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# On Alfvén wave damping in polar coronal holes

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**Abstract.** We reexamine quantitatively the linear resistive and viscous damping of Alfvén waves propagating in solar coronal holes, using recently reported observational data on the plasma density and temperature there as well as taking into account the magnetic field spreading inside the coronal hole. It is found that the latter effect results in a drastic reduction of the wave's dissipation length, thus making linear dissipation a viable mechanism of plasma heating in coronal holes. Some simple estimates related to the reflection and trapping of Alfvén waves have been also made, which show that this effect is more likely to take place under strong magnetic field relevant to the maximum phase of the solar activity cycle.

## 1. Introduction

Very recently Narain and Ulmschneider (1996) reviewed chromospheric and coronal heating mechanisms in solar and stellar environments and arrived at the conclusion that Alfvén waves and direct magnetic field dissipation are the most promising. This study is devoted to the heating of polar coronal holes by Alfvén waves via ohmic and viscous dissipation and constitutes a step towards the former option. Osterbrock (1961) was the first to analyse the effect of the magnetic field on the generation, propagation and dissipation of MHD waves that may heat the solar chromosphere and corona. He assumed a homogeneous magnetic field of 2G for quiet regions and 50 G for plage regions. It is now well established that solar magnetic fields from the photosphere to the corona are not homogeneous (Stenflo 1994 and references therein). Zwaan (1987) found that the magnetic field strength for coronal holes ranges from 1 to 7 G around minimum and 3 to 36 G near maximum of the solar activity cycle. Therefore the magnetic fields of coronal holes may be taken to vary from 1 to 36 G.

Quite recently Fisher and Guhathakurta (1995) reported electron densities and temperatures as a function of radial distance (height) using white light coronal observations aboard the Spartan 201-01 spacecraft and the ground-based K-coronameter in Mauna Loa (Hawaii) for North and South pole coronal holes. This could be used as the basic data for the present investigation. This is a step forward in comparison with Moore *et al.* (1991) who have used the assumption of isothermal hydrostatic equilibrium to study the heating of coronal holes by Alfvén waves via reflection and trapping of these waves. We also include the spreading of the coronal magnetic field inside the hole. It will be shown that the resulting reduction in the Alfvén velocity has an important effect on the dissipation length.

The paper is organized as follows : Section 2 contains theoretical details. In section 3 we present data and results. The last section is devoted to discussions and conclusions. The cgs system of units is used throughout.

## 2. Theoretical details

The Alfvén wave energy may dissipate to thermal energy via resistivity and viscosity of the coronal hole plasma. The distance in which the wave energy flux drops by a factor e, (e = 2.718...) is called damping length. For pure Alfvén waves propagating in the direction of the magnetic field in a homogeneous medium, the damping length, $L_d$, due to the above mentioned processes is given by (Osterbrock 1961, Priest 1982)

$$L_d = V_A^3 P^2/4\pi^2[(c^2/4\pi\sigma) + (\mu/\rho)], \tag{1}$$

where P is the period of the Alfvén wave, c is the speed of light in vacuum, $\sigma$ is the electrical conductivity (in $s^{-1}$), $\mu$ is the coefficient of viscosity, $\rho$ is the mass density and $V_A$, the Alfvén wave velocity, is given by

$$V_A = B/(4\pi\rho)^{0.5}. \tag{2}$$

Here B is the magnetic field strength in gauss (G). The wavelength $\lambda$, of the Alfvén wave is obtained through the relation $\lambda = V_A P$. It is quite clear from equation (1) that the damping length, $L_d$, has strong dependence on the Alfvén wave velocity ($L_d \propto V_A^3$). Therefore if the magnetic field spreads from the bottom to the top of the coronal hole the damping length could reduce drastically. Following Moore *et al.* (1991) the magnetic field may be assumed to vary as

$$B\,(x) = B_0/x^{\sigma}, \tag{3}$$

where B $(x)$ is the magnetic field at normalized height x with $x \equiv R/R_{\odot}$ and $\sigma$ is the index of spreading : $\sigma = 2$ implies radial spreading whereas $\sigma = 4$ corresponds to the observed spreading at the edges of the coronal holes. $B_0$ is the magnetic field at the bottom of the coronal hole (here it is approximated by the solar radius).

To account for another effect caused by the density and magnetic field variation we make simple estimate of the Alfvén speed variation length, $l_A$, using the expression

$$l_A = \left(\frac{1}{V_A}\frac{dV_A}{dR}\right)^{-1}, \tag{4}$$

and the observational data of Fisher and Guhathakurta (1995) for electron density. Using $l_A/V_A = P_C$ one may estimate the critical period above which the waves may get reflected and trapped in the coronal hole plasma. If $\lambda << l_A$ or equivalently $P < P_C$, no significant reflection and trapping will occur. However, if $\lambda \simeq l_A$ or $P \geq P_C$ the reflection and trapping might be significant. Both the cases, namely weak spreading ($\sigma = 2$) and strong spreading ($\sigma = 4$) are investigated.

The electrical conductivity due to electron-ion collisions is given by (see, e.g., Pneuman & Orrall 1986)

$$\sigma = 2 \times 10^7 \, T^{1.5} \qquad (s^{-1}), \tag{5}$$

where T is the temperature of the coronal hole plasma. The coefficient of viscosity may be evaluated using the expression (Lang 1980)

$$\mu = 2 \times 10^{-15} \, T^{2.5} \, A_i^{0.5}/q^4 ln\Lambda \qquad (gmcm^{-1}s^{-1}). \tag{6}$$

Here $A_i$ is the atomic weight of the positive ion, q is the charge number and $ln\Lambda$, the Coulomb logarithm, is given by

$$\Lambda = 1.3 \times 10^4 \, T^{1.5}/n_e^{0.5}, \tag{7}$$

where $n_e$ is the electron number density in $cm^{-3}$.

The formulae for the electrical conductivity and viscosity, given above, are appropriate for a collisional gas (e.g. Spitzer 1962). Thus this is only an acceptable approximation for waves with wavelength long compared to the proton mean free path, $\lambda_{mfp}$. Following Hollweg (1975), this condition may be written as

$$\lambda_{mfp} = (2kT_p/m_p)^{0.5} \, t_{coll,\,p} << \lambda, \tag{8}$$

where $T_p$ is the proton temperature, $m_p$ is the proton mass, k is the Boltzmann constant and $t_{coll,\,p}$ is the self-collision time of protons and is given by (Spitzer 1962, eqn 5-26)

$$t_{coll,\,p} = 0.5 \, T_p^{1.5}/n_p \qquad (s), \tag{9}$$

where $n_p$ is the proton number density and the Coulomb logarithm is taken to be 23.

## 3. Data and results

The data used in this investigation are as follows :

1. Electreon density and temperature are taken from Fisher and Guhathakurta (1995) for the radial distance range from 1.5 to 5.0 $R_O$.
2. The magnetic field strength at the base of coronal holes is assumed to be 5G (corresponding to Solar Minimum) and 35G (corresponding to Solar Maximum) (Zwaan 1987).
3. The Alfvén waves of period 300s and 1000s have been considered.
4. $A_i = 1.0067$, $q = 1$ and $m_p = 1.67 \times 10^{-24}$ corresponding to hydrogen plasma.

The results obtained are represented in the following way : Fig. 1 exhibits the variation of Alfvén velocity with radial distance inside the coronal hole for the different magnetic field strength ($B_0$ = 5G and 35G) and the field spreading rate ($\sigma$ = 2 and 4). It is seen that under weak spreading ($\sigma = 2$) the plasma density decreases with height and is almost balanced by the magnetic field reduction, so that the Alfvén velocity remains approximately constant in the whole distance range from $R = 1.2\ R_\odot$ to $R = 5\ R_\odot$. However, in the case of strong spreading ($\sigma = 4$) Alfvén velocity is decreasing with height by a factor of 10 in the same distance range. Fig. 2 shows how the wave dissipation length, $L_d$, varies with height for Alfvén wave with P = 300s under different values of $B_0$ (5G and 35G) and spreading ($\sigma$ = 2 and 4). The important effect which we would like to emphasize here is the significant reduction in $L_d$ due to the magnetic field spreading. For $B_0$ = 35 G and $\sigma = 4$ the damping length $L_d$ becomes of the order of $10^{10}$ cm at $R = 4\ R_O$ so even a simple linear dissipation of Alfvén wave can provide the plasma heating in coronal holes. Finally, Table 1 illustrates possible reflection and trapping of Alfvén waves generated at the bottom of coronal holes due to the variation of the Alfvén velocity with height. For example, in the case of $B_0$ = 35G and $\sigma = 2$ the critical wave period $P_C$ becomes of the order of $10^2$ at $R = 3R_\odot$ therefore waves with a longer period are likely to experience substantial reflection and hence an increased dissipation inside the coronal hole.

**Table 1.** Alfvén velocity variation length and critical period.*

| | $l_A/R_\odot$ | | $P_C(x)$ | |
|---|---|---|---|---|
| $x = R/R_\odot$ | $\sigma = 2$ | $\sigma = 4$ | $B_0 = 5G$<br>$\sigma = 4$ | $B_0 = 35G$<br>$\sigma = 2$ |
| 1.5 | 1.94 | 1.71 | 556 | 40 |
| 2.0 | 43.57 | 1.41 | 646 | 700 |
| 2.5 | 28.00 | 1.57 | 1115 | 438 |
| 3.0 | 12.63 | 2.10 | 2188 | 206 |
| 3.5 | 6.06 | 1.37 | 1862 | 95 |
| 4.0 | 2.99 | 1.47 | 2414 | 44 |
| 4.5 | 71.00 | 2.51 | 6250 | 1167 |

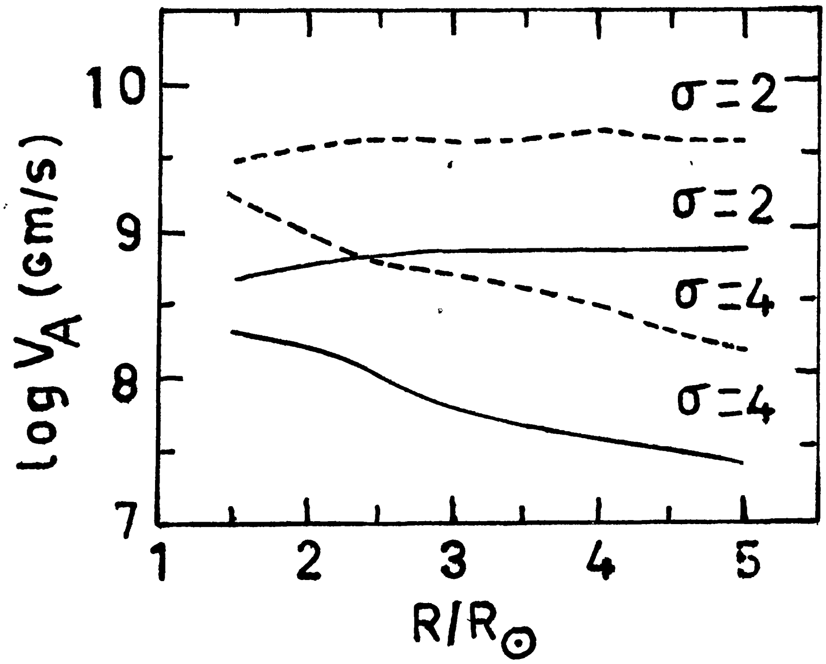

**Figure 1.** Variation of Alfvén velocity (in cm/s) with radial distance for the magnetic fields $B_0$ = 5G with spreading rate $\sigma$ = 2,4 (solid lines) and $B_0$ = 35G with $\sigma$ = 2, 4 (dotted lines).

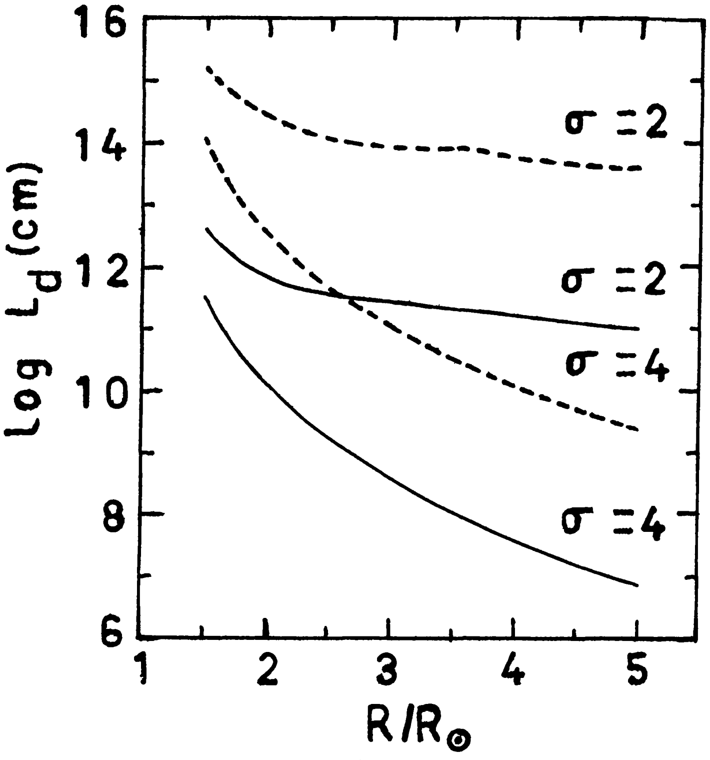

**Figure 2.** Variation of damping length (in cm) with radial distance for P = 300s at magnetic fields $B_0$ = 5G with spreading rate $\sigma$ = 2,4 (solid lines) and $B_0$ = 35G with $\sigma$ = 2, 4 (dotted lines).

## 4. Discussion and conclusions

It would not be out of place to say a few words about the data :

Fisher and Guhathakurta (1995) present for the first time the measurements of the radial density gradient of coronal holes in the polar regions over the range 1.16 $R_\odot$ - 5.0 $R_\odot$. According to them the uncertainty in electron density from 1.16 $R_\odot$ to 1.3 $R_\odot$ is 15%, from 2 to 4 $R_\odot$ the uncertainty is 17% and from 4 to 6 $R_\odot$ is 23%. The uncertainty in the region from 1.4 to 1.8 $R_\odot$ is the greatest (30% - 35%). They find that the scale height temperatures for the North and South polar coronal holes increase from the coronal base (at 1.16 $R_\odot$) to a sharp peak around 1.8 $R_\odot$ and then steadily decrease with height (radial distance). The temperature at the base is ~ 0.8 MK (1 MK = $10^6$ K). The scale height temperature estimates involve the idea that in coronal region heating occurs over the height range 1.4 – 2.6 $R_\odot$. Consequently our results also have these uncertainties.

According to Moore *et al.* (1991) Alfvén waves excited by granulation (convection cells) have periods in the range 100 - 1000s. The power spectrum of these waves peaks near 300s. For a model coronal hole of temperature 1 MK they found that Alfvén waves having periods greater than 300s are trapped and those having periods smaller than 300s escape to solar wind. In view of this we have considered Alfvén waves of the period range 300 - 1000s, for which applicability of the fluid description in calculating the damping rate (Equation 8) can be approximately justified even for the case of weak magnetic field ($B_0 = 5G$). Indeed, according to Fisher and Guhathakurta (1995), the plasma temperature and density in the polar coronal hole vary from $T \approx 10^6$ K and $n_e = 1.2 \times 10^6$ cm$^{-3}$ at $R = 1.2\ R_\odot$ to $T = 0.8 \times 10^6$ K and $n_e = 10^4$ cm$^{-3}$ at $R = 5.0\ R_\odot$. Hence the plasma mean free path, $\lambda_{mfp}$ is about $3 \times 10^9$ cm at $R = 1.2\ R_\odot$ and $10^{11}$ cm at $R = 5.0\ R_\odot$. For the Alfvén wave with period $P = 300$s, the wavelength is approximately constant for weak spreading ($\sigma = 2$), and is equal to $1.5 \times 10^{10}$ cm for $B_0 = 5G$ and $\lambda = 1.2 \times 10^{11}$ cm for $B_0 = 35G$. In the case of strong field spreading ($\sigma = 4$), $\lambda$ varies from $1.5 \times 10^{10}$ cm to $6 \times 10^9$ cm for $B_0 = 5G$ and from $1.2 \times 10^{11}$ cm to $4 \times 10^{10}$ cm for $B_0 = 35G$. Therefore the above calculated wave dissipation rate might be overestimated for the case of weak field ($B_0 = 5G$) at far distances ($R > 3R_\odot$), but gives a good approximation for the damping length in the whole distance range for strong magnetic field ($B_0 = 35G$).

It is clear from the Fig. 1 that for weak spreading ($\sigma = 2$) the Alfvén velocity remains almost constant from 2 to 5 $R_O$ but it decreases monotonically when the spreading is strong ($\sigma = 4$). Since the plasma thermal pressure in the coronal hole is small compared to the magnetic one, these Alfvén velocities greatly exceed the sound speed there.

An examination of Fig. 2 shows that the damping length, $L_d$, decreases with radial distance for the weak ($\sigma = 2$) as well as for the strong ($\sigma = 4$) spreading. The rate of decrease of the damping length is relatively faster in the case of strong spreading. For $B_0 = 5G$ and $\sigma = 4$ the damping length is smaller than the solar radius for the distance range 2 to 5 $R_\odot$, the corresponding distance range for $B_0 = 35G$ and $\sigma = 4$ is from 3.25 $R_\odot$ to 5 $R_\odot$. Thus in the case of strong spreading of the magnetic field the linear resistive and viscous dissipation mechanism is quite effective in heating coronal holes, particularly beyond 1.5 $R_\odot$.

A close look at Table 1 shows that the Alfvén speed variation length, $l_A$, for the strong spreading ($\sigma = 4$) varies from 1.4 to 2.6 $R_O$. The minimum critical period for this case is 40s which implies that Alfvén waves having period larger than 40s will suffer significant reflection and will be trapped to heat the coronal plasma inside it. These trapped Alfvén waves may dissipate to thermal energy (heat) also via intermittent magnetic levitation (Moore *et al.* 1992).

It may now be concluded that the linear resistive and viscous dissipation is a viable mechanism for heating polar coronal holes provided there is a strong spreading ($\sigma = 4$) in the coronal hole magnetic field.

We wish to investigate the nonlinear damping of Alfvén waves in a future article.

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