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Steady part of rotation and toroidal component of magnetic field in the solar convective envelope

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Abstract. With reasonable assumptions and approximations, we solve analytically the full set of Chandrasekhar's (1956) MHD equations in the convective envelope, for steady part of rotation and toroidal component of the magnetic field which may vary on diffusion time scales. Depending upon the two assumptions made about the boundary condition at the lower boundary, viz., base of the convection zone, we have the following two solutions.

In the first solution, we assumed that the Sun may be rotating uniformly at base of the convection zone. Except in the magnitudes of rotational velocities near the polar regions, the first solution yields similar rotational isocontours as that of helioseismology. Near the poles, the resulting solution yields faster rotation rates compared with that of the rotation rates inferred by helioseismology. Moreover, the chi-square fit of the resulting rotational results with the helio-seismologically inferred rotational results is also not so good. These results lead to the second solution in which we made the assumption that the Sun may be rotating differentially at base of the convection zone.

Compared with the helioseismic inferred results, the second solution yields almost similar rotational isocontours everywhere in the convective envelope. The chi-square fit improved significantly in this solution. This result suggests that, at base of the convection zone, *differential rotation is more likely* than uniform rotation.

Correspondingly, it was necessary to solve the toroidal component of the magnetic field. Except near base of the convection zone, both the solutions yield similar field structure for the steady component of the toroidal magnetic field. This consists of a two-zone like field structure of intensity ~ $1G$ near the surface and a four-zone like field structure of intensity ~ $10^4 G$ near base of the convection zone. However, near base of the convective envelope, compared to the field structure obtained from the first solution, the resulting field structure from the second solution is closer to the sites of sunspot formation.

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Implications of these results in the context of lithium depletion and solar activity phenomena are briefly discussed.

Key words : sun-internal : sun-rotation : sun-toroidal magnetic field

1. Introduction

Nearly 400 years ago (soon after the discovery of the sunspots), Sun's rotation was discovered from the movements of Sunspots over the Sun's disk. Pioneers of this discovery were Goldshmit (1587-1615); Galileo Galilee (1564-1642); Thomas Harriet (1560-1621) and Schiener (1575-1650).

Systematic study of the Sun's rotation was started after the 18th century onwards. It was Richard Carrington (1826-1875) and Gustav Sporer (1822-1895), who undertook the long series of observations of the apparent motion of sunspots. They confirmed that on the surface Sun rotates differentially.

Presently, Sun's surface rotation is determined from the measurements of positions of the sunspots (Newton & Nunn 1951; Balthasar & Wohl 1980; Godoli & Muzzucconi 1979; Gilman & Howard 1984) and the magnetic features as tracers (Wilcox & Howard 1970; Stenflo 1977; Snodgrass 1983). It is also measured from the Doppler shift in the spectral lines (Howard & Harvey 1970; Livingston & Duvall 1979; Scherrer, Wilcox & Svalgaard 1980).

Though, the Sun's surface rotation is known with better accuracy, the sun's internal rotation was poorly understood until the advent of helioseismology. Helioseismologically inferred results show that the Sun rotates differentially with a weak dependence on radius throughout its convection zone and has a nearly uniform rotation in the radiative core (Christensen-Dalsgaard & Schou 1988; Brown et al. 1989; Dziembowski, Goode & Libbrecht 1989; Gough et al. 1993; Thompson 1990; Schou et al. 1992; Korzennik et al. 1995; Antia, Basu & Chitre 1998; Corbard et al. 1998; Howe 1998; Mauro, Dziembowski & Paterno 1998).

The differential rotation in the convective envelope has been modeled earlier in the frame work of hydrodynamics by taking into account the meridional circulation, Reynolds stresses and viscous stresses. Basically, these works can be classified (Chan & Mayr 1994) as 'semi analytical' and 'latitude-dependent heat transport' models. Semi analytical models are based on the earlier works of Wasiutynski (1946), Biermann (1951) and Kippenhahn (1963). In these models, it is proposed that the anisotropic Reynolds stresses in the momentum equation plays the dominant role in maintaining the differential rotation. Based on this idea, many models were developed for the explanation of the surface differential rotation (Kohler 1970) and the internal differential rotation (Pileva 1985; Pulkkinen et al. 1993; Kuker, Rudiger & Kichatnov 1993). Another school of thought is based on the idea that the interaction of rotation with the convection leads to meridional flow which in turn generates and maintains the solar differential rotation (Gilman & Miller 1981; Glatzmaier 1985; Brummel, Hurlburt & Toomre 1998; Elliot et al. 1998; Robinson & Chan 2001).

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The aim of present study is to solve analytically and consistently the full set of Chandrasekhar's (1956) MHD equations for steady part of rotation and toroidal component of the magnetic field by taking into account the effects of important agencies, viz., *eddy viscosity also*. We assume that dominant part of the Sun's rotation at the present epoch could be mainly 'steady' (which may vary on diffusion time scales, i.e., ~ billion yr), superposed on which is a 'fluctuating' part whose amplitude of variation is very small over the solar cycle. In fact, observational evidences (Gough et al. 1993; Woodard & Libbrecht 1993) show the amplitudes of such variations in the internal rotation to be ~ 4 *nHz*. Very recent results (Howe, Komm & Hill 1999) confirm the earlier inferred results. This amplitude of variation is very small compared to the large-scale rotation which is ~ 460 *nHz*. Though conventional belief that the *observed differential rotation is a consequence of compressible convection in a rotating body*, these observed inferences suggest other way that large-scale rotational structure is not much affected by the movement of the solar plasma due to short term variations in pressure or density scale structures. This is one of the main reason that we are reasonable and safe in adopting the assumption of incompressibility in the following study. Thus we consider the Sun's rotation at the present epoch is approximately steady which may be of primordial origin (Alfvén 1942; Cowling 1953). In this study, we have not computed diffusion time scales for the steady part of rotation and toroidal component of magnetic field. Since, form of the equations for rotation and toroidal magnetic field are same as that of equation used in solving the poloidal component of the magnetic field (Hiremath & Gokhale 1995a), we expect similar order of diffusion time scales.

By neglecting viscous terms in Chandrasekhar's equations (1956), Nakagawa (1969) modeled the differential rotation in an incompressible spherical shell of infinite electrical conductivity in the presence of an absolutely steady toroidal magnetic field, without the presence of any poloidal field. Though, it is reasonable to assume that either the rotation or the toroidal magnetic field is absolutely steady, it is unreasonable to assume that poloidal component of the magnetic field is completely absent. In case of the Sun, presence of such a steady part of the magnetic field of primordial origin is neither ruled out nor detected observationally.

In the present study, we consider the terms related to eddy viscosity (Nakagawa & Swarztrauber 1969) and magnetic eddy diffusivity in the full set of Chandrasekhar's MHD equations for steady part of rotation. These important terms are the representative physical terms of the convection and solar convective envelope. We emphasize the property of the solar convection zone that the large magnitudes of aforementioned terms enabled us to solve analytically momentum equations. At the surface, we use boundary conditions for the rotation and toroidal magnetic field similar to those assumed by Nakagawa (1969). However, depending upon the following assumed two boundary conditions at base of the convection zone, we have two set of solutions. In the first solution, we assume that the Sun is rotating uniformly at base of the convection zone. In the second solution, we assume that the Sun is rotating differentially at the base. Except in magnitudes of rotational velocities near the polar regions, both the solutions yield rotational isocontours similar to the rotational isocontours inferred from the helioseismic data. By comparing goodness of fits with the available helioseismic inferred rotation rates (Antia, Basu & Chitre 1998), we conclude that the second solution for steady part of the rotation is closer to the real rotation. This indicates that at base of the convection zone,

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differential rotation rather than the uniform rotation is more likely to be present. Preliminary results of this study were presented elsewhere (Hiremath & Gokhale 1995b; Hiremath & Gokhale 1996; Hiremath & Gokhale 1998). In the present paper, we give mathematical details and discuss the implications of these results in the context of solar lithium depletion and solar activity phenomena.

Though aim of this paper is mainly to study the internal rotation of the Sun, for the sake of completeness and consistency of the solution of MHD equations, it is necessary to solve toroidal component of the magnetic field. This study indicates that in addition to the isotropic eddy viscosity a suitable toroidal field structure is necessary in order to maintain the solar differential rotation in the convective envelope. This view will be cleared and strengthened as the reader goes through this work. It is to be noted that reader should not be confused with steady field with time dependent part of the magnetic field which may be generated by dynamo or any unknown mechanisms. Except near base of the convection zone, both the aforementioned solutions give similar field structures. This consists of a two-zone field structure of strength (~ 1 G) near the surface and a four-zone field of strength (~ 10^4 G) near base of the convection zone. Near base of the convective envelope, compared to the first solution, the second solution yields the field structure closer to the sites of sunspot formations. It is interesting to know whether such a basic steady toroidal field structure in the convective envelope is responsible for the time dependent part of the magnetic field which is supposed to be generated by the dynamo mechanism (Parker 1993; Charbonneau & Mac Gregor 1997; Mac Gregor & Charbonneau 1997; Durney 1997; Dikpati & Charbonneau 1999) and emerge as bipolar magnetic regions towards the surface (Fan et al. 1999), though many doubts regarding the applicability of so called dynamo mechanisms for the solar case have been raised by the different studies (Bhattacharjee & Yuan 1995; Cattaneo & Hughes 1996; Cattaneo & Vainstein 1991; Gruzinov & Diamond 1994; Kleeorin, Rogachavskii & Ruzmaikin 1995; Levy 1992; Parker 1995; Piddington 1971; Piddington 1972; Piddington 1973; Piddington 1976; Piddington 1983; Vainstein & Cattaneo 1992; Andrew and Thomas 1999).

The plan of the paper is as follows. In section 2, we present assumptions made and the resulting form of Chandrasekhar's MHD equations. We give assumed boundary conditions in section 3 and, solution of the resulting form of equations in section 4. In section 5, we present the numerical results. Conclusions and discussion are presented in section 6.

2. Assumptions and resulting form of the basic equations

As in our previous work (Hiremath & Gokhale 1995a; Hiremath 1994), we assume that, in the convective envelope, the fluid is incompressible and the large-scale magnetic fields and the fluid motions are symmetric about the rotation axis. We also assume that the magnetic eddy diffusivity η and the eddy diffusivity due to viscosity ν are constants with values represented by the appropriate averages.

Following Chandrasekhar (1956), the magnetic field B and the velocity V for the axisymmetric system can be expressed

$$\mathbf{h} = -\varpi \frac{\partial P}{\partial z} \hat{\imath}_{\varpi} + (\varpi T)\hat{\imath}_{\varphi} + \frac{1}{\varpi} \frac{\partial}{\partial \varpi} (\varpi^2 P)\hat{\imath}_z, \tag{1}$$

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$$\mathbf{V} = -\varpi \frac{\partial U}{\partial z}\,\hat{\imath}_{\varpi} + (\varpi\Omega)\hat{\imath}_{\varphi} + \frac{1}{\varpi}\frac{\partial}{\partial \varpi}(\varpi^2 U)\hat{\imath}_z, \tag{2}$$

where $\mathbf{h} = \mathbf{B}/(4\pi\rho)^{1/2}$, ρ is the density, ϖ, φ, z are the cylindrical polar coordinates, with their axes along the axis of solar rotation; $\hat{\imath}_{\varpi}$, $\hat{\imath}_{\varphi}$, $\hat{\imath}_z$ are the corresponding unit vectors and; P, T, V, and U are the scalar functions which are independent of φ.

Further we make the following assumptions and approximations.

(i) Steady parts of the poloidal magnetic field P and poloidal component of the velocity field U (meridional velocity) are very weak compared to the steady part of the rotation Ω. In fact such a steady part of poloidal magnetic field is found to be ~ 1 G from the observation (Stenflo 1993) and ~ 0.01 G from theoretical calculations (Hiremath and Gokhale 1995a). Thus, by taking average density of the sun, Alfvén velocity varies between ~ 1 - 0.01 cm sec^{-1} which is very negligible compared to the dominant part of rotational velocity (~ 10^5 cm sec^{-1}). This leads us to safely assume that P is approximately zero and for the sake of making the problem simple we put $P = 0$ in the following Chandrasekhar's MHD equations. Similarly poloidal part of the velocity (meridional circulation) over the surface found to be ~ 0.001 – 0.01 times the rotation velocity. Though we can not neglect the terms of meridional velocity in MHD equations, it can not be equated with the terms of dominant part of the angular velocity Ω. We also assume that strength of steady part of toroidal field T is less than (or at most comparable to) that of the steady part of rotation. These assumptions lead to decoupling of poloidal part of velocity equation and, thus we have the following modified Chandrasekhar's (1956) MHD equations which take into account the eddy viscosity (Nakagawa and Swarztrauber 1969) also

$$\nu\varpi\Delta_5(\Delta_5 U) + [\varpi^2 U, \Delta_5 U] - \varpi\frac{\partial \Delta_5 U}{\partial t} = 0, \tag{3}$$

$$\eta\varpi\Delta_5 T + [\varpi^2 U, T] - \varpi\frac{\partial T}{\partial t} = 0, \tag{4}$$

$$\varpi^3\nu\Delta_5\Omega + [\varpi^2 U, \varpi^2\Omega] - \varpi^3\frac{\partial \Omega}{\partial t} = 0, \tag{5}$$

$$\varpi\frac{\partial}{\partial z}(T^2 - \Omega^2) = 0, \tag{6}$$

where
$$[f, g] = \frac{\partial f}{\partial z}\frac{\partial g}{\partial \varpi} - \frac{\partial f}{\partial \varpi}\frac{\partial g}{\partial z}, \tag{7}$$

and
$$\Delta_5 = \frac{\partial^2}{\partial z^2} + \frac{3}{\varpi}\frac{\partial}{\partial \varpi} + \frac{\partial^2}{\partial \varpi^2}, \tag{8}$$

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It is very crucial to be noted that in the absence of magnetic field and meridional circulations, the equations (5) and (6) are almost similar to the equations used in the axisymmetric hydrodynamic theories (Kichatinov 1990). Thus, inclusion of magnetic field in the hydrodynamical equations implies that in addition to isotropic eddy viscous stresses, a suitable structure of toroidal part of the magnetic field which contributes Maxwell stresses is necessary in order to maintain differential rotation in the convective envelope.

We neglect the Reynold stresses (~ gradient of averages of $\rho v_i' v_k'$, where v_i' and v_k' are the fluctuating parts of the velocity over the mean flow) terms in the momentum equations for the following reasons. For the steady state solution and by considering helioseismic inferences (Basu, Antia & Tripathy 1998) of the residual velocity (which is supposed to be fluctuating parts of velocity fields), we found that magnitudes of terms $\rho v_i' v_k'$ are very much smaller than the dominant terms $\varpi^3 \nu \Delta_5 \Omega$ and $[\varpi^2 U, \varpi^2 \Omega]$ in equation (5). Thus one has to be cautious (see also Tassoul 2000) in claiming that "Reynolds stress is the major driver of solar differential rotation".

The buoyancy term in the above derived equations is absent due to the fact that these equationsa are derived by taking curl of the basic MHD equations (Chandrasekhar 1956). On the other hand, buoyancy term is important when one wants to study the dynamics of localized structures like sunspots.

Assumption of incompressibility, large diffusion time scales of rotation compared to the time scale of the sound speed and, assumed form of axisymmetric velocity (eg. eqn 2) in the present study lead to $\frac{D\rho}{Dt}$ and ρdiv (**V**) terms zero in the continuity equation. Except the condition of incompressibility, in a way, vanishing of $\frac{D\rho}{Dt}$ and ρdiv (**V**) terms in the continuity equation is almost akin to the *anelastic approximation* used in some of the numerical simulations of rotation in the convective envelope (Elliot et al., 1998 Gough 1969). Assumption of incompressibility also means that there is no change in density variations of the moving plasma which implies that sound speed is very much greater than the Alfvén speed. This condition is almost true in case of the solar convective envelope.

We neglect the equation of the temperature structure for the following reasons. The heat equation implies that the rate of increase of heat for a unit volume as it moves in space is due to the net effects of energy sinks and sources. The same statement can be expressed (Priest 1982) as $\rho T \frac{DS}{dt} = -L$, where T is temperature, S is the entropy per unit mass of the plasma and L is energy loss function which includes the net effects of all the sinks and sources of energy and is defined as $L = \nabla \cdot q + L_r - j^2/\sigma - H$, where q is the heat flux due to conduction, L_r is heat due to radiation effects, j^2/σ is the ohmic dissipation due to currents, H is the term which represents all other heating sources. In case of the solar convective envelope, the first three terms (although L_r term near the surface may be effective) in L are negligible. The heating source term H includes heating due to nuclear energy, heating due to turbulent viscosity and heating due to any dissipation of waves in the medium. We can neglect the first term in case of solar convective envelope. On the other hand heating due to turbulent viscosity will be effective only when strong flows or rotational gradients in the convective envelope exist. Except strong shear in the rotational gradient at base and near surface of the convective envelope, helioseismic inferences rule out any such strong flows. Finally it is not clear whether

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dissipation of waves, especially due to ubiquitous sound waves, do contribute to the heating of the solar plasma. Thus overall one can conclude that L term in the solar convective envelope is almost zero which implies that entropy remains constant and hence the energy equation decouples from rest of momentum and induction equations.

(ii) The magnetic eddy diffusivity η and eddy diffusivity due to viscosity ν are very large (~ 10^{12} – 10^{14} cm^2/sec) in the convective envelope. This valid assumption leads to neglecting of second and third terms in the equations (3), (4) and (5) respectively. This is because the second terms ($[\varpi^2 U, \Delta_5 U]/\nu \sim U^2/\nu R_\odot^3$, $[\varpi^2 U, T]/\eta \sim UT/\eta R_\odot$, $[\varpi^2 U, \varpi^2 \Omega]/\nu \sim U\Omega/\nu R_\odot$) and the last terms ($\nu^{-1}\frac{\partial \Delta_5 U}{\partial t} \sim U/\nu R_\odot^2 \tau$, $\eta^{-1}\frac{\partial T}{\partial t} \sim T/\eta\tau$, $\nu^{-1}\frac{\partial \Omega}{\partial t} \sim \Omega/\nu\tau$, where $R_\odot$ is the radius and τ is the diffusion time scale which is assumed to be ~ 10^{10} yr) are very negligible compared to the first terms in the respective equations. Finally we have following simplified equations for probing the structure of the steady part of rotation and toroidal component of the magnetic field

$$\Delta_5(\Delta_5 U) = 0, \tag{9}$$

$$\Delta_5 T = 0, \tag{10}$$

$$\Delta_5 \Omega = 0, \tag{11}$$

$$\text{and } \frac{\partial}{\partial z}[\Omega^2 - T^2] = 0 \tag{12}$$

It is interesting to know the solution of equation (9) which yields meridional velocity in the sun's convective envelope. Since equation (9) is decoupled from the rest of the equations we plan to solve separately in near future. Along with the constraint, viz., equation (12), solution of the equations (10) and (11) yield steady part of toroidal component of the magnetic field and the rotation. The analytical solutions of equations (10) and (11) are as follows :

$$T(x,\mu) = \sum_{n=0}^{\infty} [b_n x^n + c_n x^{-(n+3)}] C_n^{3/2}(\mu), \tag{13}$$

$$\Omega(x,\mu) = \sum_{n=0}^{\infty} [V_n x^n + W_n x^{-(n+3)}] C_n^{3/2}(\mu), \tag{14}$$

where b_n, c_n, V_n, W_n (n = 0, 2, 4, ..) are constants, $x = r/R_\odot$, $R_\odot$ is the radius of the Sun, $\mu = \cos\vartheta$, ϑ is the co-latitude, $C^{3/2}{}_n(\mu)$ are the Gegenbaur polynomials of order 3/2. These Gegenbaur polynomials are related to the axisymmetric Legendre polynomials $P_n(\mu)$ in the following way

$$C_n^{3/2}(\mu) = \frac{dP_{n+1}(\mu)}{d\mu} \cdot \tag{15}$$

According to equation (12), equations (10) and (11) are subject to the constraint

$$\Omega^2 = T^2 + f(\varpi), \tag{16}$$

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where $f(\varpi)$ is an arbitrary function of $\varpi = x\sin\vartheta$. We assume $f(\varpi)$ to be in the form

$$f(\varpi) = \sum_{n=0}^{\infty} a_n \varpi^n. \tag{17}$$

The function $f(\varpi)$ and the constants b_n, c_n, V_n, W_n, a_n, ($n = 0, 2, 4, ..$) are to be determined from the boundary conditions.

Nakagawa (1969) assumed each of the functions T and Ω to be a series in Legendre polynomials and applied his boundary conditions. In the present study, T and Ω are solutions (13) and (14) of equations (10) and (11).

3. Boundary conditions

For the rotation and toroidal part of the magnetic field, we assume following boundary conditions at the surface and at base of the convection zone.

3.1 Boundary conditions at the surface

At the surface, we adopt the following boundary conditions for rotation and magnetic field, viz.,

$$\Omega = \Omega_{obs} \qquad \text{and} \qquad T = 0, \tag{18}$$

where Ω_{obs} is the observed surface rotation and is expressed in the form

$$\Omega_{obs} = A + B\cos^2\vartheta + C\cos^4\vartheta + \ldots.., \tag{19}$$

wherein A, B, C,.. are the coefficients of rotation determined from the observations.

The boundary condition $T = 0$ is derived from continuity of the current at the surface, assuming that the field outside is current free (Nakagawa & Swarztrauber 1969).

Equations (13) and the boundary condition (18) for T yield, (since Gegenbaur polynomials are mutually orthogonal),

$$c_0 = -b_0,\ c_2 = -b_2,\ c_4 = -b_4,\ \ldots, \tag{20}$$

where the coefficients b_0, b_2, ... etc have to be determined from the boundary conditions at base of the convection zone.

Thus we get the solution for the toroidal part of the magnetic field in terms of only one set of unknown coefficients (b_n), as :

$$T(x,\mu) = \sum_{n=0}^{\infty} b_n\left[x^n - x^{-(n+3)}\right] C_n^{3/2}(\mu). \tag{21}$$

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3.2 Boundary conditions at the base

As for boundary condition for the magnetic field, we consider magnetic field **h** and current (*cur*l**h**) are continuous. Since P is assumed to be zero, these conditions and from the equation (1) lead to T continuous. Similarly as for boundary condition for the velocity field, following Nakagawa and Swarztrauber (1969) we assume that tangential velocity is continuous at base of the convection zone. Since U is very much less than Ω in the convective envelope, assumption of tangential velocity and from the equation (2) lead to Ω continuous. That means convective envelope and radiative core are coupled and transfer of angular momentum takes place from one region to another region or vice versa. We assume the rotation to be of the form :

$$\Omega_{base} = A' + B' \cos^2\vartheta + .., \tag{22}$$

where A', B', ...are the coefficients in the law of rotation which are obtained from the internal rotation inferred from the helioseismology.

3.3 Constraint on the lower boundary condition

Initially, we made an attempt to solve the coefficients in equations (13, 14, 16 and 17) with the terms up to $cos^2\vartheta$ in both the boundary conditions at surface and at base of the convection zone. This gave an inconsistent solution, *i.e.,* the number of equations are more than the number of unknowns. For the existence of solution, the polynomial in $cos^2\vartheta$ expressing $\Omega(x, \mu)$ at the base must contain one term less than at the surface. Thus, *inclusion of Lorentz force puts the constraint on the boundary condition* to be taken at the lower boundary, viz., at base of the convection zone. Thus, if we consider the rotational law at the surface up to $cos^4\vartheta$, then at the lower boundary, one must consider the rotation law up to $cos^2\vartheta$ only. Physically this means that strong Lorentz force near the base of the convective envelope try to make the rotation *less differential.*

4. Analytic solution of the MHD equations

Stability of the Sun's angular momentum requires that internal rotation should increase outwards from the interior (Schatzman & Praderie 1993). This type of solution is possible only when we neglect the terms which contain negative powers of x in equation (14). It is to be noted that the validity of using aforementioned stability criterion for the present problem can be questioned simply because the study which we have undertaken is for the case of rotating, viscous and MHD fluid. It is interesting to know whether such a stability criterion exists for the case of present study. Ideal situation is to consider two terms for the solution of steady part of rotation. In fact we tried to solve by including two terms in the equation (14), however, we could not reproduce the rotational isocontours of similar magnitude as those of rotational isocontours inferred by the helioseismology. Moreover we also found out from the solution that the coefficients W_n, in equation (14) are very small compare to the coefficients V_n. Hence, we neglect the terms which contain negative powers of x in equation (14) for the solution of rotation in the convective envelope. Since, there is no such a stability constraint on the magnetic field, we consider both positive and negative powers of x in equation (13).

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In the following, we have two solutions depending upon the two boundary conditions assumed at base of the convection zone.

4.1 Uniform rotation at the base

Since, the observed dominant part of the rotation is symmetric about the equator, we consider even values of n ($n = 0$, $n = 2$ ...) in equations (13, 14, 16 and 17). We assume that the Sun may be rotating uniformly at base of the convection zone. Then equation (22) yields the following relation

$$\Omega_{obs} = A'. \tag{23}$$

At the surface, as explained in section 3.3, we are forced to consider the rotation law up to $cos^2\vartheta$ only. Using equations (14) and (19) with the terms up to $cos^2\vartheta$ yield the unknown coefficients V_0 and V_2 in terms of the surface rotational boundary values

$$V_2 = (2/15)B, \tag{24}$$

and

$$V_0 = A + (3/2)B. \tag{25}$$

Similarly, at the lower boundary, equations (14) and (22) yield the coefficient

$$V_0' = A'. \tag{26}$$

Using surface boundary values, we solve equations (16) and (17) to get the unknown coefficients

$$a_2 = 21V_2^2 - 15V_0V_2 \tag{27}$$

and

$$a_0 = V_0^2 + (9/4)\ V_2^2 - 3V_0V_2 - a_2. \tag{28}$$

By using equations (16), (17) and (21), we have the following solution for the unknown coefficients b_n, ($n = 0$, 2)

$$b_2 = \frac{a_2 \in^2}{10(\in^2 - \in^{-5})\left[V_0'^2 - (a_0 + a_2 \in^2)\right]^{1/2}}, \tag{29}$$

$$b_0 = \frac{(10V_0'^2 - 10a_0 - 9a_2 \in^2)}{15(1 - \in^{-3})\left[V_0'^2 - (a_0 + a_2 \in^2)\right]^{1/2}}, \tag{30}$$

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where $\in$ represents the depth of the convection zone.

4.2 Differential rotation at the base

In this second solution, we assume that the Sun is rotating differentially at base of the convection zone. In this case, we take differential rotation to be of the form

$$\Omega_{obs} = A' + B' \cos^2\vartheta. \tag{31}$$

At the surface, we have to consider the rotational law up to $\cos^4\vartheta$ for reasons given in section 3.3. By solving equations (14) and (19) we get the following three unknown coefficients in terms of observed rotation from the surface boundary conditions

$$V_4 = (8/15)C, \tag{32}$$

$$V_2 = (2/15)\,[B + (210/8)V_4], \tag{33}$$

and

$$V_0 = A + (3/2)V_2 - (15/8)V_4. \tag{34}$$

Similarly, using equations (14) and (31), we get two unknown coefficients at base of the convection zone

$$V'_2 = (2/15)\,B', \tag{35}$$

and

$$V'_0 = A' + (3/2)B' \in^2. \tag{36}$$

Using equations (14), (16) and (17) we have the following coefficients

$$a_4 = (225/4)V_2^2 + (53550/64)V_4^2 - (4095/8)V_2V_4 + (315/4)V_0V_4, \tag{37}$$

$$a_2 = (45/2)V_2^2 + (3150/32)V_4^2 - 15V_0V_2 - (855/8)V_2V_4 + (210/4)V_0V_4 - 2a_4, \tag{38}$$

and

$$a_0 = V_0^2 + (9/4)V_2^2 + (225/64)V_4^2 - (45/8)V_2V_4 - 3V_0V_2 + (15/4)V_0V_4 - a_2 - a_4. \tag{39}$$

By satisfying the boundary conditions at base of the convection zone, we solve the equations (16), (17) and (21) for the coefficients b_n, (n = 0, 2, ...) in terms of the coefficients a_n (n = 0,2,4, ...), in the following way

$$b_4 = \frac{1}{(\in^4 - \in^{-7})}\left[\frac{4f_3}{315f_1} - \frac{f_2^2}{315f_1^3}\right], \tag{40}$$

$$b_2 = \frac{1}{(\in^2 - \in^{-5})}\left[\frac{f_2}{15f_1} + \frac{14f_3}{315f_1} - \frac{14f_2^2}{1260f_1^3}\right], \tag{41}$$

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wherein

$$f_1 = \left[V_0'^2 + (9/4)\epsilon^4 V_2'^2 - 3\epsilon^2 V_0' V_2' - a_0 - a_2\epsilon^2 - a_4\epsilon^4\right]^{1/2}, \quad (43)$$

$$f_2 = -(45/2)\epsilon^4 V_2'^2 + 15\epsilon^2 V_0' V_2' + a_2\epsilon^2 + 2a_4\epsilon^4, \quad (44)$$

and

$$f_3 = (225/4)\epsilon^4 V_2'^2 - a_4\epsilon^4. \quad (45)$$

5. Numerical results

Knowing the coefficients V_n, a_n and b_n ($n = 0,2, ..$), we compute the steady part of rotation and toroidal component of the magnetic field using equations (14) an (13). With the assumption that dominant part of the Sun's internal rotation is steady, we compare our rotational results with the rotational data inferred from the helioseismology (Antia, Basu & Chitre 1998). In order to compare with the helioseismologically inferred rotation rates, we compute rotation rates with a step widths of 0.05 $R_\odot$ along the radius and intervals of 5° in latitude. While computing the χ^2, first we compare the whole (totally 1080 values) helioseismic inferred rotation rates with the rotation rates obtained by solving the MHD equations and then with inferred rotation rates (totally 780 values) in the region of 60° – 90° of co-latitude, where the inferred rotational results are more reliable.

For the rotation, we use the boundary values at the surface and at base of the convection zone deduced from the helioseismic data (Goode & Dziembowski 1991). We take depth of the convection zone ϵ to be 0.713 which is determined by Christensen-Dalsgaard, Gough and Thompson (1991). Since there are small variations in determinations of depth of convection zone, we varied the depth of the convection zone from $x = 0.699$ to $x = 0.715$ and found in the following almost similar results. With the two different boundary conditions, viz., at base of the convection zone, we have the following results.

Table 1. Components of the steady parts of rotation and toroidal part of the magnetic field.

	Coefficients of Rotation[a]			Coefficients of magnetic field[b]		
Solutions	V_0	V_2	V_4	b_0	b_2	b_4
Solution 4.1	2.83e-04	-5.07e-06	...	.137.29	61.91	....
Solution 4.2	2.79e-04	-9.63e-06	-1.30e-06	174.74	56.92	12.77
Components obtained by modifying the rotational boundary conditions						
Solution 4.2	2.78e-04	-1.05e-05	-1.55e-06	178.07	55.05	8.12

[a] Coefficients of rotation in rad/sec.
[b] Coefficients of magnetic field in gauss.

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Table 2. χ^2 and it's significance for all the points.

Solutions	χ^2	Significance of χ^2
Solution 4.1	5394.24	~ 0%
Solution 4.2	934.82 (608.90)	~ 99.90%

The value of χ^2 in bracket is obtained by changing values of coefficients in the rotation law.

5.1 Results from "solution 4.1"

We take the boundary values (Goode & Dziembowski 1991) $A = 462.4$ nHz and $B = -60.5$ nHz at the surface and, $A' = 438.7$ nHZ at base of the convection zone. In Table 1 we present the coefficients of steady part of rotation and toroidal component of the magnetic field. For all and for 780 computed rotation rates, the value of chi-square and the significance of chi-square are given in Table 2. and Table 3. respectively. The rotational isocontours in the meridional cross section is given in Fig. 2. For the sake of comparison, in Fig. 1, we present the isorotational contours obtained from the helioseismology (Antia, Basu & Chitre 1998).

Table 3. χ^2 and it's significance for 780 points.

Solutions	χ^2	Significance of χ^2
Solution 4.1	977.50	~ 45%
Solution 4.2	167.45 (119.7)	~ 100%

The value of χ^2 in bracket is obtained by changing values of coefficients in the rotation law.

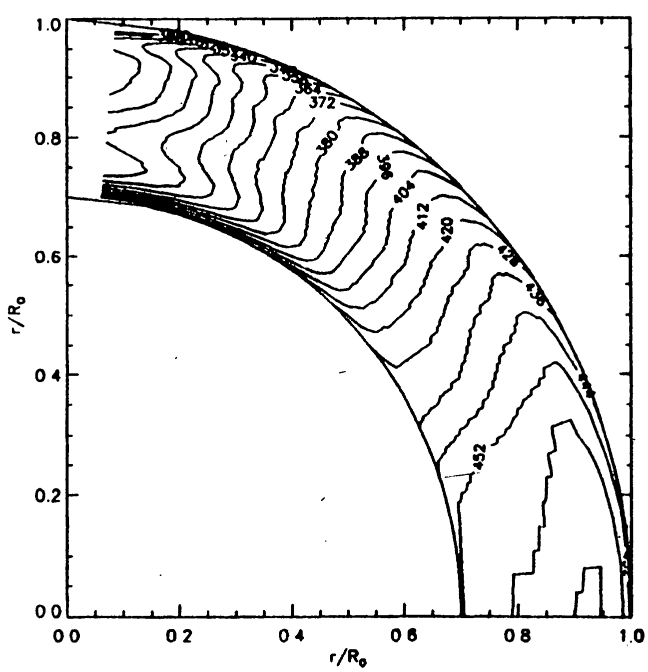

Figure 1. Meridional cross section of the solar rotational profile in the convective envelope inferred from the helioseismology (Antia, Basu and Chitre 1998). Isorotational contours are in nHz.

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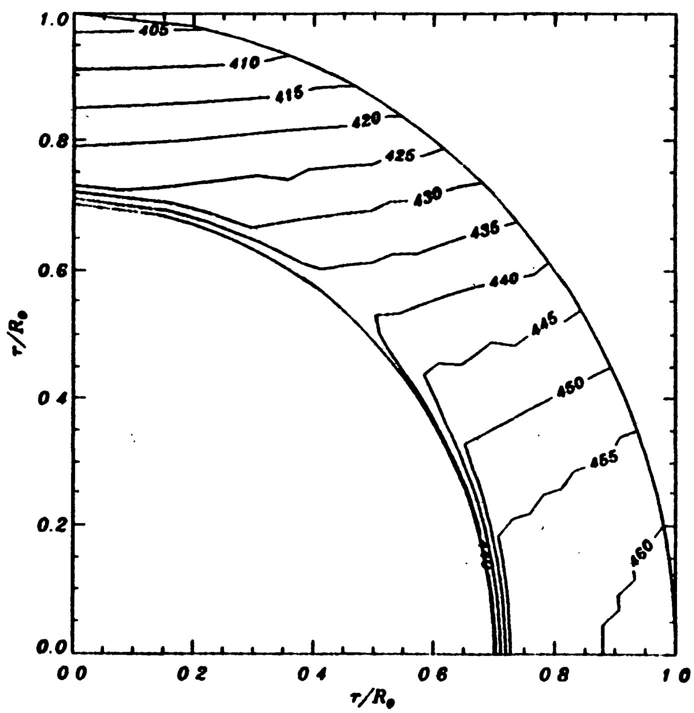

Figure 2. Meridional cross section of the rotational profile in the solar convective envelope obtained by the solution (4.1) of MHD equations. Isorotational contours are in nHz.

Note that except in magnitudes of rotational velocities at the both poles, this solution reproduces similar rotational isocontours as that of helioseismologically inferred rotational isocontours. However, the solution yields faster rotating poles compared to the helioseismic results. Also chi-square fit with the inferred helioseismic rotational rates is poor (~ 0% for all the points and ~ 45% for 780 points). This leads us to consider the second "solution 2".

In the Chandrasekhar's equations, toroidal magnetic field T is normalized to the Alfvén velocity which in turn depends upon the density structure. By taking the density values from the solar seismic model (Shibahashi, Hiremath & Takata 1998), we compute steady component of the toroidal magnetic field whose iso-gauss (i.e, ϖT) contours are presented in Fig. 3. The solution yields a two-zone like field (~ 1 G) structure near the surface and a four-zone like field (~ 10^4 G) structure near base of the convection zone. It is interesting to note that the distribution of toroidal magnetic field is concentrated in the belts where most of the Sun's flux emerges.

5.2 Results from "solution 4.2"

We consider the boundary values of rotation (Goode & Dziembowski 1991) A = 462.4 nHz, B = –60.5 nHz, C = –81.7 nHz at the surface and , A' = 438.7 nHz and B' = –11.2 nHz at base of the convection zone. In Table 1., we present the steady parts of rotation and components of toroidal magnetic field. The values of chi-square and significance of chi-square are given in Table 2. and Table 3. respectively. Compared to the previous solution (section 5.1), the chi-square value is improved further. The rotational isocontours in the convective envelope are

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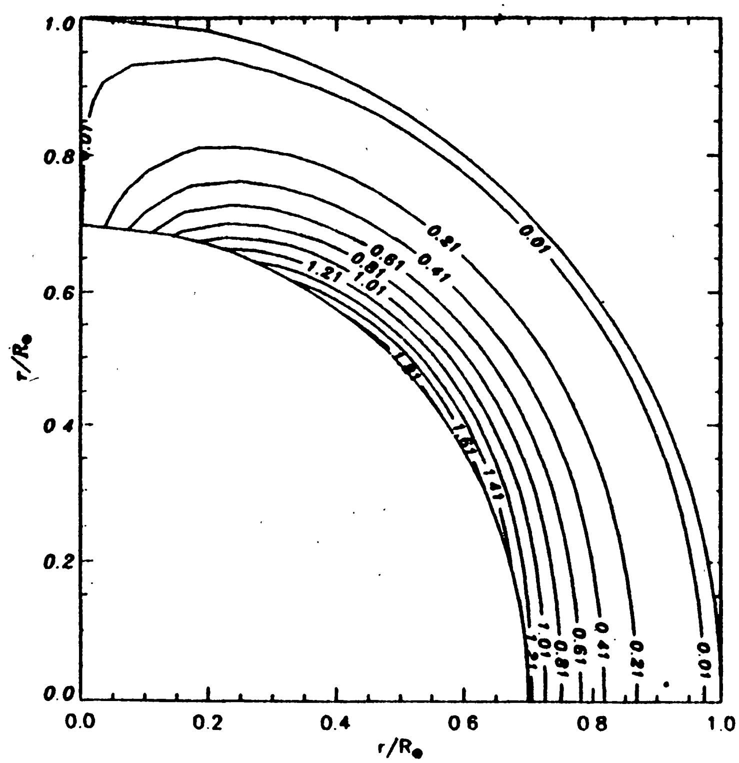

Figur 3. Meridional cross section of the steady part of the toroidal magnetic field in the solar convective envelope obtained by the solution (4.1) of MHD equations. Isogauss contours are in 10^4 G.

presented in Fig 4. Compared to the previous solution, the present solution yields nearly the same results (significance χ^2 for all the points is 99.903%) even near the polar regions and almost same results (significance of χ^2 for 780 points is ~ 100%) in the region where inferred results are more reliable. Thus, this result indicates that at base of the convection zone, *differential* rotation is more likely rather than uniform rotation. This result also agrees with the other helioseismic inferred rotation results (Thompson et a. 1996; Sekii et al. 1997; Kosovichev et al. 1997; Korzennik et al. 1997) at base of the convection zone. We present the results of isogauss contours in Fig 5. Though, this solution also yields similar field structure, it has following difference compared with the previous solution. In the present solution, at base of the convection zone, the strong toroidal belts are more closer to the sites of sunspot formations compared to the field distribution presented in Fig. 3.

Note that the resulting rotational isocontours (Fig 4.) are matching with the helioseismologically inferred rotational isocontours very well upto the middle latitudes. However, the magnitudes of computed rotation rates near the polar regions are slightly different than the magnitudes of rotaion rates inferred from the helioseismology. This could be due to the large uncertainties in the C and the B' coefficients obtained by (Goode & Dziembowski 1991). The uncertainties could be either positive or negative values added to the best fitted values. Since, C and B' coefficients are negative, we add negative uncertainties to the best fitted values. When we take into account the uncertainties in both the coefficients, the resulting solution yields

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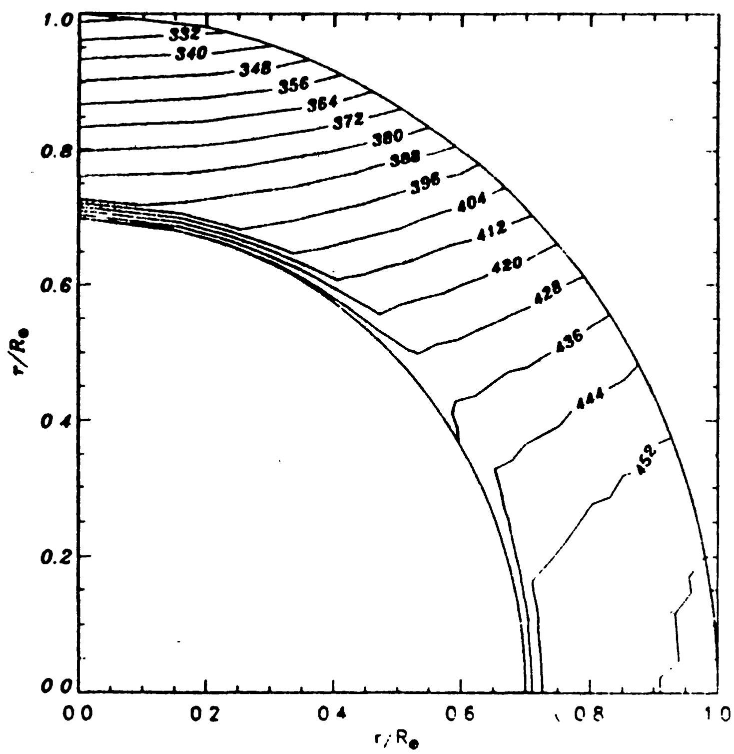

Figure 4. Meridional cross section of the rotational profile in the solar convective envelope obtained by the solution (4.2) of MHD equations with the imposition of boundary condition that base of the convection zone rotates differentially. Isorotational contours are in nHz.

perfect matching with the helioseismic rotational profile. Thus, when we consider the coefficients $C = -97.0$ nHz and $B' = -31.0$ nHz which take into account the uncertainties, we have low value of chi-square (608.90 for all the points and 119.7 for the 780 points). The resulting solutions for rotation and toroidal part of magnetic field, by changing coefficients in the boundary conditions, are presented in Fig 6. and Fig 7.

6. Concluions and discussion

The helioseismic inferences from the observed rotational frequency splittings brought up a wealth of information regarding rotation in the solar convective envelope. There are many theoretical works and simulations which try to reproduce the solar internal rotation. Using property of the solar convective envelope and with the assumption that steady part of rotation represents helioseismologically inferred rotation, depending upon used two boundary conditions at base of the convection zone, we solve analytically Chandrasekhar's MHD equations and the results are as follows.

In the first solution, which is obtained by using assumed boundary condition that the Sun may be rotating uniformly at base of the convection zone, we obtain similar rotational isocontours as those of helioseismologically inferred rotational isocontours with the important difference of

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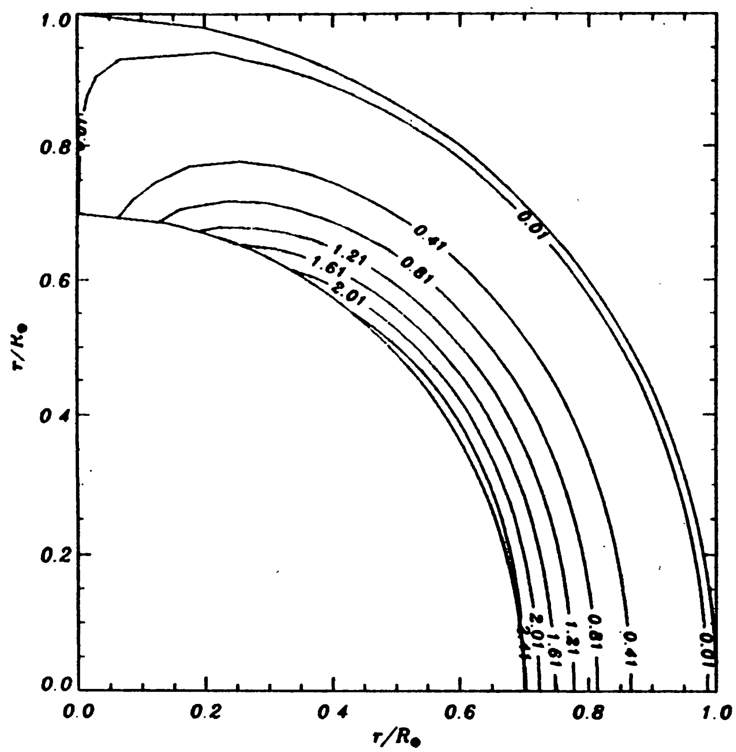

Figure 5. Meridional cross section of the steady part of the toroidal magnetic field in the solar convective envelope obtained by the solution (4.2) of MHD equations with the imposition of boundary condition that base of the convection zone rotates differentially. Isogauss contours are in 10^4 G.

faster rotating poles. These important results lead us to the second solution which we obtain from the assumed boundary condition that the Sun may be rotating differentially at base of the convection zone.

In the second solution, we obtain rotational isocontours much closer to the helioseismic inferred rotational isocontours yielding a very good χ^2 fit. This suggests that the Sun's rotation is more likely to be *differential* rather than uniform rotation at base of the convection zone. In fact, in one of our previous studies (Hiremath & Gokhale 1996) and using different helioseismologically inferred rotation rates (Dziembowski, Goode & Libbrecht 1989), we also arrived at the same conclusion.

Our conclusion that the Sun may be rotating *differentially* at base of the convection zone is mainly based on the comparison of results of the second solution (section 4.2) with the helioseismic inferred rotational results (Antia, Basu & Chitre 1998) obtained at the single epoch only. In order to study steady part of rotation and toroidal component of magnetic field and, for the sake of comparison of the these results with the helioseismic inferred results, it is necessary to compare with a suitable average rotational profile obtained from averaging over one solar cycle data. This may eliminate the fluctuating part of rotation which may contributes to the bumps and kinds in the inferred helioseismic rotational profile, especially near the surface.

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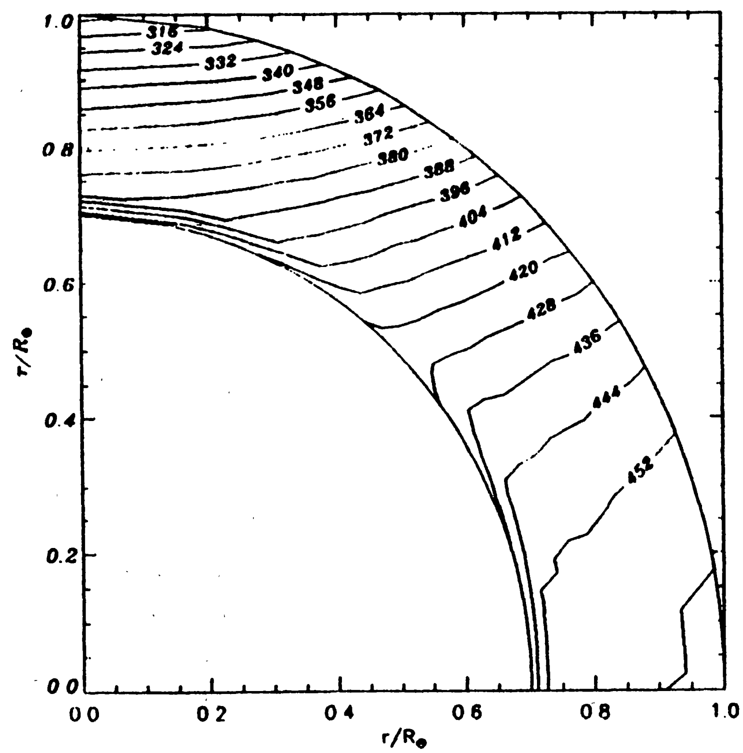

Figure 6. Meridional cross section of the rotational profile in the solar convective envelope obtained by the solution of MHD equations by modifying the coefficients of C and B' in the rotational boundary conditions. Isorotational contours are in nHz.

Though aim of the present work appears to be similar to the work presented by Nakagawa & Swarztrauber (1969) (here afterwards NS69), the following important differences in the resulting solution exist. Our solution of rotational isocontours is almost similar to the rotational isocontours as inferred by helioseismology. Where as there is a serious discrepancy between NS69 solution of rotational isocontours and helioseismologically inferred rotational isocontours. Solution of NS69 yields the direction of meridional circulation from pole to equator contrary to the observations and helioseismic inferences near the surface which show that direction of the flow is from equator to pole. This could be due to unphysical boundary conditions used in the solution of meridional circulation. In reality one should take the observed surface boundary condition for the meridional circulation and inner solution has to be obtained. We plan to solve in this manner for the meridional part of the velocity. Moreover the work of NS69 has following physical inconsistencies also : (a) unphysical boundary conditions at base of the convection zone, i.e., $P = 0$ and $T = 0$; (b) there is no clear answer for their assumptions that $\Delta_5 P = 0$ and $\Delta_5 \Omega = 0$ in the whole convective envelope. However, these equations are valid and are obtained from the reasonable assumptions and approximations (section 2) which we incorporated in our work. For example, we assumed that magnitudes of eddy viscosity and magnetic eddy diffusivity are very large in the convective envelope which is not a bad approximation. Hence we believe that our solution is closer to the reality than the solution of NS69.

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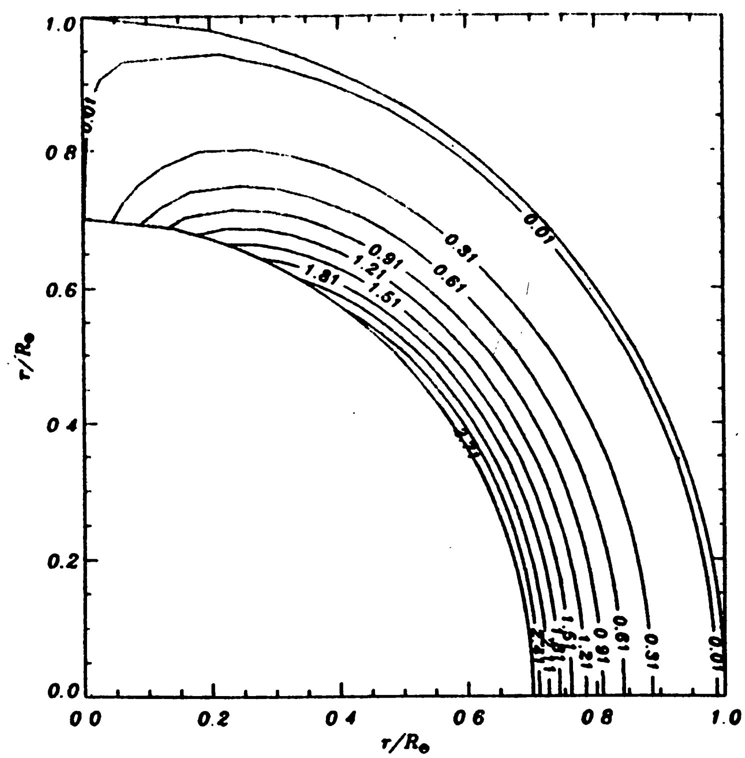

Figure 7. Meridional cross section of the steady part of the toroidal magnetic field in the solar convective envelope obtained by the solution of MHD equations by modifying the coefficients of C and B' in the rotational boundary conditions. Isogauss contours are in 10^4 G.

The aforementioned result that sun may be rotating *differentially* at base of the convection zone has implication for understanding the problem of lithium depletion in the Sun. If we assume that there is no rotational discontinuity at base of the convection zone, aforementioned result implies that radiative core may have a weak (it is very difficult to ascertain how much weak the differential rotation in the radiative core from the p mode rotational splittings) differential rotation superposed on the strong uniform rotation, both of which could be of primordial origin. At the present epoch, differential rotation appears to be weak in the radiative core. However, in the past epoch it could have been strong enough to induce the rotational mixing (Zahn 1983; Zahn 1992; Pinsonneault 1994) that might have lead to lithium destruction at base of the convection zone.

In the present study, we have compared the results of internal rotation with the internal rotation obtained from the inversion of helioseismic data. However, most of the helioseismic inversions depend mostly on the assumed free parameter used in the computations. For example, different free parameters in the inversions may yield slightly different topological rotational isocontours (Sekii 1995). Thus, we should use the rotational data, which is free from such modelling. Another way of checking our results is to compute directly rotational frequency splittings using solar kernels and our resulting rotational profile. These computed rotational splittings can be compared with the observed rotational splittings a_1, (l = 1,3, ...).

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Ritzwoller and Lavelly (1991) have shown that, in the inversion methods, if differential rotation is expanded in terms of the selected basis functions, viz., derivative of axisymmetric Legendre function of degree l, rather than commonly used ad hoc basis functions, then several significant problems currently facing in the inversions will disappear. It is interesting to note that the basis functions $C_n^{3/2}(\mu)$ in the rotational solution which are derivatives of axisymmetric Legendre functions of degree $(l + 1)$ are similar to the basis functions used in the Ritzwoller and Lavelly (1991) method. We propose here that, it would be physically more appropriate if we use the $C_n^{3/2}(\mu)$ basis functions of degree $(l + 1)$ in the Ritzwoller and Lavelly (1991) method of inversion, rather than the basis functions which are of degree l only.

Note that the equations used in the solution of solar rotation can be extended for the case of studying the stellar internal rotation, especially, late type main sequence stars which are supposed to be fully convective. Recently, we (Hiremath 1999) have undertaken such a study for understanding the internal rotation of AB Doradus. Thus, this study will be useful in understanding the internal rotation of the late type stars which in turn will be helpful in understanding the rotation of the core. In fact this study can be used as the feedback (by calculating the rotational splittings using the respective modeled stellar structure) for understanding the rotational splittings obtained from the asteroseismology. In this study, we have taken the depth of the convection zone $\in$ as a free parameter. However, if one knows the surface boundary conditions, by varying the boundary conditions at the base and by iteration one can determine the extent of the thickness of the convective envelope also.

Aim of the present work is mainly on the study of internal rotation of the Sun. However, present solution of MHD equations also yields steady part of the toroidal magnetic field in the solar interior. This consists of a two-zone structure of toroidal field (~ 1 G) near the surface and a four-zone structure of toroidal field (~ 10^4 G) near the base of the convection zone. Apart from these components in the second solution, we get an additional component ($n = 4$) with eight-zone structure of toroidal field. However, combination of all the components give similar field structure as that of the first solution.

Note that near base of the convection zone, the latitude belts of strong toroidal fields are concentrated where sunspots are supposed to be originated. The obtained field structure may give answer to the following important observational results viz., (i) the maximum flux eruption during the solar cycle is concentrated between the heliographic latitudes 5° and 75° (Howard & LaBonte 1981), (ii) the time variation of even degree splittings has strongest peak amplitudes near the belts of $50^\circ - 60^\circ$ heliographic latitude (Woodard & Libbrecht 1993) and, (iii) the dominant modes of MHD oscillations which may be contributing to the solar cycle and activity phenomena (Stenflo & Vogel 1986; Gokhale & Javaraiah 1992).

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2001BASI...29..169H

References

Alfvén H., 1942, Arkiv For Mathematik, Astron Och Fys, 28A, No. 6

Andrew M.J., Thomas M.J., 1999, ApJ, 523, 827

Antia H.M., Basu S.M., Chitre S.M., 1998, MNRAS, 298, 543

Bhattacharjee A., Yuan Y., 1995, ApJ, 449, 739

Balthasar H., Wohl H., 1980, A&A, 92, 111

Basu S., Antia H.M., Tripathy S.C., 1998, in the proceedings of the SOHO 6/GONG 98 meeting, ESA SP-418, p. 705

Biermann L., 1951, Zeitschr.f Astrophys., 28, 304

Brown T.M., Christensen-Daisgaard J., Dziembowski W.A., Goode P., Gough D.O., Morrow C.A., 1989, ApJ, 343, 526

Brummel N.H., Hurlburt N.E., Toomre J., 1998, ApJ, 493, 955

Cattaneo F., Vainstein S.I., 1991, ApJ, 376, L21

Cattaneo F., Hughes D.W., 1996, Phys. Rev. E., 54, 4532

Chan K.L., Mayr H.G., 1994, Solphys, 152, 283

Chandrasekhar S., 1956, ApJ, 124, 231

Charbonneau P., MacGregor K.B., 1997, ApJ, 486, 502

Christensen-Dalsgaard J., Gough D.O., Thompson M.J., 1991, ApJ, 378, 413

Christensen-Dalsgaard J., Schou J., 1998, (ESA SP-286), eds. V. Domingo and E.J. Rolfe, 149

Corbard T., Berthomieu G., Provost G., Blanc-Feraud L., 1998, proceedings of the SOHO6/GONG98 Workshop, ESA Sp-418, vol 2, 747

Cowling T.G., 1953, The Sun, ed. Kuiper G.P., The University of Chicago press, 532

Dikpati M., Charbonneau P., 1999, ApJ, 518, 508

Durney B.R., 1997, ApJ, 486, 1065

Dziembowski W.A., Goode P.R., Librecht K.G. 1989, ApJL, 337, L53

Elliot J.R. Miesh M.S., Toomre J., Glatzmaier G.A., 1998, Proceedings of the SOHO 6/GONG 98 Workshop, Edtors : Korzennik S., Wilson A, ESA SP-418, p. 765

Fan Y., Zweibel E.G., Linton M.G., Fisher G.H., 1999, ApJ, 521, 460

Gilman P.R., Miller W.A., 1981, ApJ, 46, No 2, 211

Gilman P.A., Howard R., 1984, ApJ, 283, 385

Glatzmaier G.A., 1985, ApJ, 291, 300

Gokhale M.H., Javaraiah J., 1992, Solphys, 138, 399

Goode P.R., Dziembowski W.A., 1991, Nature, 349, 223

Gough D.O., 1969, J. Atmos. Sci., 26, 448

Gough D.O., Kosovichev A.G., Sekii T., Libbrecht K.G., Woodard M.F., 1993, Inside the Stars, IAU. Coll., 137, eds. Weiss, W.W. and Baglin Annie, 93

Gruzinov A., Diamond P.H., 1994, phys. Rev. Let., 72, 1651

Hiremath K.M., 1994, Ph.D., Thesis, Bangalore University, India

Hiremath K.M., Gokhale M.H., 1995a, ApJ, 448, 437

Hiremath K.M., Gokhale M.H., 1995b, in Proc. of 4th SOHO Workshop: Helioseismology, Vol. 2, ed. B. Battrick (ESA Publ. Division, Nordwijk), 97

Hiremath K.M., Gokhale M.H., 1996, Bull. Astr. Soc. India, 24, 345

Hiremath K.M., Gokhale M.H., 1998, Bull. Astrom. Soc. India, 26, 161

Hiremath K.M., 1999, Bull. Astron. India, 28, 243 Astronomical Society of India meeting

Howard R., Harvey J., 1970, Solphys, 12, 21

Howard R., LaBonte B.J., 1981, Solphys, 74, 131

Howe R., 1998, proceedings of the SOHO6/GONG98 Workshop, ESA SP-418, vol 2, 669

© Astronomical Society of India • Provided by the NASA Astrophysics Data System

2001BASI...29..169H

Howe R., Komm R., Hill F., 1999, ApJ, 524, 1084

Kichatinov L.L., 1990, in the proceedings of IAU Symp 138, "Solar Photosphere : Structure, Convection, and Magnetic Fields", ed. Stenflo, J.O., Kluwer : Dordrecht, p. 297

Kippenhahn R., 1963, ApJ, 137, 664

Kleeorin N., Rogachavskii Ruzmaikin A., 1995, A&A, 297, 159

Kohler H., 1970, Solphys, 13, 3

Korzennik S.G., Rhodes E.J., Johnson N.M., Rose P., Cacianni A., 1995, in GONG 94: Helio and Astero-seismology, ASP, Conf. Ser, vol. 76, eds. Ulrich, R.K. Rhodes E.J., & Dappen W., 12

Korzennik S.G., et al., 1997, in IAU 181, p. 211

Kosovichev A.G., et al., 1997, in IAU 181, p. 203

Kuker M., Rudiger G., Kitchatinov L.L., 1993, A&A, 279, L1

Levy E.H., 1992, in "The Solar Cycle ASP Conf. Ser.", vol 27, p. 139

Livingston W.C., Duvall T.L., Jr. 1979, Solphys, 61, 219

Mac Gregor K.B., Charbonneau P., 1997, ApJ 486, 484

Mauro M.P., Dziembowski W.A., Paterno L., 1998, proceedings of the SOHO6/GONG98 Workshop, ESA Sp-418, vol 2, 759

Nakagawa Y., 1969, ApJ, 157, 881

Nakagawa Y., Swarztrauber P., 1969, ApJ, 155, 295

Newton H.W., Nunn M.L., 1951, MNRAS, 11, 413

Parker E.N., 1993, ApJ, 408, 707

Parker E.N., 1995, in the proceedings of GONG 94 : Helio & Astero-Seismology ASP Conference Series, Vo. 76, eds. R.K. Ulrich E.J., Rhodes Jr., & Dappen W., p. 3

Piddington J.H., 1971, Proc. Astron. Soc. Australia, 2, 7

Piddington J.H., 1972, Solphys, 22, 3

Piddington J.H., 1973, Astrophys. Space. Sci, 24, 259

Piddington J.H., 1976, in "Basic Mechanisms of Solar Activity", eds. Bumba V., & Kleczek J., IAU Symp, 71, (Dordrecht : Reidel), p. 389

Piddington J.H., 1983, Astrophys. Space. Sci., 90, 217

Pileva N.A., 1985, ApJ, 291, 300

Pinsonneault M.H., 1994, in Cool Stars, Stellar Systems and the Sun, ASP Con Ser, vol. 64, ed. Caillault J.P., 254

Priest E.R., 1982, Solar Magnetohydrodynamics, D. Riedel Publishing Company, Dordrecht, Holland

Pulkkinen P., Touminen I., Brandenburg A., Nordlund A., & Stein R., 1993, A & A, 267, 2565

Ritzwoller M.H., Lavelly E.M., 1991, ApJ, 369, 557

Robinson F.J., Chan K.L., 2001, MNRAS, 321, 723

Schatzman E.L., Praderie F., 1993, in The Stars, Springer-Verlag, 221

Scherrer P.H., Wilcox J.M., Svalgaard L., 1980, ApJ, 241, 811

Schou J., Christensen-Dalsgaard J., Thompson M.J., 1992, ApJ, 385, L59

Sekii T., 1995, in GONG 94 : Helio and Astero-Seismology, ASP conf.ser. vol. 76, eds. Ulrich R.K., Rhodes E.J., & Dappen W, 67

Sekii et al., 1997, in IAU 181, p. 189

Shibahashi H., Hiremath K.M., Takata M., 1998, proceedings of the SOHO6/GONG98 Workshop, ESA SP-418, vol 1, 537

Snodgrass H.B., 1983, ApJ, 270, 288

Stenflo J.O., 1977, A & A, 61, 797

Stenflo J.O., Vogel M., 1986, Nature, 319, 285

Stenflo J.O., 1993, in "Solar Surface Magnetism", eds. Rutten R.J., & Shrijver C.J., (Kluwer) Proc. NATO Advanced Research Workshop, Scocsterberg, The Netherlands

Tassoul J.L., 2000, in Stellar Rotation, Cambridge Astrophysics Series, 36, p. 89

© Astronomical Society of India • Provided by the NASA Astrophysics Data System

2001BASI...29..169H

Thompson M.J., 1990, Solphys, 125, 1
Thompson M.J., et al., 1996, Science, 272, 1300
Vainstein S.I., Cattaneo F., 1992, ApJ, 393, 165
Wasiutynski J., 1946, Astrophys. Norw., 4, 1
Wilcox J.M., Howard R., 1970, Solphys, 13, 251
Woodard M.F., Libbrecht K.G., 1993, ApJ, 402, L77
Zahn J.P., 1983, in Astrophysical Processes in Upper Main Sequence Stars, eds. Hauck, B & Maeder, A, 253
Zahn J.P., 1992, A & A, 265, 115

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