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Non-linear oscillation of inter-connected satellites system under the combined influence of the solar radiation pressure and dissipative force of general nature

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Abstract. The non-linear oscillation of inter-connected satellites system about its equilibrium position in the neighabourhood of main resonance $\omega = 1$, under the combined effects of the solar radiation pressure and the dissipative forces of general nature has been discussed. It is found that the oscillation of the system gets disturbed when the frequency of the natural oscillation approaches the resonance frequency.

Key Words : Satellites, non-linear oscillation, solar radiation pressure, forces of general nature.

1. Introduction

This paper deals with the study of non-linear effects due to combined influence of the solar radiation pressure and some perturbing forces of general nature on the motion and stability of two satellites connected by a light, flexible and inextensible string in the central gravitational field of the Earth.

Musen (1960) has considered the influence of the solar radiation pressure on the motion of an artificial Earth's satellite neglecting the effects of the Earth's shadow on it. Radzievskii and Artemev (1962), have jointly studied in their paper the influence of the solar radiation pressure on the variation of the perigee distance of an artificial Earth satellite, considering the effect of the Earth's shadow. The effects of the solar radiation pressure on the motion of an artificial Earth's satellites, Moon satellites, space craft and some dust particles in the circumterestrial dust cloud etc., has been considered by Vinogradova and Radzievskii (1965), Kripichnikov (1968), Divari and Klikh (1968), Brayant (1961), Minorsky et al. (1976).

Beletsky (1969) and Beletsky and Novikova (1969) studied the motion of the system of two satellites connected by a light, flexible and inextensible string in the central gravitational field of force relative to the center of mass, which is assumed to move along a Keplerian elliptical orbit. This study assumed that the two satellites are moving in the plane of the center of mass.

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This problem was dealt with in its more general form by Singh (1971, 1973), studying it both in the two and the three dimensional cases. Sinha and Singh (1987, 1988) studied the effects of solar radiation pressure on the motion and stability of the inter-connected satellite system, in circular as well as in elliptical orbit. His study was limited to linear effects only. Narayan and Singh (1987, 1990 and 1993), studied non-linear oscillation of an inter-connected satellites system under the influence of solar radiation pressure. Manaziruddin and Singh (1990, 1992), studied the effects of small external forces on the planar oscillation of a cable connected satellites system in orbit.

Neverthless a distant satellite beyond gravitational field of the Earth, apart from solar radiation pressure, it could still be expected to be affected by general nature external forces. These general nature external force could arise due to the dissipation of the energy generated on account of friction of bodies in the atmosphere by tidal forces, gravitational radiation etc. These forces though small, can significantly affect the oscillation of the system under consideration. These forces could be modeled as frictional forces with small dissipation coefficient. Further, the forces generated by the multiple moments and the absorption of gravitation waves at resonance frequency pyragas et al. (1978), could be characterized as external periodic forces having a slowly varying frequency. These forces then could be estimated by certain model assumptions. Thus, in order to study the non-linear oscillation of a distant scientific probe on realistic basis, it is essential to consider the combined influence of the solar radiation pressure, frictional forces and periodic force. The effects of the Earth shadow on the non-linear oscillation of the system has also been considered.

2. Equation of Motion

The influence of solar radiation pressure together with the effects of the Earth's shadow on the motion of two satellites connected by a light, flexible and inextensible string, under the action of the central gravitational field of the Earth, has been considered.

The analysis of the motion and stability of the system has been restricted to the two dimensional case, we have assumed that the satellites are moving in the orbital plane of the center of mass of the system. The equations of motion of one of the satellites under the solar radiation pressure, Sinha & Singh (1987), together with the effects of the Earth's shadow in Nechvile's coordinates system (1926) as shown in Figure 1 can be put in the form :

$$x'' - 2y' - 3x\rho + \rho^3 A\cos\epsilon\,\delta_r \cos(v - \alpha) = \lambda_a \rho^4 x \qquad 2.1$$

$$y'' + 2x' - \rho^3 A\cos\epsilon\,\delta_r \sin(v - \alpha) = \lambda_a \rho^4 y$$

where $\lambda_a = \dfrac{\rho^3 \lambda}{\mu}$

$$A = \frac{P^3}{\mu}\left[\frac{B_1}{m_1} \quad \frac{B_2}{m_2}\right]\delta_r$$

Where λ denotes Lagrange's multiplier and μ denotes the product of gravitational constant and the mass of the Earth. Here B_1 and B_2 are the absolute values of the forces due to the direct solar radiation pressure exerted on masses m_1 and m_2. Let v be the true anomaly of the center of mass of the system in elliptical orbit and α be the angular separation of the solar position vector projected on the orbital plane from the orbit perigee. Here $\in$ is the inclination of the osculating plane of the orbit of the center of mass of the system with the plane of ecliptic as shown in Figure 1.

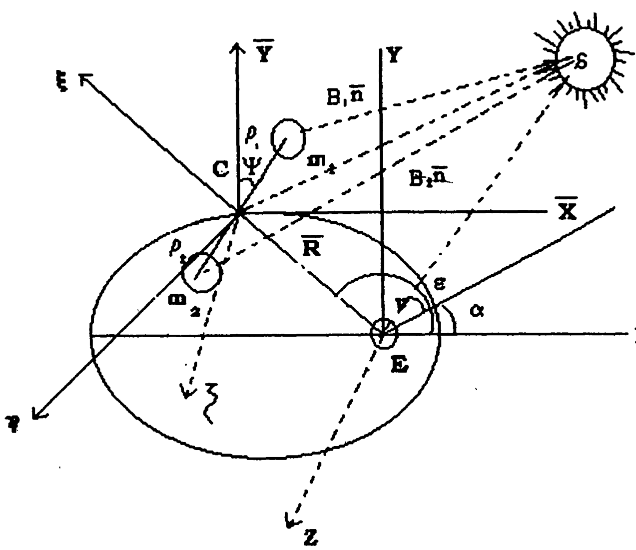

Figure 1. Rotating frame of reference.

In this case, the condition for constrained is given by the inequality

$$x^2 + y^2 \leq \frac{1}{\rho^2} \tag{2.2}$$

$$\text{where } \rho = \left(\frac{R}{P}\right) = \left(\frac{1}{1 + e \cos \mathrm{v}}\right) \tag{2.3}$$

Here p and e are focal parameter and eccentricity of the orbit of the center of mass and prime(/) denotes differentiation with respect to v. Here δ_r stands for Earth's shadow function. The system of two satellites has been allowed to pass through the shadow beam during its motion. We have assumed that the shadow beam is cylindrical, the angle θ has been assumed to be between the axis of cylinder (shadow beam) and the line joining the Earth's centre and the end point of the orbit of the center of mass, within the Earth's shadow, considering the positive direction towards the motion of the system. Hence, the system will start to be influenced by the perturbative force of the solar radiation pressure, when it will make an angle θ with the axis of the cylinder. Then it will remain under the influence of the solar radiation pressure till it will make an angle $(2\pi - \theta)$ with the axis of the cylindrical shadow beam when the effects of the solar radiation pressure will come to an end. The shadow function δ_r is taken to be 1 (one) when the system is under the influence of the solar radiation and 0 (zero), when it is within the shadow beam. Hence, the small secular and long periodic effects of the solar pressure together with the effects of the Earth's shadow on the system as shown in Figure-2 may be analyzed by averaging the periodic terms in the equation of motion with respect to v, it will take the equivalent value.

$$\frac{1}{2\pi}\left[\underset{\delta_r=0}{\int_{-\theta}^{\theta} A\cos(v-\alpha)dv} + \underset{\delta_r=1}{\int_{\theta}^{2\pi-\theta} A\cos(v-\alpha)dv}\right] = -K\cos\alpha$$

and
$$\frac{1}{2\pi}\left[\underset{\delta_r=0}{\int_{-\theta}^{\theta} A\cos(v-\alpha)dv} + \underset{\delta_r=1}{\int_{\theta}^{2\pi-\theta} A\cos(v-\alpha)dv}\right] = -K\sin\alpha$$

where $K = \dfrac{A\sin\theta}{\pi}$ (2.4)

Therefore, the small secular and long periodic effects of the solar pressure on the motion of the system may be described by the average equations of motion given below :

$$x'' - 2y' - 3x\rho - \rho^3 K\cos\epsilon\cos\alpha = \lambda_a\rho^4 x \quad (2.5)$$
$$y'' + 2x' - \rho^3 K\cos\epsilon\sin\alpha = \lambda_a\rho^4 y$$

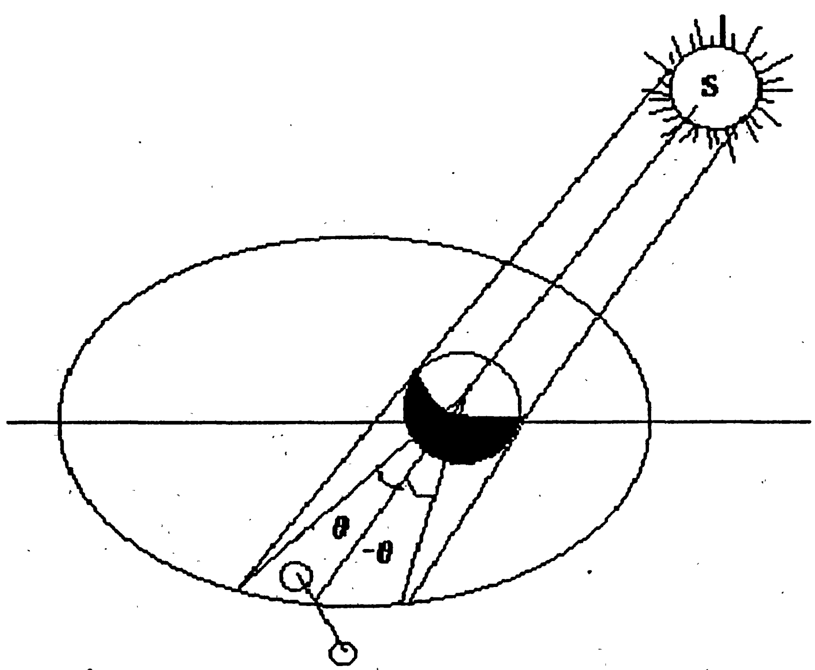

Figure 2. Satellites under the effects of Earth shadow.

Here x-axis is in the direction of position vector joining the center of mass of the system and the attracting center and the y-axis is along the direction normal to it and in the direction of motion of the satellite as shown in Figure 1. As free motion is bound to be converted into constrained motion with the lapse of time Sinha and Singh (1987), we shall analyse the constrained motion. In the case of constrained motion the equality sign holds in the (2.2), i.e. The system will be moving along the circle of variable radius given by

$$x^2 + y^2 = \frac{1}{\rho^2}$$

In order to discuss the non-linear oscillation of the system, it is required to transform the equations (2.5) into polar form by substituting $x = (1 + e \cos v) \cos \psi$ and $y = (1 + \cos v) \sin \psi$ and solving with respect to ψ and λ_a we obtain :

$$(1 + e \cos v)\psi'' - 2e \sin v \, \psi' + 3 \sin\psi \cos\psi + \rho^3 K \cos \in \sin(\psi + \alpha) = 2e \sin v \qquad (2.6)$$

The equation (2.6) is the equation of motion of a dumbell satellite in the central gravitational field of the Earth under the influence of the solar radiation pressure together with the effects of the Earth shadow. The equation determining the Lagrange's multiplier is given by :

$$(1 + e \cos v)^4 (\psi' + 1)^2 + (1 + e \cos v)^3 (3\cos^2\psi - 1) - K \cos \in \cos(\psi + \alpha) = -\lambda_a \qquad (2.7)$$

The non-linear oscillation described by the equation (2.6) takes place, so long as the inequality given below is satisfied

$$(1 + e \cos v)^4 (\psi' + 1)^2 + (1 + e \cos v)^3 (3\cos^2\psi - 1) - K \cos \in \cos(\psi + \alpha) \geq 0 \qquad (2.8)$$

Here ψ is the angular deviation of the line joining the satellites with stable position of equilibrium.

The essential feature of the paper is that, we have considered the influence of some phenomenological factors on the non-linear oscillation of the system under the solar radiation pressure. The main effects have been found by taking into account the additional terms $\gamma\psi'$ and $E \sin \nu v$ to the equation of the system. The presence of the phenomenological friction force and a small periodic force results in the following equation

$$(1 + e \cos v)\psi'' - 2e \sin v \, \psi' + 3 \sin\psi \cos\psi + \rho^3 K \cos \in \sin(\psi + \alpha)$$
$$= 2e \sin v + \gamma\psi' + E \sin \nu v \qquad (2.9)$$

Where γ and E are some phenomenological parameters characterizing the dissipative and periodic terms, which are assumed to be of the order of e and ν is the frequency of the external periodic force. However, these parameter can be determined by proceeding from specific model assumptions concerning these forces. Assuming 'e' to be a very small parameter instead of zero, we shall study the non-linear oscillation of the system (2.9) about the stable position of equilibrium given by

$$\phi = 0; \quad \psi_0 = \frac{K \sin \alpha}{3 - K \cos \alpha} \qquad (2.10)$$

where the above mentioned parameters are the stationary values of the transformation of modified potential energy in polar spherical co-ordinates system in a moving frame of reference. The equilibrium position is found to be stable in the sense of Liabunov (1959).

Substituting $\psi = \psi_0 + \eta$, retaining the terms upto the third order the expansion of $\sin\eta$ and $\cos\eta$, setting $E = eE_1$ and $\gamma = e\gamma_1$ in (2.8). Assuming, the solar radiation pressure parameter and non-linearity is of the order of e, we get :

$$\eta'' + \omega^2\eta = e[2\sin v + 2\eta'\sin v - \eta''\cos v - \beta_1\eta^3 - \beta_2\eta^3 - K_1\eta + \frac{K_2\eta^2}{2} + K_3\eta^2 + \gamma_1\eta' + E_1\sin \nu v]$$

$$+ e^2\left[3K_2\cos v - \frac{3K_2}{2}\eta^2 + 3K_3\eta^2\right]$$

$$\text{where } \omega^2 = \left[3 - \frac{K^2\sin\alpha.\cos\epsilon}{(3 - K\cos\alpha)}\right]$$

$$\beta_1 = \frac{3}{e},\ \beta_2 = \frac{K\cos\epsilon\ \sin\alpha}{e(3 - K\cos\alpha)}$$

$$K_1 = \frac{K^2\sin^2\alpha\cos\epsilon}{(3 - K\cos\alpha)e}$$

$$K_2 = \frac{K^2\cos\epsilon\ \sin\alpha\cos\alpha}{e(3 - K\cos\alpha)}$$

$$K_3 = \frac{K\cos\epsilon\ \sin\alpha}{e} \qquad (2.11)$$

3. Non-linear oscillation of inter-connected satellites system in the neighbourhood of main resonance ω = 1

The presence of two sine forces, one due to the moment of central force and other due to external periodic force, given rise to two main resonances $\omega = \nu$ and $\omega = 1$. We shall construct below the asymptotic solution of (2.11) exploiting the method of Bogoliubov and Mitropolsky (1961) in the neighbourhood of main resonance $\omega = 1$, in the form :

$$\eta = a\cos(v + \theta) \qquad (3.1)$$

where the amplitude 'a' and the phase 'θ' are given by the system of differental equations :

$$\frac{da}{dv} = \frac{\gamma a}{2} - \frac{2e}{(\omega+1)}\cos\theta$$

$$\frac{d\theta}{dv} = \omega - 1 + \left[\frac{K^2 \sin^2 \alpha \cos \epsilon}{2\omega(3 - K\cos\alpha)} + \frac{3a^2}{4\omega} + \frac{K\cos\epsilon\ a^2}{4\omega(3 - K\cos\alpha)}\right] + \frac{2e}{a(\omega + 1)}\sin\theta \qquad (3.2)$$

It is clear from (3.2) that there is no effect of external periodic force E sin νv on the amplitude and the phase of the oscillatory system in the first approximation at this overtone. But the presence of dissipative force and the solar radiation pressure introduces a correction in the amplitude of the system.

Now we shall examine the stationary regimes of oscillation of the system under the combined influence of above mentioned perturbative forces.

The stationary state of oscillation is defined by :

$$\frac{da}{dv} = \text{and } \frac{d\theta}{dv} = 0$$

Hence from the equation (3.2) may be represented, correct to the second order in the form :

$$2a\delta_e(a) - 2e\cos\theta = 0$$

$$(\omega_e^2 - 1)\,a + 2e\sin\theta = 0 \qquad (3.3)$$

$$\text{where } \omega_e(a) = \left[\omega + \frac{K^2\sin^2\alpha\cos\epsilon}{2\omega(3 - K\cos\alpha)} + \frac{3a^2}{4\omega} + \frac{K\cos\epsilon\ a^2}{4\omega(3 - K\cos\alpha)}\right]$$

$$\delta_e(a) = \frac{\gamma}{2\omega}$$

where $\omega_e(a)$ is the equivalent frequency of the system when the external forces are supposed to be absent. After eliminating the phase 'θ', we get :

$$\omega_e^2(a) = 1 \pm \sqrt{\frac{4e^2}{a^2} - \gamma^2} \qquad (3.4)$$

To obtain this relation in the neighbourhood of resonance frequency, we introduce the following relation

$$\omega = 1 + \delta \qquad (3.5)$$

Assuming that the quantity δ is small we transform the relation (3.4) into a more convenient form :

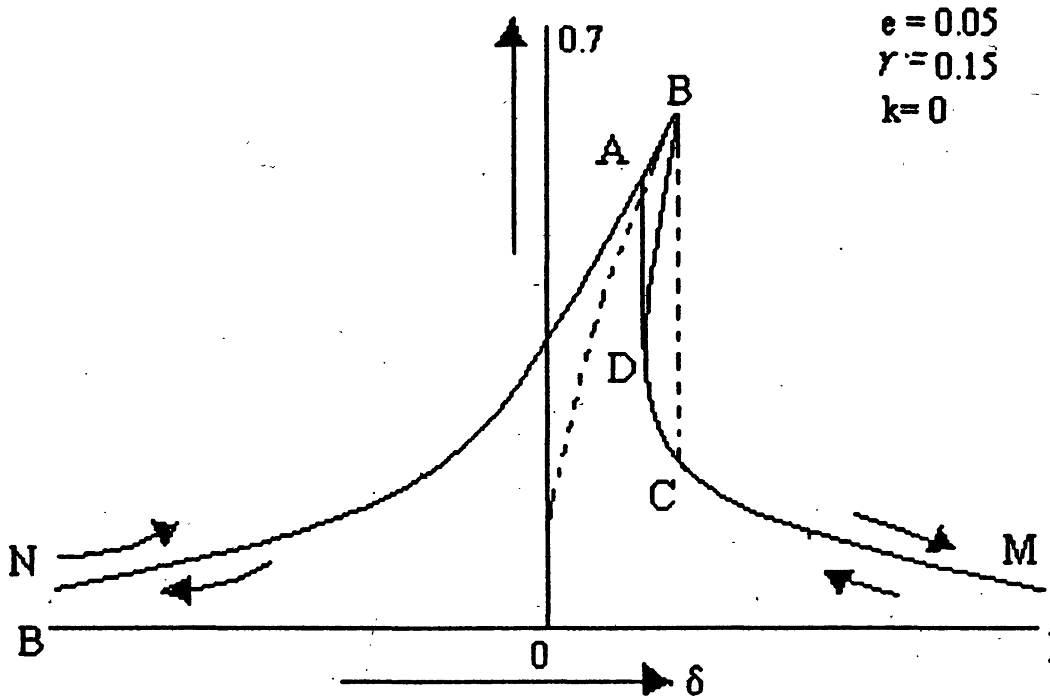

Figure 3. Resonance curve in the case of main resonance ω = 1 under the Earth's shadow.

$$\delta = \frac{1}{2}\left[\frac{3}{2} + \frac{K\cos \in}{2(3 - K\cos\alpha)} + \frac{3K^2\sin^2\alpha\cos\in}{4(3 - K\cos\alpha)\omega^2} + \frac{K^3\sin^2\alpha\cos^2\in}{8(3 - K\cos\alpha)^2}\right]a^2 \quad (3.6)$$

$$\pm\frac{1}{2}\sqrt{\frac{4e^2}{a^2} - \gamma^2}$$

A schematic representation of behaviours of the relation (3.6) in the range of the parameter γ, when the system is under the Earth shadow K = 0, is given in Figure-3 Manaziruddin and Singh (1990). The dotted line in the figure represents the skeleton curve. We notice, here, that when δ decreases, the amplitude of oscillation increases along MCD but it has a sudden rise at D and passes to A and further decrease with the decrease of δ along the curve BAN. On the other hand when δ increases, the amplitude increases and gets an abrupt break at B when it jumps to C and further goes along the curve DCM. Thus the section NAB and DCM of response curve will correspond to the stable amplitude. The section BC of the response curve is unstable. The behaviour of the system is almost the same when the effect of the solar radiation pressure is taken into account with the same value of γ′with a slight change in amplitude as shown in Figure 4.

We shall determine the relation that must exist between the parameters γ and K, (solar radiation pressure) of the system for the effect under consideration to occur. Proceeding with the equation (3.6) and differentiating with respect to δ, we get :

$$\frac{da}{d\delta} = \frac{2(La^3 - \delta a)}{[3L^2a^4 - 8\delta La^2 + \gamma^2 + 4\delta^2]} \quad (3.7)$$

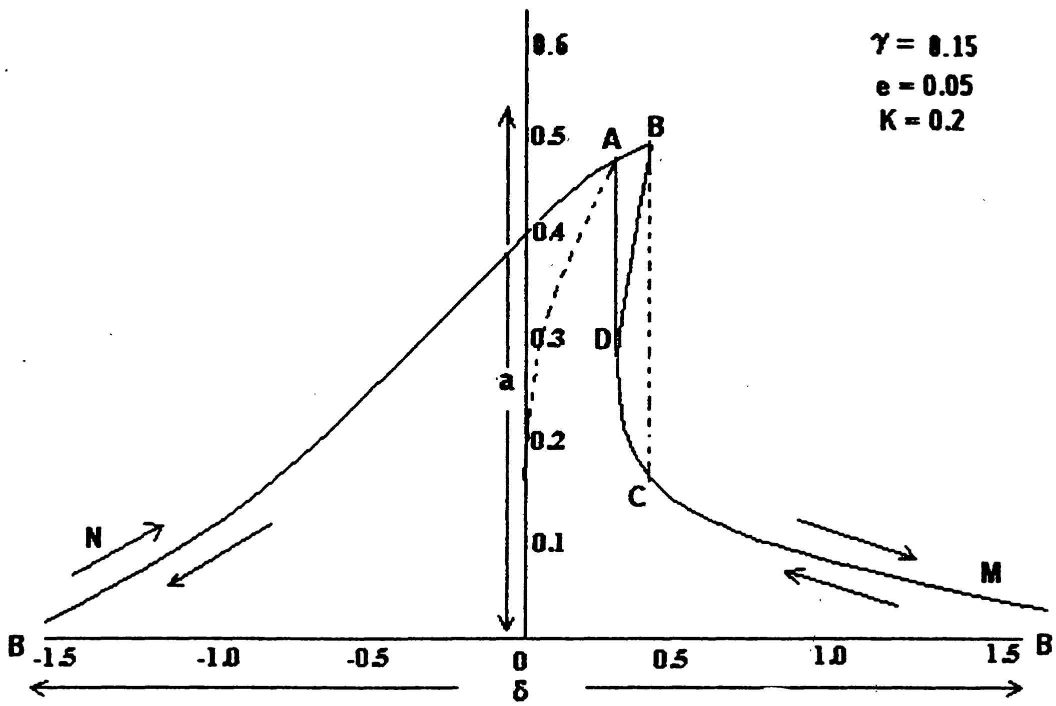

Figure 4. Resonance curve in the case of main resonance ω = 1 under the combined influence of solar radiation pressure and dissipative force.

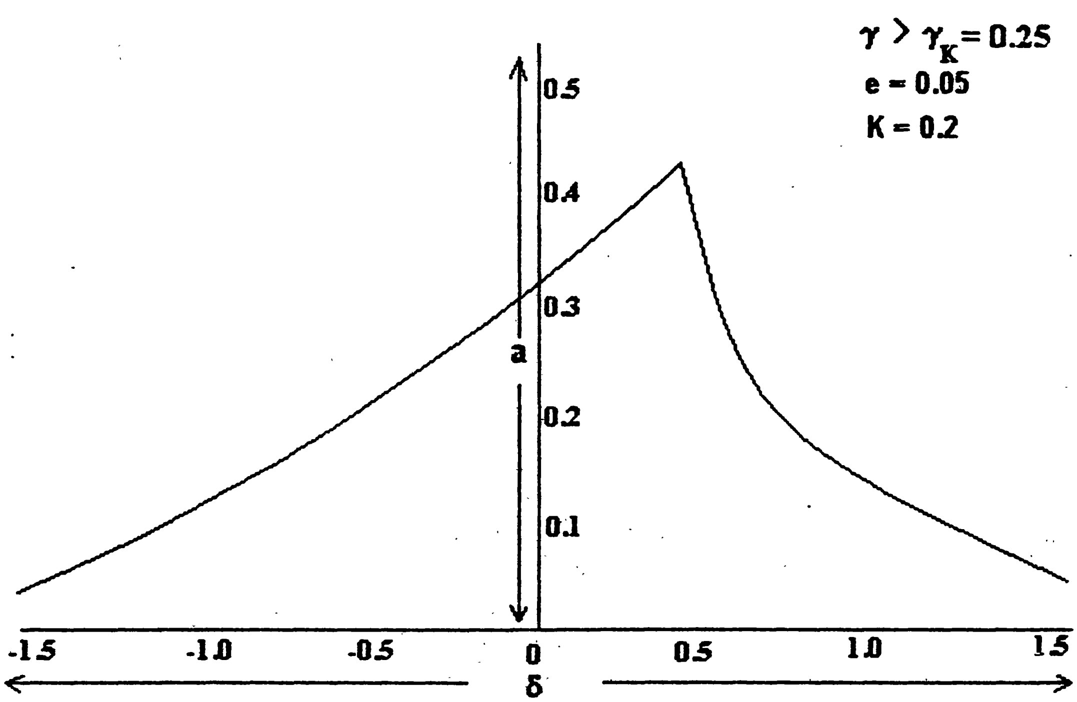

Figure 5. Resonance curve in the case of main resonance ω = 1 under the combined influence of solar radiation pressure and dissipative force.

The necessary condition for the unstable branch to exist is $\frac{da}{d\delta} = \infty$. This results in the relation :

$$3a^4L^2 - 8L\delta a^2 + (4\delta^2 + \gamma^2) = 0 \qquad (3.8)$$

$$\text{where } L = \left[\frac{3}{2} + \frac{K\cos\in}{2(3 - K\cos\alpha)} + \frac{3K^2\sin^2\alpha\cos\in}{4(3 - K\cos\alpha)} + \frac{K^3\sin^2\alpha\cos^2\in}{8(3 - K\cos\alpha)^2}\right]$$

Here, we observe from equation (3.7) that both the roots δ are positive; that is, the effect under study is always possible at a frequency greater than the resonance frequency.

The maximum value of the amplitude is defined by the condition $\frac{da}{d\delta} = 0$. Thus, we obtain $a_{max} = \frac{2e}{\gamma}$. Also, we obtain from (3.7), the critical value γ_k of γ given by

$$\gamma^3_k = \frac{6}{\sqrt{3}}\left(e^2L\right)$$

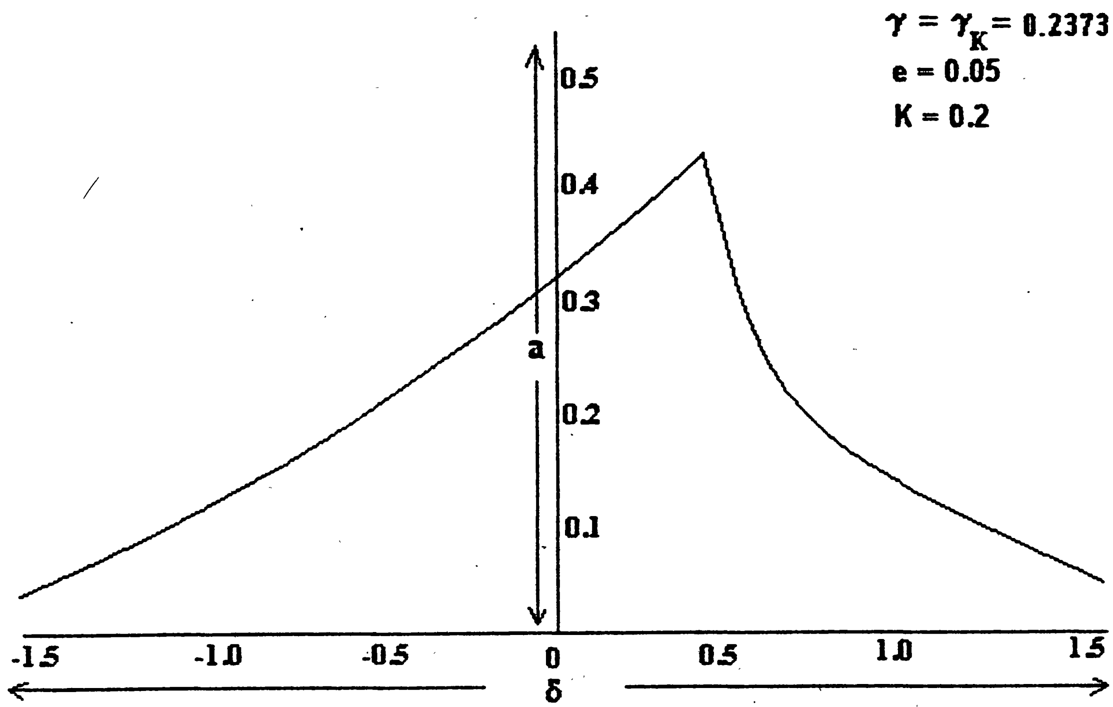

Figure 6. Resonance curve in the case of main resonance ω = 1 under the combined influence of solar radiation pressure and dissipative force.

The breakdown of the oscillation is possible only for those values of $\gamma < \gamma_K$; as shown in the Figure 4. The curves for different values of $\gamma = \gamma_K$ and $\gamma > \gamma_K$ represent a symmetrical curve with its maximum directed towards the positive side of δ as shown in Figure 5 and Figure 6. In this case there is no discontinuity, in the amplitude of the oscillating system.

The behaviour of the system can be analysed further by tracing the graph between different values of γ and the maximum $\left|\frac{da}{d\delta}\right|$.

Now the maximum $\frac{da}{d\delta}$ is obtained by the condition $\frac{d^2a}{d\delta^2} = 0$, which gives:

$$\delta^4 + 4La^2\delta^3 + (2\gamma^2 + 1 - 22L^2a^4)\delta^2 + (36L^3a^6 - 4La^2\gamma^2 - 4La^2)\delta + (6a^4L^2\gamma^2 + \gamma^4 - 15a^8L^4 + 3L^2a^4 - 4L^2a^4r^2) = 0 \quad (3.9)$$

Now solving the equation (3.9), we have obtained the values δ and substituting in equation (3.7) and tracing the curve for different value of γ and maximum $\left|\frac{da}{d\delta}\right|$, as shown in Figure 7.

We arrive at the conclusion that when th values of γ increases the maximum values of $\left|\frac{da}{d\delta}\right|$ goes on decreasing, steadily hence the system is found to be stable.

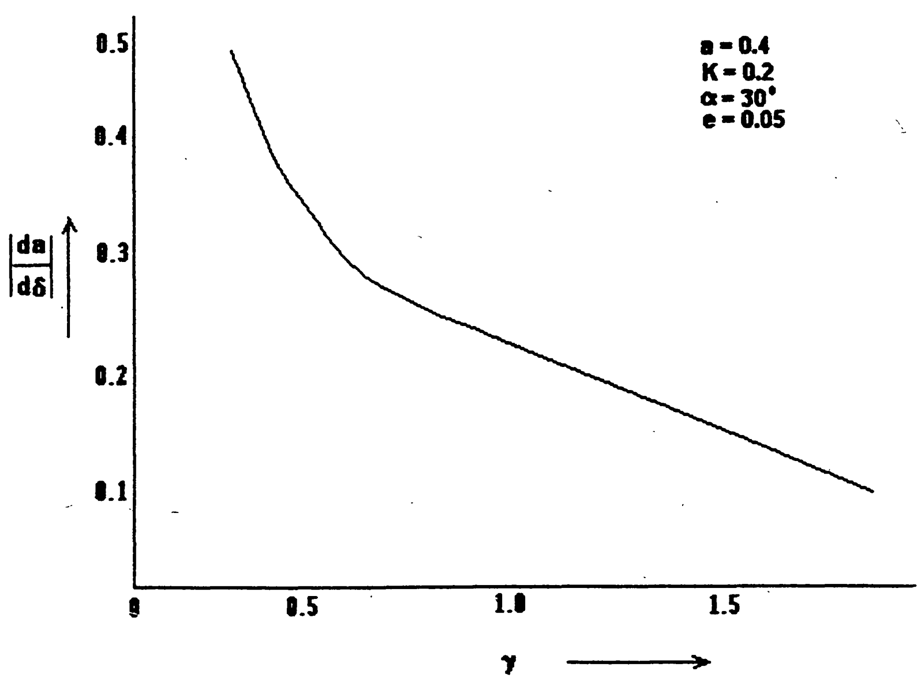

Figure 7. Relation between γ and maximum $\left|\frac{da}{d\delta}\right|$

4. Conclusion

The combined effects of solar radiation pressure and the forces of general nature on the non-linear oscillation of an inter-connected satellites system in elliptical orbit has been discussed. The effect of the Earth shadow has been taken into consideration on the system.

We conclude that the amplitude of the oscillation of a system of two inter-connected satellites suffers discontinuity in the neighbourhood of main resonance $\omega = 1$; under the effect of the Earth shadow. The discontinuity in the amplitude of oscillation occurs at a frequency greater than the resonance frequency.

The system exhibits the same behaviour, when the effects of the solar radiation pressure is taken into account on the system for the same value of γ only there is change in amplitude. The break and jump in the amplitude is possible only for those values of dissipative co-efficient γ which are less than the critical value γ_k.

References

Beletsky V.V., Novikova E.T., 1969, Kosmicheskiya Issledovania 7, 377.

Beletsky V.V., 1969, Kosmicheskiya Issledovania 7, 827.

Bogoliubov N.N., Mitropolsky Y.A., 1961, Asymptotic Methods in the theory of Non linear oscillation, Hindustan publishing corporation, Delhi.

Bryant R.W., 1961, Astron, Journs. 66(8), 430.

Dewari N.N., Klikh Yu. A., 1968. Sov. Astro A.J., 11(4), 672.

Kozai Y., 1961, Smithsonian, Astrophys. Obs-special report, 56.

Kripichnikov S.N., 1968, Sov. Astr. A.J. 12(3), 535.

Manaziruddin, Singh R.B., 1992, Celest. Mech. 53, 219.

Manaziruddin, Singh R.B., 1990, Proc. Nat. Acad. Sci. India 60(A).

Minorsky N., Popov V.I., Yankov I.O., 1976 Cos Res., 14.

Musen P.J., 196, Geophys. Res., 65, 1931.

Pyragas K.A., Zhdanov V.I., Alexendrov A.N., and Pyragas., L.E.: 1978, Astrophys. Space Sci. 57, 305.

Radzievski V.V., Artemiev A.V., 1962 Sov. Astr. A.J. 5(5), 758.

Liapunov A.M., 1959, Sobrania, Sachimeiviya, Vol.-2 N. Moscow (Russian).

Nechvile V., 1926, Acad Paris, Compt. Rend, 182, 310.

Narayan A., Singh R.B., 1987, Proc. Nat Acad. Sci. India 57.(A).

Narayan A., Singh R.B., 1990, Proc. Nat Acad. Sci. India 62.(A).

Narayan A., Singh R.B., 1992, Proc. Nat Acad. Sci. India 62.(A)-II.

Singh R.B., Demin V.G., 1972 Celest Mech. 6, 268.

Singh R.B., 1973, Astronautica Acta 18, 301.

Sinha S.K., Singh R.B., 1987, Astrophys. Space Sci. 129, 233.

Sinha S.K., Singh R.B., 1988, Astrophys. Space Sci. 140, 49.

Vinogradova V.P., Radzievskii V.V., Sov. Astr. A.J., 9(2), 334.