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Flat Spectrum Gamma Ray Burst Afterglows

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Abstract. Temporal behaviour of GRB afterglow light curve is derived for the case where the electron energy distribution is relatively hard, with the power-law index p lying between 1.0 and 2.0. It is shown that the expected behaviour will be the same as that for $p > 2.0$ if the upper cutoff in the electron energy distribution evolves in direct proportion to the bulk Lorentz factor of the blast wave.

Keywords : Gamma Ray Burst – Afterglow – Radiation Mechanism – Theory

1. Introduction

Detailed observations of afterglows of Gamma Ray Bursts over the last four years have established that they exhibit power-law broadband spectra and power-law temporal decay of their light curve. The generally accepted model for the afterglow, called the fireball model, explains this emission as being due to synchrotron emission from a relativistically expanding blast wave which accelerates electrons to large Lorentz factors, with a power-law energy distribution (see Piran 1999 for a review). Recently in several cases the light curve of the afterglow has been seen to undergo a break into a steeper power law, a behaviour that is expected if the burst is beamed into a narrow solid angle (see Rhoads 1999, 2001). Theoretical predictions for the spectral and temporal evolution have been made in detail for both isotropic (Wijers, Rees & Mészáros 1997; Waxman 1997a,b,c; Sari, Piran & Narayan 1998; Wijers & Galama 1999) and beamed (Rhoads 1997,1999; Sari, Piran & Halpern 1999) fireballs, and these have enjoyed considerable success in modelling the behaviour of observed afterglows.

Nearly all theoretical work in the literature so far assume that the energy distribution of the injected electrons is a power-law:

$$N(\gamma_e) \propto \gamma_e^{-p}, \quad (\gamma_m < \gamma_e < \gamma_u) \tag{1}$$

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(where γ_e is the Lorentz factor of the electron and γ_m and γ_u are the lower and upper cutoff of the energy distribution respectively), with an index p larger than 2.0. This assumption simplifies the derivations, since the particles at the low-energy end dominate both the total number and the energy content for such a steep energy distribution. The values of p derived from observations of most afterglows do indeed fall above 2.0, thereby allowing meaningful comparison being made between theoretical predictions and observations in these cases.

However, at present no compelling argument is known as to why the energy distribution of the accelerated electrons must always be so steep. Indeed a fairly large dispersion is seen in the spectral index distribution of shock-accelerated electrons in Galactic shell supernova remnants, which include several cases where p is inferred to be less than 2.0 (cf. Green 2000). Moreover, in Crab-like nebulae, where the particle energy distribution is shaped possibly by a relativistic standing shock (Rees & Gunn 1974, Kennel & Coroniti 1984), the value of p is almost always found to be less than 2.0. In the context of GRB afterglows, $p \sim 1.5$ has been invoked for GRB 000301c (Panaitescu 2001) and for GRB 010222 (Sagar et al 2001, Cowsik et al 2001).

Clearly, modelling of such a hard spectrum afterglow at present suffers from the handicap that theoretical predictions specific to such energy distributions are not available in the literature. Panaitesu (2001) makes a detailed case study of GRB 000301c with $p \sim 1.5$, but does not provide general results that are easily applicable to other cases. The aim of this paper is therefore to extend the predictions of the fireball model to the case of $p < 2.0$. In this paper I will address only the most commonly used spectral and dynamical regimes, namely slow cooling, adiabatic evolution for both isotropic and beamed bursts. I will consider a range of p between 1.0 and 2.0 and make certain simplifying assumptions that allow easy analytical treatment. I will also assume that the afterglow is optically thin over the entire range of frequencies of interest. A more detailed and exhaustive study of hard spectrum fireball models will be reported in a future publication (D. Bhattacharya & K.M. Basu, in preparation). In what follows I will adopt, wherever applicable, expressions from Rhoads (1999) and Wijers & Galama (1999) with slight modification in notation.

2. Adopted results

I list below the expressions already available in the literature which I adopt for the purposes of the present paper. These expressions are not affected by the change of energy distribution index p.

2.1 Dynamics

I will confine myself to the adiabatic regime of expansion of the blast wave. Rhoads (1999) presents results for dynamical evolution both before and after the light curve break, which corresponds to a time t_b (in the frame of the observer) when the initially tightly collimated ejecta begins to expand predominantly sideways. The expressions corresponding to times before t_b can also be used to represent isotropic bursts by setting the initial solid angle of collimation to 4π.

From Rhoads (1999) I adopt the following expressions for the dynamical evolution of the blast wave:

$$\Gamma = 2^{-5/4}\left(\frac{3E_0}{\pi\theta_0^2c^2\rho}\right)^{1/8}\left(\frac{1+z}{ct}\right)^{3/8} \quad (t < t_b) \tag{2}$$

$$\Gamma = \Gamma_b\left(\frac{t}{t_b}\right)^{-1/2} \quad (t > t_b) \tag{3}$$

$$t_b = (1+z)\left(\frac{3}{\pi}\right)^{1/3}\frac{5^{8/3}}{64}\frac{c}{c_s}\left(\frac{E_0}{\rho c_s^5}\right)^{1/3}\theta_0^2 \tag{4}$$

$$\Gamma_b = \frac{2c_s}{5c}\frac{1}{\theta_0} \tag{5}$$

where Γ is the bulk Lorentz factor of the blast wave, Γ_b the value of Γ at $t = t_b$, E_0 is the total energy of the blast wave, θ_0 is the initial opening angle of the collimated ejecta, ρ is the ambient density and c_s is the sound speed of the postshock medium, also taken to be the speed of lateral expansion. c, as usual, is the speed of light. For a spherical blast wave, the appropriate expressions can be obtained by substituting $\pi\theta_0^2$ with 4π. The time t is measured in the frame of the earthbound observer. z is the redshift of the afterglow.

2.2 Magnetic Field

Using the expressions in Rhoads (1999) the evolution of the postshock magnetic field in the blast wave (as measured in the comoving frame) can be written as:

$$B = \left(8\pi\frac{5c}{3c_s}\varepsilon_B\rho\right)^{1/2}\Gamma c \tag{6}$$

where ε_B is the fraction of the postshock thermal energy converted into magnetic energy.

2.3 Radiation

As explained by Wijers & Galama (1999), the observed location of the peak of the synchrotron spectrum radiated by a single electron of Lorentz factor γ_e is

$$\nu(\gamma_e) = \frac{0.286}{1+z}\frac{e}{\pi m_e c}\Gamma B\gamma_e^2 \tag{7}$$

Integrated over the power-law energy distribution of electrons, one obtains a power-law radiation spectrum for the whole afterglow, with the peak lying at

$$\nu_m = \frac{x_p}{1+z}\frac{e}{\pi m_e c}\Gamma B \gamma_m^2 \tag{8}$$

where γ_m is the lower cutoff of the energy distribution (see eq. (1)). The factor x_p is a function of the index p of the energy distribution. For $1.0 < p < 2.0$ the value of x_p lies between ~ 2.0 and ~ 0.65 (Wijers & Galama 1999). Here e and m_e are the charge and the mass of the electron respectively. The received flux per unit frequency at this peak of the afterglow spectrum is given by

$$F_m = \Gamma \mathbf{N}_e \phi_p \frac{\sqrt{3}e^3 B}{m_e c^2}\frac{1+z}{\Omega d^2} \tag{9}$$

where $\mathbf{N}_e$ is the total number of radiating electrons, Ω is the solid angle in which the radiation is beamed and d is the luminosity distance to the afterglow from the observer. ϕ_p is a p-dependent factor, and lies between 0.4 and 0.6 for $1.0 < p < 2.0$ (Wijers & Galama 1999). According to Rhoads (1999) F_m works out to be, for $t < t_b$,

$$F_m = \sqrt{10\pi}\frac{\phi_p \epsilon_B^{1/2}}{\mu_e m_p}\frac{e^3}{m_e c^3}\sqrt{\frac{c}{c_s}}\frac{\rho^{1/2}E_0}{\pi\theta_0^2}\frac{1+z}{d^2} \tag{10}$$

where μ_e is the mean molecular weight of the ambient medium and m_p is the proton mass. Additional factors of order unity will need to be inserted in eqs. (8)–(10) to represent the result of integration over different parts of the fireball. Eq. (10) shows that F_m is independent of time, i.e. the flux at the peak is constant. If we denote this value as F_m^0, then the evolution after t_b can be written as (Rhoads 1999)

$$F_m = F_m^0 \left(\frac{t}{t_b}\right)^{-1} \tag{11}$$

The electron energy above which synchrotron cooling is important within the expansion time corresponds to the Lorentz factor (Wijers & Galama 1999)

$$\gamma_c = \frac{6\pi m_e c}{\sigma_T \Gamma B^2 t} \tag{12}$$

where σ_T is the Thomson scattering cross section.

Using eq. (7) and the expressions of the comoving magnetic field given above one obtains the expression for the cooling frequency from γ_c:

$$\nu_c = \frac{0.286 \times 384 c^{1/2}}{(1+z)^{5/2}(40)^{3/2}}\frac{e m_e}{\sigma_T^2}\left(\frac{c_s}{c}\right)^{3/2}\epsilon_B^{-3/2}\frac{\theta_0}{\rho E_0^{1/2}}t^{-1/2} \quad (t < t_b) \tag{13}$$

$$\nu_c = \nu_c(t_b) = \text{constant} \quad (t > t_b). \tag{14}$$

3. Results for Flat Spectral Index

We now have all the pieces necessary to compute the evolution of the afterglow spectrum for a hard ($p < 2$) energy distribution of electrons. For $1 < p < 2$ and $\gamma_u \gg \gamma_m$ the only way this modifies the evolution is by changing the evolution of γ_m with time. As in Sari, Piran & Narayan (1998) we note that the postshock particle density and energy density are $4\Gamma n$ and $4\Gamma^2 n m_p c^2$ respectively, where n is the number density of the ambient medium. Assuming a fraction ε_e of the postshock thermal energy goes into power-law electrons, these quantities can be equated to integrals over the electron energy distribution:

$$\int_{\gamma_m}^{\gamma_u} N(\gamma_e)d\gamma_e = 4\Gamma n \quad (15)$$

$$\int_{\gamma_m}^{\gamma_u} \gamma_e m_e c^2 N(\gamma_e)d\gamma_e = \varepsilon_e 4\Gamma^2 n m_p c^2 \quad (16)$$

Clearly, for $1 < p < 2$ the dominating limit in the first integral is γ_m while that in the second integral is γ_u. Using the fact that $\gamma_m \ll \gamma_u$ one then obtains

$$\gamma_m = \left[\varepsilon_e \left(\frac{2-p}{p-1}\right) \frac{m_p}{m_e} \Gamma \gamma_u^{p-2}\right]^{1/(p-1)} \quad (17)$$

This is the key element that causes the difference of evolution between the hard spectrum and steep spectrum afterglows. To recall (e.g. from Sari, Piran & Narayan 1998), for $p > 2$, the value of γ_m evolves as

$$\gamma_m = \varepsilon_e \left(\frac{p-2}{p-1}\right) \frac{m_p}{m_e} \Gamma \quad (18)$$

The integral in eq. (16) is carried out over the injected energy spectrum of electrons, which is an unbroken power-law up to γ_u. This, therefore, is a measure of the total energy the acceleration process injects into relativistic electrons, and for the purposes of this paper I assume that this is a constant fraction (ε_e) of the postshock thermal energy. The spectrum of the *accumulated* electrons, however, would steepen to $\gamma_e^{-(p+1)}$ beyond the cooling break γ_c (eq. 12) because of the radiation losses suffered after acceleration. Over the period of interest, γ_c would in general be much less than γ_u (see, e.g. Gallant & Acherberg 1999, Gallant, Achterberg & Kirk 1999), so for $p < 2$, only a small fraction of the total injected energy will remain in the accumulated electrons. Dai and Cheng (2001) have computed the evolution of the afterglow assuming that the ratio (ε_c) of this remaining energy to the postshock thermal energy stays constant with time. While the constancy of either ε_e or ε_c as defined above is a questionable assumption, the degree of difficulty in arranging a physical situation to maintain a constant ε_c is certainly greater. We therefore derive our results assuming a constant ε_e, although the final results will be general enough for application to either case.

Eq. (17) shows that the evolution of γ_m in the hard spectrum case depends on how γ_u changes with time. This depends on the details of the particle acceleration process in the ultrarelativistic blast wave, which have so far not been very well understood (see the review by Bhattacharjee and Sigl (2000) and references therein). Broadly speaking, the maximum energy achieved by an electron in the acceleration process would be limited either by radiation losses within the acceleration cycle time or by the cycle time itself exceeding the age of the blast wave. These quantities depend on the shock parameters as well as the upstream magnetic field strength. Since the evolution of most of the shock parameters can be expressed as power-law dependences on Γ, for the purposes of this paper I make the simplifying, but perhaps not very unreasonable assumption that γ_u for a given afterglow is a function of Γ alone, and parametrize this dependence as a power law:

$$\gamma_u = \xi\Gamma^q \tag{19}$$

where ξ is a constant of proportionality. The value of q, however, may not be constant with time, and may depend on the dynamical regime. For example, in the simplest acceleration models (cf. Gallant and Achterberg 1999), $q \sim 0.5$, independent of dynamical regime, if the acceleration is limited by radiative losses; but if the age t of the blast wave limits the acceleration, $\gamma_u \propto \Gamma t$, which yields a dynamics dependent q.

Eq. (19) yields

$$\gamma_m = \left[\epsilon_e \left(\frac{2-p}{p-1}\right) \frac{m_p}{m_e} \xi^{p-2}\right]^{1/(p-1)} \Gamma^{(1+pq-2q)/(p-1)} \tag{20}$$

The dependence of γ_m on Γ reduces to that for $p > 2$ (eq. 18) if $q = 1$, i.e. if the upper cutoff energy also is directly proportional to the bulk Lorentz factor of the shock. In this case all the results derived for the temporal behaviour of $p > 2$ afterglows will also be applicable to those with $p < 2$.

It is now straightforward to obtain the dependence of ν_m on time by inserting eq. (20) in eq. (8), and using the appropriate expressions for Γ and B. The result is

$$\begin{aligned} \nu_m = & \frac{1}{1+z} \frac{x_p}{\pi} \frac{e}{m_e} \left[\frac{40\pi}{3} \frac{c}{c_s} \epsilon_B \rho\right]^{1/2} \left[\epsilon_e \left(\frac{2-p}{p-1}\right) \frac{m_p}{m_e} \xi^{p-2}\right]^{2/(p-1)} \times \\ & \left[2^{-5/4} \left(\frac{3E_0}{\pi\theta_0^2 c^2 \rho}\right)^{1/8} \left(\frac{1+z}{c}\right)^{3/8}\right]^{2(p+pq-2q)/(p-1)} t^{-\frac{3}{4}\frac{p+pq-2q}{p-1}} \end{aligned} \tag{21}$$

for $t < t_b$ and

$$\begin{aligned} \nu_m = & \frac{1}{1+z} \frac{x_p}{\pi} \frac{e}{m_e c} \left[\epsilon_e \left(\frac{2-p}{p-1}\right) \frac{m_p}{m_e} \xi^{p-2}\right]^{2/(p-1)} \times \\ & B_b \Gamma_b^{(p+1+2pq-4q)/(p-1)} \left(\frac{t}{t_b}\right)^{-(p+pq-2q)/(p-1)} \end{aligned} \tag{22}$$

for $t > t_b$.

Noticing now that below and above the cooling break the afterglow flux is given by

$$F_\nu = F_m \left(\frac{\nu}{\nu_m}\right)^{-(p-1)/2} \qquad (\nu_m < \nu < \nu_c) \tag{23}$$

$$F_\nu = F_m \left(\frac{\nu_c}{\nu_m}\right)^{-(p-1)/2} \left(\frac{\nu}{\nu_c}\right)^{-p/2} \quad (\nu_c < \nu < \nu_u) \tag{24}$$

(where ν_u is $\nu(\gamma_u)$), we can easily obtain the time dependence of afterglow flux by inserting the time dependence of F_m, ν_m and ν_c. The final results are

$$F_\nu \propto \nu^{-(p-1)/2} t^{-3(p+pq-2q)/8} \quad (\nu_m < \nu < \nu_c) \tag{25}$$

$$F_\nu \propto \nu^{-p/2} t^{-[3p+2+3q(p-2)]/8} \quad (\nu_c < \nu < \nu_u) \tag{26}$$

for $t < t_b$ and

$$F_\nu \propto \nu^{-(p-1)/2} t^{-[p+2+q(p-2)]/2} \quad (\nu_m < \nu < \nu_c) \tag{27}$$

$$F_\nu \propto \nu^{-p/2} t^{-[p+2+q(p-2)]/2} \quad (\nu_c < \nu < \nu_u) \tag{28}$$

for $t > t_b$. As one can verify, these expressions reduce to the familiar expressions for $p > 2$ by setting $q = 1$. Further, in the case of constant ε_c (Dai and Cheng 2001) γ_c plays the role of γ_u, and the corresponding results can be obtained by inserting the dependence of γ_c on Γ in the above equations: $q = -1/3$ for $t < t_b$ and $q = -1$ for $t > t_b$.

4. Conclusions

I have presented above the expected behaviour of GRB afterglow light curves when the index p of the power-law energy distribution of electrons lies in the range $1.0 < p < 2.0$. The results presented here correspond to the optically thin, adiabatic, slow cooling regime. The total energy content in such an energy distribution is dominated by the upper cutoff Lorentz factor γ_u, and hence the evolution of γ_u influences the evolution of the light curve. I derive the light curve behaviour assuming a simple power-law dependence of γ_u on the bulk Lorentz factor Γ of the blast wave. It follows that the behaviour of the light curve for $1 < p < 2$ will be similar to that for $p > 2$ if $\gamma_u \propto \Gamma$.

It ought to be remembered that the broken power-law description of the afterglow spectrum and light curve presented here represents only the asymptotic behaviour, in reality the transitions between different regimes are expected to be smooth. Moreover, for relatively hard electron energy distributions considered here, synchrotron cooling is expected to cause a pile-up of particles at the cooling break γ_c (cf. Pacholczyk 1970) and hence some excess emission (i.e. a local peak) near the cooling frequency ν_c may be observed. Some of the results presented above find application in modelling the light curve and spectrum of GRB 010222 afterglow (Sagar et al 2001, Cowsik et al 2001).

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