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Tilt of COBE can constrain aspects of superstring geometry*

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Abstract. Superstring theories for which experimental evidence is meagre are considered a most promising approach to understand the quantum nature of gravity and its unification with other fundamental interactions. The geometric structure of these theories goes beyond the usual Riemannian one of general relativity. In the low energy limit they are well known not to simply reduce to Einstein's theory, but inevitably contain an antisymmetric field associated with space time torsion and a dilation scalar field. Although equivalence principle tests and evidence from the binary pulsar would make their effects very small these fields could have dominated in the inflationary epoch and left their imprints in the COBE spectrum. This approach enables us to quantify quite precisely the relative strength of the antisymmetric field and consequent deviation from general relativity at that epoch.

A popular and promising line of approach to reconcile general relativity with quantum physics possibly leading to a unified picture of fundamental interactions is contained in superstring theories for which the experimental evidence is very meagre despite the rich geometrical and physical structure of these theories. It is generally agreed that the distinctive effects predicted by these theories should have manifested themselves in the conditions prevailing during the early inflationary phase of cosmological evolution. These first instants of creation, by their production of a spectrum of long wavelength gravitational waves, characteristic of these theories, could have left a detectable imprint on the cosmic microwave background [1]. The measured anisotropies give direct information about the inhomogenities and the way the inhomogenities amplitude vary with wavelength (quantified by the so called power law index) tell us about the variations in the space time metric and the relative contributions of the different entities producing such variations. As we shall see, deviation from a scale invariant spectrum can be related to the way the universe was expanding during the inflationary phase which in turn is determined by the additional geometric structure (and the associated fields) present in the gravity of superstrings. This enables us to quantify the deviation from general

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relativity at that epoch. These constraints are much more stringent than bounds from primordial nucleosynthesis and post-newtonian effects.

The content of superstring theories are such that their particle spectrum, amplitudes, dimension of space-time etc. are essentially determined by their own algebraic structure originating from the internal geometric property of the string whose length is related to the Regge slope [2]. The effective string action involves the spin-2-graviton field h, the antisymmetric tensor field A (introduced in all superstring theories) and the dilation scalar ϕ. It is usually expressed as:

$$S_{eff}(h, A, \phi) = S^{(o)} + \alpha' (S^{(1)}) + \alpha'^2 (S^{(2)}) + ... \text{ etc.} \tag{1}$$

(α' being the Regge slope related to string tension. The string tension is the basic constant in string theories, the gravitational constant being derived from string tension [3].

From the geometric view point, the structure of the interaction is interpreted in terms of geometric entities like the metric tensor and affine connection, defined through a d-bein field $V^{\alpha}_{\mu}(x)(a = 1 ... d)$ and its inverse so that $g_{\mu\nu} = V^{\alpha}_{\mu} V_{a\nu}$, $V^{\alpha}_{\mu}V^{b\mu} = \eta^{\alpha,b}(\eta^{00} = -\eta^{ii} = -1)$. The effective action $S^{(0)}$ is then written as (in d dimensions) :

$$S^{(0)} = -\int d^dx g^{1/2}[R + \exp[-2\kappa\phi/(d-2)^{1/2}H^2] + (1/2)\partial_\mu\phi\partial^\mu\phi \tag{2}$$

where the antisymmetric field strength H is defined as

$$H_{\mu\nu\rho} = \kappa(\partial_\mu A_{\nu\rho} + \partial_\nu A_{\mu\rho} + \partial_\rho A_{\mu\nu})$$

In addition to the usual Spin-2 part of gravity, this third-rank gauge-invariant antisymmetric tensor field defined as the exterior derivative of a potential, i.e. [3,4]:

$$H_{\mu\nu\sigma} \equiv A[_{\mu\nu,\sigma}],$$

has been a ubiquitous feature of all superstring theories including N = 8 supergravity in 11 dimensions and in N = 1 supergravity.

Scherk and Schwarz first observed [3, 4], that the antisymmetric tensor field strength appears in the effective Lagrangian only as torsion in the connection of generalised curvature. There are also indications from non-linear σ models [5] and supersymmetry [7,8] that the torsion requirement is natural.

Thus this antisymmetric field arises naturally in a gravitation theory with a torsion potential and in fact the above effective string action can be completely rewritten in terms of the Hilbert Lagrangian as:

$$S^{(0)}(h,A) = -\frac{1}{16\pi\kappa}\int d^d x\sqrt{g}R(\hat{\Gamma})$$

$$= -\frac{2}{\kappa}\int d^d x\sqrt{g}[R(\Gamma) + \frac{1}{2}H_{\mu\nu\rho}H^{\mu\nu\rho}]$$

but with the connection [5, 4, 3]

$$(\hat{\Gamma})^{\rho}_{\mu\nu} = G^{\rho}_{\mu\nu} + \frac{1}{\sqrt{12}}H^{\rho}_{\mu\nu} \qquad (5)$$

which contains torsion. The curvature scalar is the same but defined with $\hat{\Gamma}$. Thus the string action can be written generally solely in terms of the Riemann tensors with torsion with the effective substitution $R \to R + H_{\alpha\beta\gamma}H^{\alpha\beta\gamma}$. Thus the A field strength occurring in all string theories turns out to be the torsion or 'supertorsion' associated with the string modified space time geometry. Just as redefining, $A_{\mu\nu\rho}$, involves action with torsion, the dilation ϕ can be related to propagating torsion as $Q \to \partial_\mu\phi$ as outlined below. Essentially this arises through a duality transformation $H_{\mu\nu\beta} = \in_{\mu\nu\alpha\beta}\partial^\alpha H$, (satisfying $\partial_\rho(\in^{\rho\mu\nu\alpha} H_{\mu\nu\alpha}) = 0$, giving the replacement of the form: $R \to R + 1/2\partial^\mu\phi\partial_\mu\phi$. Again the torsion $H^h_{ij} = 1/2\ (\Gamma^h_{ij} - \Gamma^h_{ji})$ can be written as:

$$H^{\kappa}_{ij} = \frac{1}{2}q\,(\delta^{\kappa}_{i}\,\Phi_{,j} - \delta^{\kappa}_{j}\Phi_{,i}) \qquad (6)$$

where Φ is a torsional potential, which may be expressed as [9]

$$\Phi = ln\psi$$

Here the quantity q in eq. (6), to be more explicitly identified later is defined to be constant at different space time points and can be identified with a kind of gauge coupling of the H field.

With these substitutions, the modified curvature, scalar takes the form:

$$R = R\,(\{\}) - 6q^2\Phi^{,h}\Phi_{,h} + 6q\Phi^{,h}_{,h} \qquad (8)$$

(with { } being the usual GR connection).
With the corresponding action given by

$$\delta\int[R(\{\}) - 6q^2\Phi^{,k}\Phi_{,k} + \kappa L_m]\sqrt{-g} \qquad (9)$$

Making the conformal transformations:

$$\psi = \phi$$

$$g_{ij} = \phi \bar{g}_{ij}$$

$$L(g_{ij}) = \phi^{-2} \bar{L}(\bar{g}_{ij}) \quad (10)$$

We have

$$\delta L = \delta \int \bar{\Phi} \ \bar{R}(\{\}) + m\phi^{\iota-2}\phi^{\prime k}\phi^{\prime}_{,k} + \kappa\phi^{0}\bar{L}] \sqrt{-g} d^{d}x \quad (11)$$

with

$$m = \left(\frac{3}{2} - \frac{2}{3} q^2\right)$$

Eq. (11) has the form of a Brans-Dicke type of theory with $\omega = \left(\frac{2}{3} q^2 - \frac{3}{2}\right)$ and an effective gravitational constant $G_{eff} \approx [f(\phi)]^{-1}$.

The field equations following from the action given by Eq.(11) can be shown [9] (after appropriate formal transformations) to give rise to a power-law type of inflation of the scale factor given by

$$R(t) \alpha \ t^{q^2/3} \quad (12)$$

The net inflationary factor in R(t) can be expressed as a ratio of $[\phi(t_f)/\phi(t_i)]^{3k/q^2}$ where t_f and t_i correspond to the start and end of inflation and k is a numerical factor.

The density perturbation spectrum is tilted from scale invariance to a power law:

$$P(k) \alpha \ k^n; \quad n = 1 - \frac{2}{P - 1} = \frac{q^2 - 9}{q^2 - 3} \quad (13)$$

q = 0 would give the usual scale invariant spectrum. If all microphysical parameters are time-independent during inflation, then all of the fluctuations are produced with the same amplitude on average and one has a scale invariant (n = 1) spectrum of perturbations. In GR both bubble and perturbation spectra are scale invariant. Breaking of or deviation from scale invariance (due to the antisymmetric tensor string geometry in the above case) would remove short-scale power. While the COBE measurements constrains the amplitude on large scales, one has tight limits as to how much power can be removed on short scales [10]. As the density perturbation spectrum is now proportional to $[f(\phi)]^{1/2}$, one expects perturbations leaving horizon at later times to be smaller.

Power-law inflation of the type given by eq. (12) also generate gravitational waves [11] and their relative contribute to large angle microwaves background anisotropics is independent of the multiple and in the case given by [12] the ratio.

$$R \approx 300/\delta q^2 + 9 \qquad (14)$$

To obtain constraints we can for instance utilize the Cobe 10 deg. 2σ result $\sigma^2 \approx 10^{-5}$. Depending weakly on whether one has hot dark matter (HDM) or cold dark matter (CDM) bubbles, q is constrained to be 1 less than 5.1 or n ~ 0.75. For HDM q can be slightly higher q < 5.5. Constraint is more severe with an admixture of dark matter or a cosmological constant dominated flat model because more short scale power is removed. The following table shows q for various n values. (n = 1 is the scale invariant GR result).

n	q
0.75	5.1
0.8	5.75
0.85	5.80
(implied also by QDOT)	
0.6	4.5

Roughly speaking the coupling of the antisymmetric gauge field H of the string action is ~ G_{eff}/q at the inflationary epoch (see the defining equations)..., thus one can relate H^i_{jo} to the temporal variation of $f(\phi)^{-1}$ during the expansion (eqs. (11) and (12).

$$H^i_{jo} = \frac{1}{2}(\dot{f}(\phi) / f(\phi))\delta^i_j$$

The constraints on q from primordial nucleosynthesis and post Newtonian effects are much less severe [14]. (From nucleosynthesis q < 17 and post Newtonian effects give q < 19).

In short an analysis of the tilt in the COBE spectrum can constrain some important aspects of superstring geometry.

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