cosmological solutions. This reveals that the physical meaning and essence of using Einstein's equations in studies of cosmological problems consists in neglecting the gravitational field correlations.

To take into account the gravitational field correlations and to understand their effect and role in the dynamics of the universe, the equations of macroscopic gravity are to be solved. One of the most important questions here is to clear up the status of the homogeneous and isotropic (Friedmann-Robertson-Walker) models in cosmology. Given a hydrodynamic stress energy tensor $\langle \mathbf{t}^{\alpha(\mathrm{micro})}_{\beta} \rangle$ (or upon calculating it for a given microscopic gravitating matter model) and an equation of state, there is the following system of partial differential equations in the simplest case then only the correlation tensor $Z^{\alpha}{}_{\beta[\gamma}{}^{\mu}{}_{\underline{\nu}\sigma]}$ is taken into account : the field equations (5.2), or (5.4), for the macroscopic metric tensor $G_{\alpha\beta}$, the equations (4.5) for $Z^{\alpha}{}_{\beta[\gamma}{}^{\mu}{}_{\underline{\nu}\sigma]}$ together with the field equation (6.2) with a given, or assumed, stress-energy tensor of macroscopic gravitational field $T^{\alpha(\mathrm{grav})}_{\beta}$, and the equations (4.8) for $U^{\alpha\beta}$.

This is the system for that part of macroscopic gravity which is related to the Riemannian induction fields $G_{\alpha\beta}$, or $M^{\alpha}{}_{\beta\gamma\delta}$, and correlation fields $Z^{\alpha}{}_{\beta[\gamma}{}^{\mu}{}_{\underline{\nu}\sigma]}$ and $U^{\alpha\beta}$, the part being closed in both geometry and field equations[14]. The second part of macroscopic gravity is non-Riemannian and related to the average fields which are represented by the connection $\Pi^{\alpha}{}_{\beta\gamma}$, or the curvature $R^{\alpha}{}_{\beta\gamma\delta}$, the affine deformation tensor $A^{\alpha}{}_{\beta\gamma}$ and the non-metricity object $N_{\alpha\beta\gamma}$. To find all these objects after resolving the first part one should solve the equation (4.2) for $A^{\alpha}{}_{\beta\gamma}$ with a determined correlation tensor $Q^{\alpha}{}_{\beta\gamma\delta}$ with subsequent calculations of $\Pi^{\alpha}{}_{\beta\gamma} = \overline{\digamma}^{\alpha}{}_{\beta\gamma} - A^{\alpha}{}_{\beta\gamma}$, $N_{\alpha\beta\gamma}$ and $R^{\alpha}{}_{\beta\gamma\delta}$.

Due to the physical structure of macroscopic gravity as a classical macroscopic theory with one-point averages it can be applied for the description of the universe since decoupling matter and radiation when the evolving universe can be considered as being in an equilibrium state.

Acknowledgements

It is a pleasure for me to thank Dennis Sciama for hospitality and support during my visits to SISSA and ICTP where the work has been done in part, and Boris Dubrovin, George Ellis and Reza Tavakol for many discussions and encouragement. I would like to express my gratitude to Ian Roxburgh for his hospitality in the School of Mathematical Sciences, QMW. I am thankful to Jorge Devoto, Chris Isham, Nelly Konopleva, Andrzej Krasiński, Roy Maartens, Malcolm MacCallum, David Matravers, Alexei Nesteruk, Kamilla Piotrkowska, and Bill Stoeger for discussions. I thank Henk van Elst for careful reading of the manuscript and discussion. The author was supported by a Royal Society fellowship.

[14] The non-trivial fact of decoupling of the induction and average field parts of the presented formulation of macroscopic gravity is a consequence of two assumptions : (A) the averaged microscopic tensor $\bar{r}^{\alpha}{}_{\beta\gamma\delta}$ is supposed to be again a curvature tensor of a metric affine connection geometry with curvature $R^{\alpha}{}_{\beta\gamma\delta} = \bar{r}^{\alpha}{}_{\beta\gamma\delta}$ (see Section IV), which allows one to apply Schouten's classification [37] of such geometries where the metric and non-metric parts *do decouple*, and (B) the correlations between metric and connection are assumed to vanish (4.6), which preserves in fact the decoupling mentioned in (A). It should be emphasized here that the rule (4.6) brings nevertheless the non-trivial correlations between metric and curvature (5.1). On the other side, it is that rule that makes it possible to write down the averged Einstein equations (5.2) in the form of Einstein's equations (5.4). Such a structure of the theory of macroscopic gravity is one of the simplest possible formulations, and any generalization either (A) or (B) leads to more sophisticated macroscopic space-time geometries and field equations, physical content and interpretation of which are much more difficult [32].

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