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Linear polarization of binaries II. Phase function : $w\tilde{Q}(\mu)\tilde{Q}'(\mu')$

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Abstract. This paper presents a method of calculating linear polarizations in close binaries whose surfaces are distorted due to tidal and rotational forces. Limb-darkening effect has been taken into account. Particles of different sizes are embedded in the outer atmosphere. The law of differential rotation of the primary is considered in analytic form: $\Omega = b_1 + b_2 w^2 + b_3 w^4$, where b_1, b_2 and b_3 are constants and w is the distance of a point P (r,θ,ϕ) from the axis of rotation of the primary. The atmosphere is assumed to be non-grey, plane-parallel and the phase function is $w\tilde{Q}(\mu)\tilde{Q}'(\mu')$. Calculations are done with respect to rest frame fixed at the centre of the primary star for several functions as : mass-ratio (q) between the secondary and the primary, polar radius (r_p) of the primary, wave-length (λ) of the incident light, radius of a particle (1) and angle of inclination (β) with respect to the line of sight. It is noticed that polarization increases with an increase of the radius r_p steadily; polarization increases with an increase of the radius of the particle (1), polarization increases with an increase of the mass-ratio q. The method of solution has been applied to several late type binaries to calculate disk integrated linear polarization of light emitted by them. When the mass-ratio q = 0, the general problem reduces to the calculation for a rotationally distorted single (primary) star.

Key Words : binary, linear polarization, generalised Rayleigh scattering, late type.

1. Introduction

In the paper (Barman 1999) we have calculated linear polarization with grey atmosphere considering Rayleigh scattering. Here we would like to introduce phase function $w\tilde{Q}(\mu)\tilde{Q}'(\mu')$ (explained in the text) which can explain both Rayleigh scattering and scattering slightly different from Rayleigh scattering. This can be applied to some late type binaries like red variable stars (Shawl 1974) where both Rayleigh scattering and slightly different Rayleigh scattering occur.

We consider that the components of the binary are distorted due to the rotational and tidal effects. We assume that the components have extended atmosphere which contains solid iron particles of different sizes. The particles can absorb and emit radiation; the limb-darkening effect has been taken into account. The model contains several parameters as: (i) wavelength of the incident light (λ), (ii) the size of particle (1), (iii) the mass ratio of binary components (q), (iv) the polar radius of the primary component (r_p), (v) the angle of inclination (β) and (vi) the effective temperature of the atmosphere (T). The model can be applied to a single star by taking the mass ratio q equal to zero. The method has been applied to some late type binaries to find polarization with some related properties in them.

We describe the theoretical model in Section 2, the results and applications are described in Sections 3a and 3b and the conclusions are drawn in Section 4.

2. Theoretical model

For a Roche model of mass m the non-uniform velocity Ω at any point P (r, θ, ϕ) considered here is given by (Barman 1999)

$$\Omega = b_1 + b_2 w^2 + b_3 w^4 \quad \text{with} \quad w = r \sin\theta \tag{1}$$

where r is non-dimensional. Equation of an equipotential surface of a star distorted due to tidal and rotational effects is given by (Barman 1997)

$$1 / r + q / A + ½.\ Q_6 r^2 \sin^2\theta - 1/r_p - q / (1 + r_p^2)^{.5} = 0 \tag{2}$$

where $$A = (1 - 2\ r \cos\phi \sin\theta + r^2)^{.5} \text{ and } q = m_2 \backslash m_1 \tag{2a}$$

Q_i (i = 1,..,7) are given in Appendix 1; r_p is the polar radius; r is in units of R where R is the distance between the centres of gravities of the components; b_1^2, b_2^2, b_3^2 are in units of Gm_1/R^3; and the local surface gravity g is given by

$$g = (g_r^2 + g_\theta^2 + g_\phi^2)^{.5} \tag{3}$$

where $$g_r = 1/r^2 + q\ (r - \cos\phi \sin\theta) / A^3 - Q_7\ r \sin^2\theta,$$

$$g_\theta = -\ q \cos\phi \cos\theta / A^3 - Q_7 r \sin\theta \cos\theta,\ g_\phi = q \sin\phi / A^3 \tag{3a}$$

Equation (2) enables us to find the radius of any point on the surface of stars for any values of r, θ, ϕ.

(a) The radiative transfer equation

The radiative transfer equation in vector form, in a semi-infinite plane-parallel atmosphere can be written as (considering the equation to be azimuth independent)

$$\mu \frac{d\tilde{I}(\tau, \mu)}{d\tau} = \tilde{I}(\tau, \mu) - \frac{1}{2} \int_{-1}^{1} \tilde{P}(\mu, \mu') \tilde{I}(\tau, \mu')\, d\mu' \tag{4}$$

$$\text{where } \tilde{I}(\tau, \mu) = \begin{bmatrix} \tilde{I}_l(\tau, \mu) \\ I_r(\tau, \mu) \end{bmatrix} \tag{4a}$$

where $\tilde{I}_l(\tau, \mu)$ and $\tilde{I}_r(\tau, \mu)$ refer respectively to the intensities polarized with the electric vector vibrating parallel and perpendicular to the principal meridian, and $\tilde{P}(\mu, \mu') = w\tilde{Q}(\mu)\tilde{Q}'(\mu')$ (Burniston and Siewert 1970, Siewert 1972) where

$$\tilde{Q}(\mu) = \frac{3(c+2)^{1/2}}{2(c+2)} \begin{bmatrix} c\mu^2 + (2/3)(1-c) & (2c)^{1/2}(1-\mu^2) \\ (1/3)(c+2) & 0 \end{bmatrix} \tag{4b}$$

$\tilde{Q}'(\mu')$ denotes the transpose of $\tilde{Q}(\mu')$; and w is the albedo for single scattering. Equation (4) is applicable to several studies of the scattering of the polarized light. Here $c \in [0^\sigma, 1]$ and $w \in [o, 1]$. For c=1 and w=1 Equation (2) yields conservative Rayleigh scattering while $c \neq 1$ and $w \neq 1$ yields scattering slightly different from Rayleigh scattering. Equation of the radiative transfer (4) has been solved following the processes adopted by Chandrasekhar (1960).

The Q_{scat} and Q_{abs} of a particle in an atmosphere is given by (cf Hulst 1959, p.144 & 270)

$$Q_{scat} = \frac{8}{3} x^4 \left(\frac{m^2-1}{m^2+2}\right)^2 \left[1 + \frac{6}{5} \cdot \frac{m^2-1}{m^2+2} x^2 \right] \tag{5a}$$

$$Q_{abs} = 4x \frac{m^2-1}{m^2+2} + \frac{4}{15} x^5 \left[\frac{m^2-1}{m^2+2} \right]^2 \cdot \frac{m^4 + 27m^2 + 38}{2m^2+3} \tag{5b}$$

where m is the complex refractive index of the particle and $x = 2\pi 1 / \lambda$, where 1 is the radius of the particle and λ is the wavelength of the incident light. So $Q_{ext} = Q_{scat} + Q_{abs}$; and hence the albedo for single scattering w is given by

$$w = Q_{scat} / Q_{abs}, \tag{6}$$

(b) The disk integrated linear polarization

Following the idea of Harrington & Collins (1968) we consider two rotations. One is the rotation of x - y - plane about the z-axis through an angle α and the other rotation of x′ - z′-plane about the y′-axis through an angle β' ($=\pi/2-\beta$) where β is termed as an angle of inclination. If ñ be the unit vector normal to the surface of the star, the components of ñ along the x, y, z - axes are $\tilde{n}.\tilde{x}''$, $\tilde{n}.\tilde{y}''$, $\tilde{n}.\tilde{z}''$ respectively. If the component $\tilde{n}.\tilde{x}''$ is the line of sight $\tilde{n}.\tilde{1}$ then the observer will see the unit vector ñ projected on the y″-z″ - plane. If we denote the angle between this projection and the z″-axis by 5, then the tangent of this angle is given by (see Appendix 2) :

$$\tan\delta = (\tilde{n}.\tilde{y}'') / (\tilde{n}.\tilde{z}'') \tag{7}$$

The effect of limb-darkening can be estimated by using a quadratic law (cf. Kopal 1959, p. 156 & 182)

$$I(\tilde{n}.\tilde{1}) = \frac{I(0,\mu)}{I(0,1)} = 1 - u_1 - u_2 + u_1(\tilde{n}.\tilde{1}) + u_2(\tilde{n}.\tilde{1})^2 \tag{8}$$

where $u_1 = 0.65$ and $u_2 = -0.0226$.

Following Harrington and Collins (1968) and Peraiah (1970), the amount of polarization with limb-darkening effect can be written as

$$P = \frac{\iint [g_o (1 - \tau_o) + \tau_o g] (I_r - I_l) I(\tilde{n}.\tilde{1}) \cos 2\delta \, ds}{\iint (I_r + I_L) g (\tilde{n}.\tilde{1}) \, ds} \qquad (9)$$

where $ds = gr^2 \sin\theta \, d\theta \, d\phi / g_r,$ (9a)

$g_o = \iint g \, ds / \iint ds,$ (9b)

and τ_o is calculated from equation (Kopal 1959, p.177)

$$\tau_o = \frac{hv}{4kT (1 - \exp(- hv / kT))} \qquad (9c)$$

where h is the Planck constant, k is the Boltzman constant, v is the frequency of light and T is the temperature. From Equation (9) the polarization versus the wavelength have been calculated for some different values of the parameters of the inclination angle β, the radius of particle 1, the polar radius r_p and the mass-ratio q, and the results are presented in Figs. 1 - 4. The method has been applied to some late-type binaries to find polarizations with some related properties in them (see Table 1).

Table 1. The polarizations with some related properties of some Late Type Binaries for 1=0.1μ and .11μ, λ = 0.395μ, β = 90°; T–3500°C for YY Gem, T=4500°C for VW Cep and T=4900°C for RZ Com.

Binaries	q	b_1	b_2	b_3	I	Ωe/Ωp	r_e/r_p	1(μ)	P%
YY Gem	1.0	.1404	.00038	-.00071	77°.4	1.0	1.0	.10	0.4546
								.11	0.5406
VW Cep	.3264	.2863	.00506	-.00307	67°.4	1.0032	1.0031	.10	0.5657
								.11	0.6672
RZ Com	.4824	.3129	.00496	-.00417	65°.6	1.0024	1.0027	.10	0.5609
								.11	0.6619

3. Discussion of the results and its application

In this paper we consider the combined effects of gravity darkening, rotational distortions and the scattering controlled by the iron particles which together cause the intrinsic linear polarization. The polariation is effected by the limb-darkening; the refractive index of the particle is wavelength dependent. For the double integration of Equations (9) and (9b), 234 points on the surface of a hemisphere have been taken. The optical depth of the atmosphere for the emergent radiation is taken to be zero.

As a check if we put c = 1 and w = 1 in Equation (4), all the deductions of the radiative transfer equation reduce to the deductions for Rayleigh scattering obtained by Chandrasekhar (1960, p. 234 § 68); moreover when q = 0, $b_2 = 0$, $b_3 = 0$ i.e. a single star (primary) is rotating uniformly, then our expressions reduce to those of Harrington and Collins (1968). From Figs. 1 - 4 we see in general that the polarization depends on the wavelength of light and it increases towards the ultraviolet region; the polarization is maximum at the equator and least at the pole.

1) From Fig. 1 we see that the polarizations for c=w=1 (Rayleigh scattering) are much larger than those for other values c=w= .99, .9, .5, .1 (generalised Rayleigh scattering). While the polarization at the equator for c=w=1 is 5.09%, the polarizations at the equator for c=w=.99 is 2.84%, for c=w= .9 is 1.69% (see Fig. 1 for other parameters).

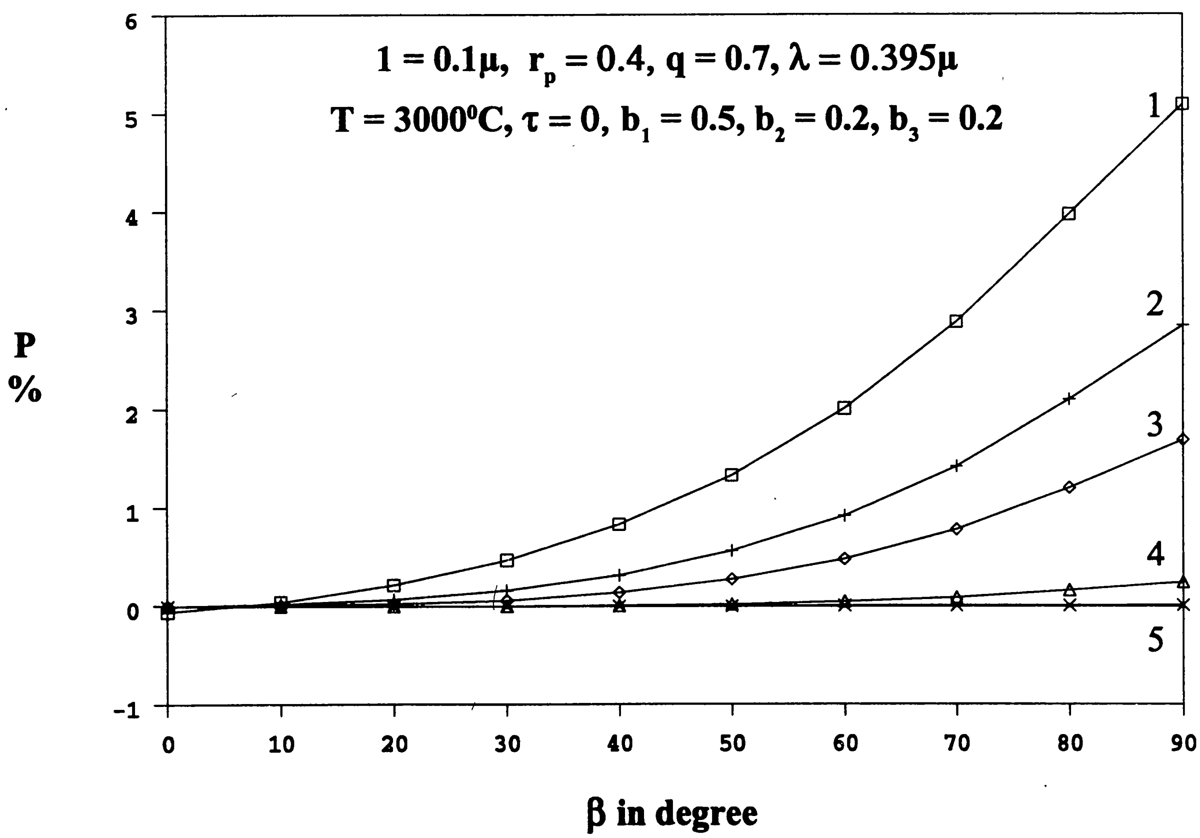

Figure 1. Polarization vs. β for (1) c = w = 1, (2) c = w = 0.99, (3) c = w = 0.9, (4) c = w = 0.5 & (5) c = w = 0.1.

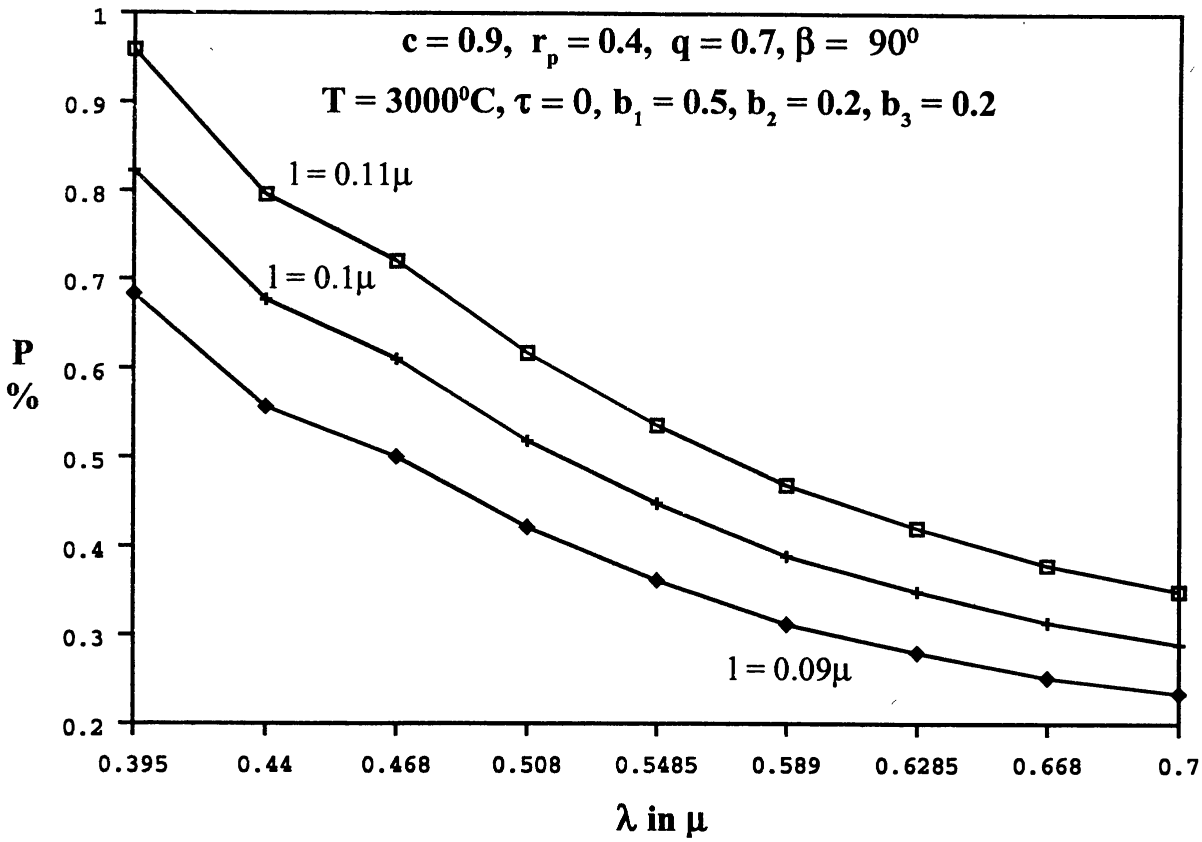

Figure 2. Polarization vs. wavelength for l = 0.09μ, 0.1μ, 0.11μ.

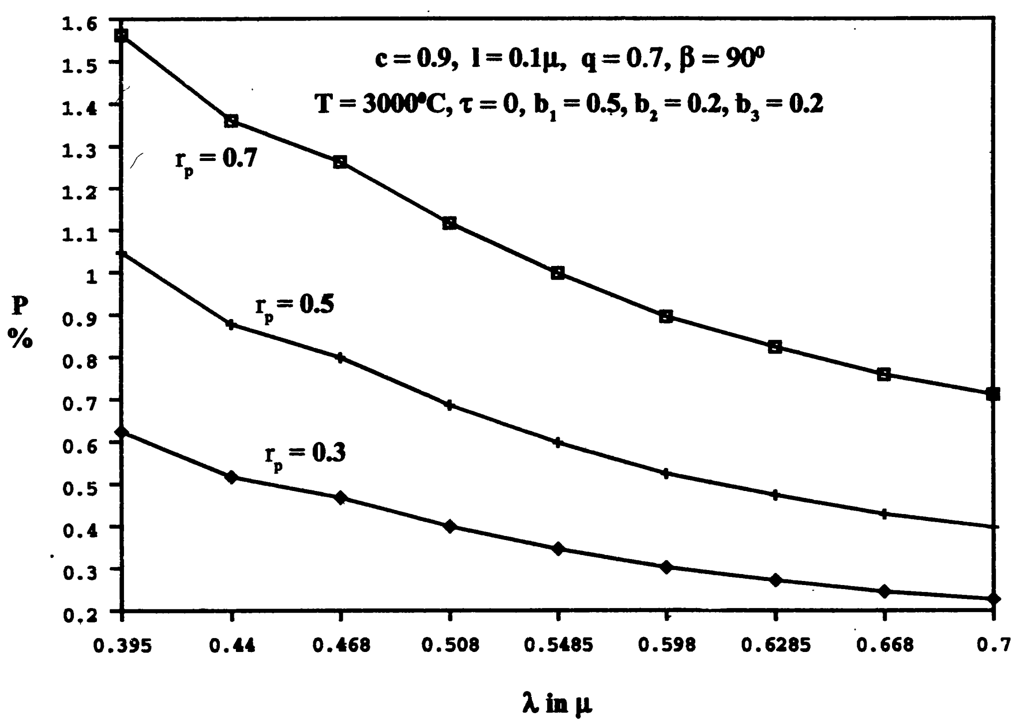

Figure 3. Polarization vs. wavelength for r_p = 0.3, 0.5 & 0.7.

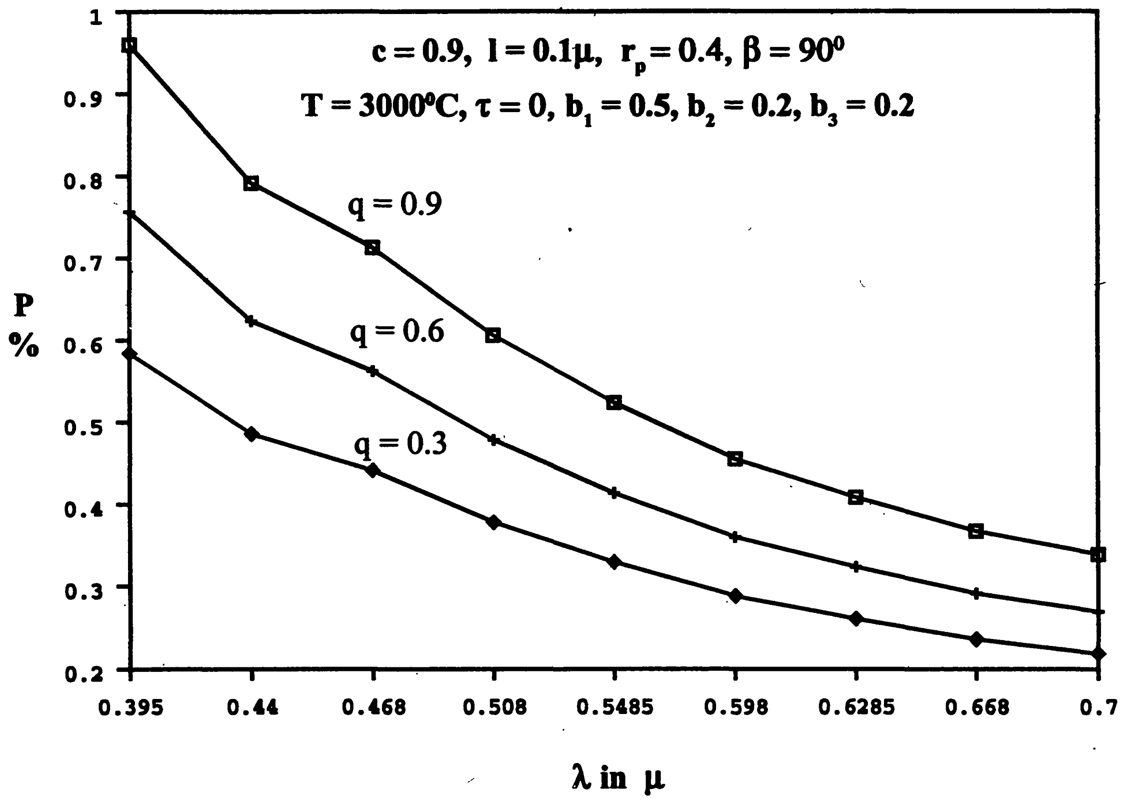

Figure 4. Polarization vs. wavelength for q = 0.3, 0.6 & 0.9.

2) From Fig. 2 we see that as the radius 1 of a particle increases from .09μ to .11μ, the polarization increases at λ = .395μ from .68% to .96% (i.e., .28%) which is larger than it increases towards λ = .7μ. The polarization increases with the increase of 1, since particles of larger sizes ($2\pi 1$ / λ) create more distortions in the extended envelope.

3) From Fig. 3 we see interestingly that as the polar radius r_p increases from r_p = 0.3 to to r_p = 0.7, the polarization also increases and the increase at λ = .395μ is from 0.62% to 1.56% (i.e., 0.94%) which is largerr than the increase towards λ = 0.7μ. Harrington and Collins (1968) have computed polarization in grey atmosphere as a function of the fraction of breakup velocity. Barman in the paper (1999) have shown that the results for r_p=0.2 agrees excellently with theirs but as r_p increases the polarization also increases. As r_p increases r_e also increases and hence the distorted surface area also increases. It then proves that if the distorted surface area increases, the polarization also increases.

4) From Fig. 4 we see that as the mass-ratio q increases, the polarization also increases. Peraiah (1976) and Barman and Peraiah (1991, 1992) have also shown this, though in Peraiah's paper the increase is larger since he has considered the cuvature effect in his model. As q increases, the distortion increases due to tidal forces from the secondary component.

Application to some Late-Type Binaries : Using Tables 7-1 and 7-14 of Kopal (1959) and the processes adapted by Barman (1997), we calculate the constants b_1, b_2, b_3 and the quantities - the mass-ratio q, the position angle I, the ratio of the angular velocities $\Omega e/\Omega p$, ratio of the radii r_e/r_p and the polarization P at β=90^0 for some late-type binaries (see Table 1).

4. Conclusions

We can now conclude that : (1) the polar radius r_p, in addition to the angle β and the mass ratio q, plays an important role to change the polarization, (2) if the linear polarization obeys Rayleigh and generalised Rayleigh scattering as in some red variable stars, we can consider the phase function $w\tilde{Q}(\mu)\tilde{Q}(\mu')$, (3) since the wavelength dependent polarization changes with time, it is then expected that the particles in the envelope change their sizes continuously by some unknown mechanism.

Appendix 1

The expressions for Q_1 to Q_7 are given by :

$$Q_1 = b_1^2, \quad Q_2 = b_1 b_2 \sin^2\theta$$

$$Q_3 = (1/3)\,(b_2^2 + 2\, b_1\, b_3)\sin^4\theta$$

$$Q_4 = (1/2)\, b_2\, b_3 \sin^6\theta$$

$$Q_5 = (1/5)\, b_3^2 \sin^8\theta$$

$$Q_6 = Q_1 + Q_2 r^2 + Q_3 r^4 + Q_4 r^6 + Q_5 r^8$$

$$Q_7 = Q_1 + 2\, Q_2 r^2 + 3\, Q_3 r^4 + 4\, Q_4 r^6 + 5\, Q_5 r^8$$

Appendix 2

$g(\tilde{n}.\tilde{x}'')$, $g(\tilde{n}.\tilde{y}'')$ and $g(\tilde{n}.\tilde{z}'')$ are given by :

$$g(\tilde{n}.\tilde{x}'') = g_r G_1 + g_\theta G_2 + g_\phi G_3$$

$$g(\tilde{n}.\tilde{y}'') = g_r G_4 + g_\theta G_5 + g_\phi G_6$$

$$g(\tilde{n}.\tilde{z}'') = g_r G_7 + g_\theta G_8 + g_\phi G_9$$

$$G_1 = \mathrm{Sin}\theta\ \mathrm{Sin}\beta\ \mathrm{Cos}\,(\phi - \alpha) + \mathrm{Cos}\theta\ \mathrm{Cos}\beta$$

$$G_2 = \mathrm{Cos}\theta\ \mathrm{Sin}\beta\ \mathrm{Cos}\,(\phi - \alpha) - \mathrm{Sin}\theta\ \mathrm{Cos}\beta$$

$$G_3 = \mathrm{Sin}\beta\ \mathrm{Sin}(\phi - \alpha)$$

$$G_4 = \mathrm{Sin}\theta\ \mathrm{Sin}(\phi - \alpha)$$

$$G_5 = \mathrm{Cos}\theta\ \mathrm{Sin}(\phi - \alpha)$$

$$G_6 = \mathrm{Cos}\,(\phi - \alpha)$$

$$G_7 = -\mathrm{Sin}\theta\ \mathrm{Cos}\beta\ \mathrm{Cos}(\phi - \alpha) + \mathrm{Cos}\theta\ \mathrm{Sin}\beta$$

$$G_8 = -\mathrm{Cos}\theta\ \mathrm{Cos}\beta\ \mathrm{Cos}(\phi - \alpha) - \mathrm{Sin}\theta\ \mathrm{Sin}\beta$$

$$G_9 = \mathrm{Cos}\beta\ \mathrm{Sin}(\phi - \alpha)$$

$$\zeta = \mathrm{Tan}^{-1}\,(g(\tilde{n}.\tilde{y}'') \,/\, g(\tilde{n}.\tilde{z}''))$$

$$(\tilde{n}.\tilde{I}) = g\,(\tilde{n}.\tilde{I}) \,/\, g$$

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