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Do flat rotation curves include non-Keplerian motions?

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Abstract. In an earlier paper (Prabhu and Krishan 1994) a model of the flat rotation curves of galaxies has been given. This model attempts to explain the rotation curves without invoking dark matter, instead it depends on certain properties of a turbulent medium. It is felt that some issues related to Keplerian motion have remained unaddressed. In this paper, by going into the local Keplerian frame, the equations for turbulent motions are derived. In our earlier work we used these equations to obtain the turbulent energy spectrum. Thus the contributions of Keplerian and turbulent motions are required to account for the flat rotation curves of galaxies.

1. Introduction

In an earlier paper (Prabhu and Krishan 1994) an alternate way of modeling the flat rotation curves of spiral galaxies was proposed. This model is based on the recently explored properties of a turbulent medium which supports the formation of large scale structures from small scale structures (Levich and Tzvetkov 1985; Sulem *et al.* 1989; Moffat and Tsinober 1992; Hossain 1994). The basic idea underlying the inverse cascade of energy is the interaction between small and large scales in a driven anisotropic helically turbulent medium. The presence of helicity in the flow retards the flow of energy to small scales. A galaxy, with several forms of energy inputs from supernovae explosions and stellar winds, seems to be an appropriate system to explore the role of turbulent processes. Moreover the fact that the fluid model of a galaxy does elucidate many of its salient features, is well established (Hunter 1983). The rotation curves of spiral galaxies may then be viewed as large scale coherent motions suitably driven by small scale energy input mechanisms. Using a part of the complete energy spectrum in a helically turbulent medium derived in (Krishan 1991; Krishan and Sivaram 1991), the velocity field of a galaxy was suggested to consist of a combination of rigid rotation (i.e. $V \alpha l$) and Kolmogorov spectrum ($V \alpha l^{1/3}$) in the inner region and a combination of Keplerian motion ($V \alpha l^{-1/2}$) and a flat branch of the turbulent spectrum ($V \alpha \sqrt{ln l}$) [in the outer region]. With this model, rotation curves of 22 galaxies were fitted and the corresponding values of the velocity coefficients for all the branches were given. The fits looked quite good. In all previous work, while deriving the turbulent energy spectrum, the role of gravity was not mentioned, except for assuming that it cannot account for the entire rotation curve. In this paper, we start with the equation of motion including the gravitational and external forces representing the effect of turbulence. By going into the local Keplerian frame, equations for turbulent motions are derived. These equations have been used to derive the turbulent energy spectrum, in our earlier work.

2. Galactic circulation

The galactic fluid undergoes several types of motion under the action of gravitational and nongravitational forces. In order to examine the influence of these forces, it is best to write the fluid equation of motion in a rotating frame, which is given by:

$$\frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \vec{\nabla})\vec{V} = 2\vec{V} \times \vec{\Omega} - \frac{1}{\rho} \vec{\nabla} p + \vec{g} - \vec{\Omega} \times (\vec{\Omega} \times \vec{r}) + \vec{F}_{ext} + \vec{F}_{diss} \quad (1)$$

where $\vec{V}$ is the velocity in a frame rotating with angular velocity $\vec{\Omega}$, ρ is the mass density, p is the pressure, $\vec{g}$ describes the gravitational field, $2\vec{V} \times \vec{\Omega}$ is the coriolis force; $\vec{\Omega} \times (\vec{\Omega} \times \vec{r})$ is the centrifugal force, $\vec{F}_{ext}$ represents all the nongravitational forces, as e.g. stresses due to turbulence and $\vec{F}_{diss}$ is the dissipative force. Now, if there is no other motion in the rotating frame i.e. if $\vec{V} = 0$ and there are no other forces except for the gravitational and the centrifugal forces, one gets

$$\vec{g} = \vec{\Omega} \times (\vec{\Omega} \times \vec{r}). \quad (2)$$

Equation (2) represents the Keplerian motion. Since in this case the rotation velocity $V \alpha r^{-1/2}$, it cannot explain the flat portion of the rotation curve. What is then done is to assume that the mass contained within a radius r varies as r, which then leads to V being a constant. But since one does not observe this mass which increases linearly with distance in the outer regions of a galaxy, it is called the dark matter, which influences the gravitational force without modifying the radiation in any way.

We explore an alternative possibility. We go into the Keplerian frame i.e. we choose $\vec{\Omega}$ from $\vec{g} = \vec{\Omega} \times (\vec{\Omega} \times \vec{r})$, then equation (1) becomes,

$$\frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \vec{\nabla})\vec{V} = 2\vec{V} \times \vec{\Omega} - \frac{\vec{\nabla} p}{\rho} + \vec{F}_{ext} + \vec{F}_{diss} \quad (3)$$

Since, there are nongravitational forces and there are motions other than the Keplerian, and equation (3) describes such a situation, there is no justification for putting $\vec{V} = 0$. Equation

(3) describes motions additional to the Keplerian motion. These additional motions may be driven by supernovae explosions and stellar winds. Conventionally, these motions are believed to give rise to turbulent broadening of the spectral lines and attempts are made to subtract them out in order to determine the circular motion in a galaxy. We emphasize that these random turbulent motions can be transferred to more organized large scale motions under conditions conducive to the inverse cascade of energy in a helically turbulent medium. These upscaled motions are exhibited as rotation. In equation (3), F_{ext}, for example can be identified with the driving forces due to supernovae explosions and stellar winds. It is difficult to give it an explicit form, but we assume that F_{ext} maintains a continuous state of small scale (l) random turbulent motions describable by a velocity spectrum of the Kolmogorov type i.e. the velocity $V \alpha l^{1/3}$. The question, then, one can ask, is if motions at these small spatial scales (l) can be transferred to large spatial scales (L). The answer is yes, if the small scale motions are helical in nature. If the system has a net helicity, then by a process akin to the α - effect for the generation of magnetic field, small scale motions can be inverse cascaded to large scales. It may happen that the system has helical fluctuations, but the net helicity is zero. In such a case, as shown earlier, (Levich and Tzvetkov 1985; Krishan & Sivaram 1991; Prabhu and Krishan 1994) the quantity I, defined as:

$$I = \int < \gamma(x)\ \gamma(x + r) > d^3x$$

is also an invariant of an ideal three dimensional hydrodynamic system in addition to the total energy. Here $\gamma = \vec{V} \cdot \vec{\omega}$ is the helicity density and $\vec{\omega} = \vec{\nabla} \times \vec{V}$ is the vorticity. Using Kolmogorovic arguments, the velocity spectrum in the inertial range for the I invariant (Prabhu and Krishan 1994) can be derived and it is found to be:

$$V(L)\ \alpha\ (ln L)^{1/2}.$$

The flat nature of this law combined with the Keplerian motion helps to account for the flat portions of rotation curves as shown in Prabhu and Krishan (1994). Thus, a circular (strictly helical) motion with a nearly constant velocity is a result of the inverse cascade process in a turbulent medium. The energy for the maintenance of this motion is provided by the small scale turbulent motions which result from supernovae explosions and stellar winds. That, the Reynold's stresses, produced by small scale motions become a source term for the generation of large scale motions has been shown in more detail in the appendix of Prabhu and Krishan (1994) and in Krishan (1993). Summarizing, the complete rotation curve consists of two parts, the inner and the outer. The inner part exhibits the combined effects of the rigid rotation and the small scale Kolmogorovic turbulence ($l^{1/3}$). These two contributions result in an increase of velocity with distance (or scale). At the transition to outer regions or at large scales the contributions of the Keplerian and the inverse cascaded helical motions ($\alpha\ (ln L)1/2$) maintain the near-constancy of the velocity. The complete energy spectrum in a turbulent medium is given in our earlier papers. The lnL branch at even larger spatial scales turns again into a Kolmogorovic branch ($l^{1/3}$). One can ask what is the largest scale of motion that can exist in a turbulent medium? It is fixed by the amount of energy available in small spatial scales. The larger this energy, the larger the scale that can be sustained. At this scale the normal direct cascade will then take over.

Conclusion

In our earlier attempts to account for the large scale structure of universe in general and the flat rotation curves in particular through the role of inverse cascade of energy in an asymmetrical turbulent medium, we had not explicitly addressed the contributions from the Keplerian motions. Here, we have pointed out that all considerations of turbulence must be applied to the fluid equations written in the local Keplerian frame, i.e. to equation (3). This gives a turbulent energy spectrum. The rotation curves are then reproduced by combining this energy spectrum with the Keplerian motion due to the visible mass.

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