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The collapse of voids in a Roberston-Walker universe with nonzero spatial curvature

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Abstract. We consider the model of a spherical void (or its precursor) with heat conducting perfect fluid in Region I with Maiti metric (1982) surrounded by a spherical shell filled with radiation (Region II) having Vaidya metric. The combination is embedded in Robertson-Walker (RW) Universe in Region III. Earlier, Mandal and Banerji (1998) took RW Universe with zero spatial curvature and showed that to an observer, a little away from the boundary in Region III, the void appears to collapse provided the arrow of time is future directed in all the regions. In this paper we show that the same result holds also for the case of non-zero spatial curvature of Region III. The rate of collapse is the fastest for k = +1, moderate for k = 0, and the slowest for k = -1.

Key words : General Relativity, Cosmology, Voids.

1. The Model

Mandal and Banerji (1998) took the model of the void in Fig. 1. They took k=0 in Region III. This paper is a generalisation of their paper with K = ± 1.

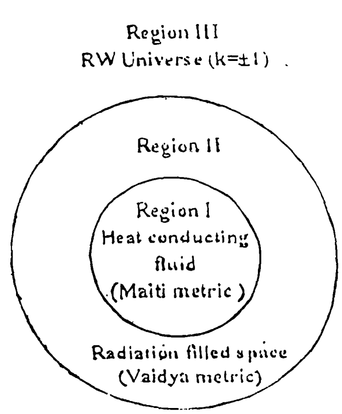

Figure 1. Three regions of space-time.

Region I : Core of the Void : Maiti (1982) metric for thermally conducting fluid :

$$ds_I^2 = [1+\frac{a}{1+\xi r_I^2}]^2 dt_I^2 - \frac{R^2(t)}{(1+\xi r_I^2)^2}(dr_I^2 + r_I^2 d\theta^2 + r_I^2 \sin^2\theta d\phi^2) \quad (1)$$

Region II : Envelope of the Void : Vaidya (1953) metric :

$$ds_{II}^2 = (1-\frac{2m(v)}{r_2})dv^2 + 2dv\, dr_2 - r_2^2 (d\theta^2 + \sin^2\theta\, d\phi^2) \quad (2)$$

Region III : RW Universe :

$$ds_{III}^2 = dt_3^2 - \frac{S^2(t_3)}{(1+ k\, r_3^2 / 4)^2} (dr_3^2 + r_3^2 d\theta^2 + r_3^2 \sin^2\theta\, d\phi^2) \quad (3)$$

We assume also a linear equation of state for the perfect fluid in Region III :

$$p = \gamma\, p,\ 0 \le \gamma \le 1/3 \quad (4)$$

2. Matching conditions

Matching the first and second fundamental forms on the boundary between Regions II and III we obtain the following equations keeping the time lines future directed in both regions [Goldwirth & Katz(1995)] :

$$r_2 = \frac{S(t_3)r_3}{(1+ k/4\, r_3^2)} \quad (5)$$

$$\dot{t}_3^2 - \frac{S^2(t_3)}{(1+ k/4\, r_3^2)^2}\dot{r}_3^2 = 1 \quad (6)$$

$$\left[1 - \frac{2m(v)}{r_2}\right]\dot{v}^2 + 2\dot{r}_2\dot{v} = 1 \quad (7)$$

$$\frac{r_3\, S\, \dot{t}_3}{(1+k/4\, r_3^2)} - \frac{kr^3\, S\, \dot{t}_3}{2(1+k/4\, r_3^2)^2} + \frac{S^2 r_3^2\, \dot{r}_3\, S'}{(1+k/4\, r_3^2)^3} = \left[1 - \frac{2m(v)}{r_2}\right]\dot{v}\, r_2 + \dot{r}_2 r_2 \quad (8)$$

$$\frac{m(v)\dot{v}}{r_2^2} - \frac{\ddot{v}}{v} = \frac{S\, \dot{t}_3 \ddot{r}_3}{1+k/4\, r_3^2} - \frac{k\, r_3\, \dot{r}_3^2 S\dot{t}_3}{2(1+k/4\, r_3^2)^2} + \frac{2\dot{r}_3 \dot{t}_3^2\, S'}{1+k/4\, r_3^2} - \frac{S\dot{r}_3 \ddot{t}_3}{1+k/4\, r_3^2} - \frac{S^2\, \dot{r}_3^3\, S'}{(1+k/4\, r_3^2)^3} \quad (9)$$

A dash represents derivative with rfespect to t_3 and dot that with respect to the proper time τ of the bounding surface.

3. Motion of the boundary

The solution of the above equations (5) - (9) gives the equation of motion of the boundary as seen by a comoving observer in Region III, a little away from the boundary :

$$r_3 = u_3 = 2 \tan \left[\alpha_0 - \gamma_{/2} \sin^{-1} S^{(1+3\gamma)/2}\right] \text{ for } k = +1 \qquad (3.9\text{ a})$$

$$= u_3 = 2 \tanh \left[\alpha_0 - \gamma_{/2} \sinh^{-1} S^{(1+3\gamma)/2}\right] \text{ for } k = -1 \qquad (3.9\text{ b})$$

$$= u_3 = 2 \left[\alpha_0 - \gamma_{/2} \frac{3(\gamma + 1)}{3\gamma + 1} S^{(1+3\gamma)/2}\right] \text{ for } k = 0 \qquad (3.9\text{ c})$$

Here $\alpha_0 > 0$

For all values of k the void or its precursor will appear to collapse as the universe expands when γ is nonzero. The rate of collapse is the fastest for $k = +1$, medium for $k = 0$ and the slowest for $k = -1$. At present $\gamma = 0$ and a void formed now or the one left over from the past will have its size unchanged.

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