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On the interchange instability of solar magnetic flux tubes

II. The influence of energy transport effects

M. Bünte[1], **S. Hasan**[2*], **and W. Kalkofen**[2]

[1] Institute of Astronomy, ETH Zentrum, CH–8092 Zürich, Switzerland
[2] Center for Astrophysics, 60 Garden Street, Cambridge, MA 02138, USA

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Abstract. We examine the interchange instability of thin photospheric magnetic flux tube models which satisfy both force and energy balance with their surroundings. The stability of the tubes is independent of the efficiency of internal convective energy transport and shows only a weak dependence on the plasma beta. The structures are susceptible to the instability in a layer 200 – 300 km deep immediately below optical depth unity in the quiet photosphere. The presence of an internal atmosphere reduces the magnetic field strength in comparison with that of an evacuated tube. While this has a stabilizing effect on the tube surface, temperature differences between interior and exterior are usually destabilizing. We find that the two effects approximately cancel each other for tubes with radii $R \lesssim 200$ km for which the stability properties are very similar to those of completely evacuated structures. For larger tubes, the temperature contrast with respect to the surroundings begins to dominate and destabilizes the tubes. Thus, despite the inclusion of energy transport effects on the tube structure, the stability problem of small tubes (with magnetic fluxes $\Phi < 10^{19} - 10^{20}$ Mx) remains. Consequences for photospheric magnetic fields are discussed.

Key words: Sun: magnetic fields – Sun: photosphere – MHD – instabilities – radiative transfer

1. Introduction

Vertical flux tubes in the solar photosphere expand with height due to the atmospheric stratification. The resulting concave magnetic surfaces are liable to the interchange instability (Parker 1975), with growth times very short compared to the lifetimes of the tubes. Although buoyancy stabilizes larger structures (Meyer et al. 1977) and might allow only limited energy release once the instability sets in, it is not clear whether this leads to a new equilibrium configuration (Schüssler 1986). Hence, small magnetic tubes with fluxes below some $10^{19} - 10^{20}$ Mx are intrinsically interchange unstable. To overcome this stability problem Schüssler (1984) suggested whirl flows surrounding the tubes. However, the magnitude of the velocities needed for stabilization can be implausibly high (well in excess of 2 – 3 km/s) for tubes in the range of typically $5 \times 10^{17} - 10^{19}$ Mx.

To allow for an analytic treatment of the problem, the above studies have generally focussed on evacuated tubes in the thin flux tube approximation (Roberts & Webb 1978), thereby neglecting the influence of the internal (magnetic) atmosphere and the magnetic tension forces. Although tension forces only marginally influence the overall structure of small flux concentrations (Knölker et al. 1988), they have a considerable influence on their stability in that they reduce the critical fluxes and whirl velocities by up to 60% (Steiner 1990; Bünte et al. 1993, henceforth denoted Paper I). The effect of the internal atmosphere, on the other hand, is much more complex, and – as shown in Paper I – may either stabilize or destabilize the tube surface depending on the prescribed pressure and temperature stratification.

In this contribution we focus on the latter aspect, the influence of the internal atmosphere. For this purpose we analyse thin flux tube models whose internal structure, rather than being prescribed, is computed self-consistently using a realistic energy equation which includes convective and radiative transport. Such models have been described in detail by Hasan (1988) and recently by Hasan & Kalkofen (1993) so that the description in Sect. 2 is restricted to their main features. In Sect. 3 we outline the stability analysis. For further details we refer to Paper I. The results are described in Section 4 where we also present an asymptotic analysis of the whirl velocity for highly evacuated tubes in order to understand the various factors influencing stability. Conclusions are given in Sect. 5.

2. Self-consistent thin tube models

We determine the structure of a cylindrical flux tube by solving the magnetohydrostatic (MHS) equations in the thin flux tube approximation, allowing for both radiative and convective

Send offprint requests to: M. Bünte
* Permanent address: Indian Institute of Astrophysics, Sarjapur Road, 560034 Bangalore, India

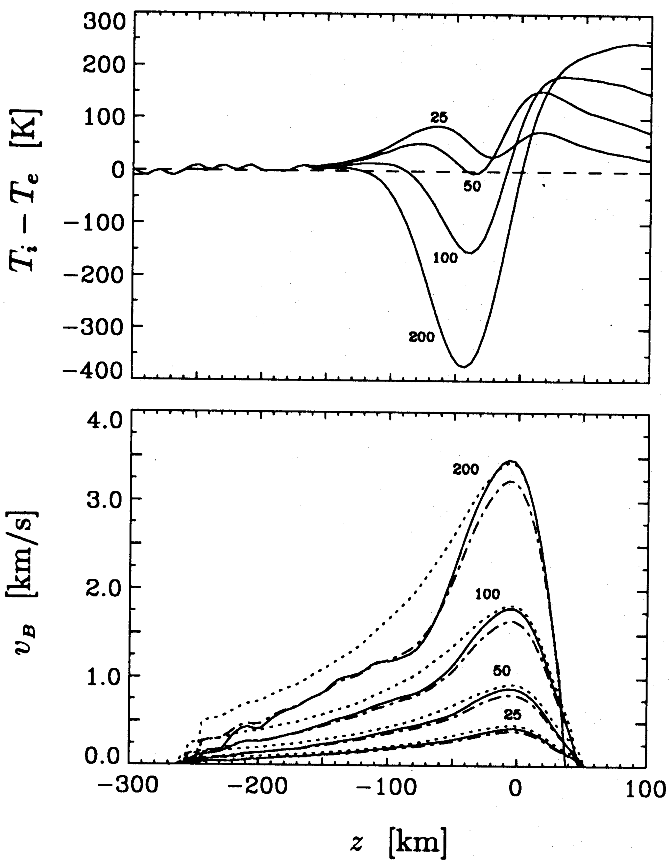

Fig. 1. Height variation of the temperature difference $\Delta T(z) = T_i - T_e$ between interior and exterior of the tube (upper panel), and the critical whirl velocity v_B (lower panel) from Eq. (3.4) (solid) for tubes with $\alpha = 0.2$, $\beta = 0.5$, and radii R = 25, 50, 100, and 200 km, respectively (see also Table 1). The dotted curve results for evacuated tubes [v_{B0} from Eq. (4.2)], the dot-dashed curve is the first order approximation to v_B from Eq. (4.1)

energy transport in the atmosphere. The radiative transfer equation is solved in the six-stream approximation, assuming grey opacity and local thermodynamic equilibrium, and the equation for convective energy transport is solved in the mixing length approach with an additional parameter $\alpha \leq 1$, which characterizes the efficiency of convection in the tube ($\alpha = 1$ in the external atmosphere). Since the equations are nonlinear in the temperature and the pressure, an iterative method is employed: First, we solve the equations of hydrostatic and energy equilibrium for a plane-parallel atmosphere in the ambient medium for constant net vertical energy flux. In this way a "quiet sun" model is constructed (henceforth denoted $\mathrm{HK_{ext}}$), which is used as the external atmosphere in all self-consistent model calculations. We then construct an atmosphere inside a flux tube that is embedded in the external atmosphere. This calculation assumes that the structure of the atmosphere inside the flux tube can be characterized by the values of the temperature, the pressure, and the intensity of the radiation field on the axis of the cylindrical flux tube (cf. Hasan & Kalkofen 1993). The calculation is based on the same multistream approximation as that employed in the external atmosphere, and it assumes a fixed value of $\beta = 8\pi p/B^2$ at the top of the models, 600 km above $\tau_c = 1$ in the external medium (p is the internal gas pressure, B the magnetic field strength, and τ_c the continuum optical depth at 5000 Å). For the initial guess, we assume equality of internal and external temperatures at the same height. The linearized MHS and transfer equations are solved (in the thin flux tube approximation) to obtain corrections to the internal temperature, pressure and mean radiation intensity (Hasan 1988). This procedure is repeated for the updated internal atmosphere until the corrections become sufficiently small. The resulting temperature differences ΔT between interior and exterior of the tubes are shown in Fig. 1 (upper panel) for tubes of radius 25, 50, 100, and 200 km assuming $\alpha = 0.2$, and $\beta = 0.5$. For these models the magnetic field strength at external optical depth unity is 1500 G.

In general, at equal geometric heights the temperature on the tube axis is higher in the photosphere (i.e. for $z > 0$) (cf. Kalkofen et al. 1986), and lower in the convection zone than in the external medium. For tubes with radius $R < 50$ km, the internal temperature is higher also in the subphotospheric layers. However, at equal respective optical depths, the temperature inside the tube is higher than in the ambient medium, at least for the relatively thin tubes studied in this paper. At external optical depth unity, this difference is typically a few hundred degrees. The temperature structure of the flux tube atmosphere is influenced mainly by radiative transfer effects in the photosphere and by convection in the deeper layers, however, it is insensitive to the value of α, i.e. the degree by which convection is inhibited in the flux tube by the magnetic field. This is due to the fact that increased convective transport within the tube reduces the radiative input from the surroundings and vice versa (see also Deinzer et al. 1984).

There still remain deviations of the theoretical models from semi-empirical results, e.g. the most recent plage and network models of Solanki & Brigljević (1992): the internal temperature of the theoretical models shows a much steeper downward increase just below the external $\tau_c = 1$ level (see Hasan & Kalkofen 1993). As a consequence, the internal $\tau_c = 1$ level lies less deep in the atmosphere so that the magnetic field strengths at this level are slightly smaller than what is expected from recent infrared measurements (Rüedi et al. 1992). Hence only limited quantitative conclusions may be drawn from the stability analysis (cf. Sect. 4.4). However, the degree of sophistication of the present models appears sufficient to study principle effects of energy transport on the stability of flux tube surfaces.

3. The interchange instability analysis

The interface between a magnetic and a non-magnetic region in stationary equilibrium is locally interchange stable if

$$\boldsymbol{n} \cdot \left[\nabla \left(p_i + \frac{B^2}{8\pi} \right) - \nabla p_e \right] > 0\,, \tag{3.1}$$

where $\boldsymbol{n}$ is the normal to the surface pointing into the magnetic region, p_e and p_i are the external and internal gas pressures, respectively, and B is the magnetic field strength at the interface. We apply this criterion to a cylindrically symmetric untwisted magnetic flux tube surrounded by a purely azimuthal non-magnetic flow (the "whirl flow"). We use cylindrical coordinates (r, ϕ, z) where the z-axis defines the symmetry axis of the vertical tube and the gravitational acceleration is given by $\mathbf{g} = -g\mathbf{e}_z$. Then, using the momentum equations and applying the thin tube approximation, the stability criterion (3.1) can be written as a biquadratic inequality for the radius $R = R(z)$ of the tube (see Paper I),

$$\left(\frac{R}{H_e}\right)^4 - \frac{8}{\kappa^2}\left[\mu - \left(\frac{v_B}{v_A}\right)^2\right]\left(\frac{R}{H_e}\right)^2 + \frac{128}{\kappa^4}\left(\frac{v_B}{v_A}\right)^2 > 0 \,, \quad (3.2)$$

where H_e is the external pressure scale height, v_B is the velocity of the external azimuthal flow at the tube boundary, v_A is given by $v_A = B(z)/\sqrt{4\pi\rho_e(z)}$, and ρ_e is the external mass density. The quantities κ and μ are defined as follows:

$$\kappa = \frac{1-\lambda\epsilon}{1-\epsilon} \,,$$

$$\mu = \frac{1}{(1-\lambda\epsilon)^2}\left\{\frac{4-m_e}{m_e} + \left[2\frac{m_e-1}{m_e} - 3\lambda + 2\frac{m_i-1}{m_i}\lambda^2\right]2\epsilon + \frac{4-m_i}{m_i}\lambda^2\epsilon^2\right\} \,, \quad (3.3)$$

where $m_{i,e} = -[\mathrm{d}H_{i,e}/\mathrm{d}z]^{-1}$ for the internal and external atmospheres, respectively, and where $\epsilon = p_i(z)/p_e(z)$ and $\lambda = H_e(z)/H_i(z)$. It should also be noted that ϵ and β are related by

$$\epsilon = \frac{\beta}{\beta+1} \,.$$

In the limit of a fully evacuated tube we have

$$\epsilon \to 0 \,, \qquad \mu \to \mu_0 = (4-m_e)/m_e \,, \qquad \kappa \to 1 \,,$$

and Eq. (3.2) reduces to the result of Schüssler (1984). From (3.2) it follows that a tube of given flux Φ is stable at some point of its surface if the whirl velocity satisfies the condition

$$v_B \geq \sqrt{(1-\epsilon)\frac{g\kappa^2R^2}{4H_e}\,\frac{8\mu H_e^2-\kappa^2R^2}{16H_e^2+\kappa^2R^2}} \,. \quad (3.4)$$

Obviously, no whirl velocities are required if the radius of the tube $R \geq R_{\min} = \sqrt{8\mu}\,H_e/\kappa$ or, equivalently, if the magnetic flux of the tube exceeds $\Phi_{\min} = \pi\sqrt{8\pi p_e(1-\epsilon)}R_{\min}^2$. The lower panel in Fig. 1 shows the height dependence of the critical whirl velocity $v_B(z)$ calculated from Eq. (3.4) (solid lines) for model tubes of various sizes, for internal convection reduction factor $\alpha = 0.2$ (cf. Sect. 2) and $\beta = 0.5$ ($\epsilon = 1/3$). The dotted lines represent the results for the corresponding evacuated tubes. If the effect of the internal atmosphere is included to first order in ϵ, the dot-dashed lines result [see Sect. 4.3, Eq. (4.1)].

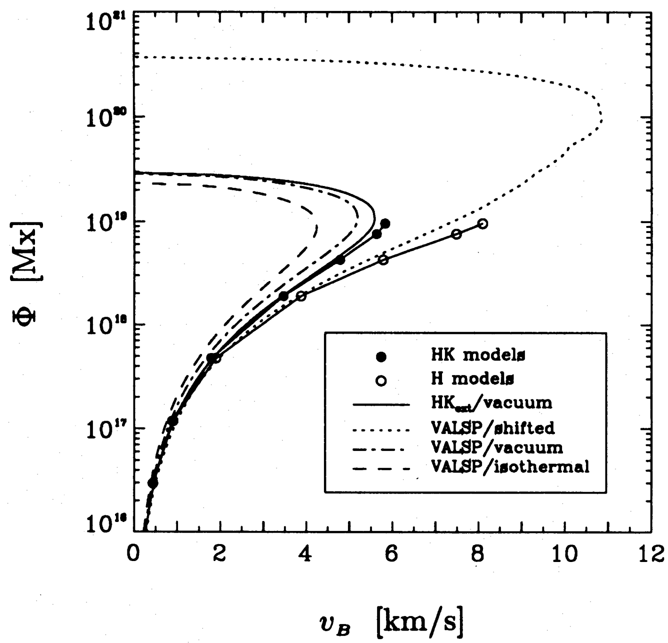

Fig. 2. Stability diagram, showing Φ vs. v_B. The open circles are for the self-consistent thin tube models of Hasan (1988; "H"), the filled circles for the models of Hasan & Kalkofen (1993; "HK"), and the solid line for the corresponding evacuated tubes. Other curves represent thin tubes embedded in the VALSP atmosphere (Spruit 1977) and internal atmospheres as indicated. See also Table 1

4. Results

4.1. The influence of α and β

We have studied the stability of approximately 40 thin flux tube models varying in magnetic flux Φ from 3×10^{16} Mx to 10^{19} Mx, covering five different values of β at the top of the models (0.1, 0.5, 1.0, 2.0, 3.0) and two values of the reduction factor α of the convective energy flux (0.2 and 0.8) inside the tubes. We find that the maximum value of v_B needed for stability is practically independent of α since the temperature structure in the tube is insensitive to changes in this parameter (Deinzer et al. 1984; Hasan 1988). Also, the influence of β on the stability criterion was found to be very small. Only marginally different values (smaller by a few percent in v_B) were obtained for higher values of β due to the reduced magnetic field strength. In view of this, we shall concentrate on the stability properties of tubes with different radii or fluxes, assuming fixed values of α and β, which we take to be 0.2 and 0.5, respectively.

4.2. Results for $\alpha = 0.2$ and $\beta = 0.5$

Figure 2 shows the results in the form of a stability diagram Φ vs. v_B. Tubes with fluxes below some $10^{19} - 10^{20}$ Mx (depending on the models considered) require a critical whirl velocity $v_B > 0$ for stabilization. Hence, each curve separates the unstable regime (between curve and ordinate) from the stable regime. The open circles are the results for the models of Hasan (1988) and the filled circles for the new models of

Hasan & Kalkofen (1993). Owing to numerical problems in constructing self-consistent model atmospheres within the tube for $R(0) > 450$ km, it was not possible to cover the higher flux range. In any case, the use of the thin flux tube approximation is questionable when the tube radius becomes too large (i.e. greater than the local pressure scale height). For comparison we have also plotted the stability curves for "crude" thin tubes, i.e. those with a *prescribed* internal atmospheric structure. The solid line is for completely evacuated tubes surrounded by the HK_{ext} model atmosphere. The dot-dashed line corresponds to the Spruit-VAL (VALSP) atmosphere (Spruit 1977).

The stability curves for these evacuated models are fairly close and differ mainly because of the slightly steeper temperature gradient of the HK_{ext} model as compared to the VALSP model in the layer just below the photosphere, mainly due to differences in opacities and the treatment of radiative transfer. The models of Hasan (1988) and Hasan & Kalkofen (1993) use opacity tables (kindly provided by R. Kurucz) for the Rosseland mean opacity and the Eddington and multistream approximations, respectively, in the treatment of radiative transfer. On the other hand, the VALSP model uses the diffusion approximation in the convection zone, which could result in temperature differences in the subphotospheric layers. The other curves are for tubes embedded in the VALSP atmosphere and differ by the choice of the *internal* atmosphere which in each case was simply prescribed without requiring energy balance (cf. Paper I). The dashed line is for the "isothermal" model where the internal temperature is identical to the external one but gas pressure and density are reduced by a constant factor $\epsilon = 1/3$, corresponding to $\beta = 0.5$. The dotted line is found when using again the identical atmosphere inside the tube but this time shifted downwards by 200 km, resulting in a tube which is cooler than the exterior at equal geometric height. In Paper I this shift was found to give the worst possible stability curve.

Note that the new models of Hasan & Kalkofen (1993) require peak velocities that are smaller (by up to 1.9 km/s) than those of Hasan (1988) for fluxes above $\approx 10^{18}$ Mx. As will be shown in the following Sects. this is due to the improved treatment of the radiative transfer in the new models, which produces maximum temperature differences of only -400 to -620 K between the interior and the exterior of the tubes, in contrast to -2000 to -2800 K in the previous models. Furthermore, Fig. 2 shows that the stability properties of the new models (filled circles) are quite close to those of the corresponding evacuated tubes (solid line; see also Fig. 1).

4.3. *The role of energy transport*

In the thin tube models considered here the interior of the tubes is coupled to the exterior by radiative energy transport. Some light may be cast on the role of the resulting atmospheric structure inside the tubes by investigating the limit of small internal gas pressure, i.e. when $\epsilon \ll 1$. Expanding the expression for the critical v_B from Eq. (3.4) in powers of ϵ and retaining only terms up to first order yields

$$v_B = v_{B0} \left\{ 1 + \left[-\frac{1}{2} - aH_e \left(\frac{1}{H_e} - \frac{1}{H_i} \right) + bH_e^2 \left(\frac{1}{H_e} - \frac{1}{H_i} \right)^2 + bH_e^2 \frac{\mathrm{d}}{\mathrm{d}z} \left(\frac{1}{H_e} - \frac{1}{H_i} \right) \right] \epsilon + O\left(\epsilon^2\right) \right\} , \qquad (4.1)$$

where v_{B0}, the vacuum value of v_B, a, and b are given by

$$v_{B0} = \sqrt{\frac{gR^2}{4H_e} \frac{8\mu_0 H_e^2 - R^2}{16H_e^2 + R^2}} ,$$
$$a = \frac{32H_e^2 + m_e R^2}{m_e \left(8\mu_0 H_e^2 - R^2\right)} - \frac{16H_e^2}{16H_e^2 + R^2} > 0 , \qquad (4.2)$$
$$b = \frac{16H_e^2}{8\mu_0 H_e^2 - R^2} > 0 .$$

From the expression for v_{B0} and the remark following Eq. (3.4) it is clear that $8\mu_0 H_e^2 - R^2 > 0$, hence that b is positive, and – using $\mu_0 = (4 - m_e)/m_e$ – that also a is positive. The first-order correction given in Eq. (4.1) is an acceptable approximation to the exact value, Eq. (3.4), only for small tubes and breaks down for larger tubes (with radius R close to $R_{\min}$) when higher-order terms become important. The first term in square brackets $(-1/2)$ arises from the finite pressure in the flux tube (so that the field strength is less than that of an evacuated tube) and is always stabilizing. The relative importance of the other terms depends on the positive factors a and b which are solely determined by the properties of the external atmosphere and the radius R of the tube. Note that $a,\ b \rightarrow \infty$ as R approaches the critical radius of the evacuated tube, $\sqrt{8\mu_0}H_e$. The second and third terms have a *destabilizing* effect if the internal atmosphere is cooler than the surroundings ($H_i < H_e$). The last term depends on the gradient of the scale heights and makes a destabilizing contribution if the external pressure scale height drops faster than the internal one.

A numerical analysis shows that for small tubes it is the reduction of the gas pressure jump across the interface which dominates in the relevant layers of the atmosphere. As shown in Fig. 1, this yields (slightly) reduced whirl velocities (dot-dashed line) and hence a stabilizing contribution of the internal atmosphere as compared with evacuated tubes (dotted line). Note that the exact result from Eq. (3.4) (solid line) is well approximated by (4.1) except in the region of the peak velocity. For larger tubes with enhanced temperature differences (the open circles in Fig. 2), however, the other terms start to dominate the first one, and the net effect of the internal atmosphere is destabilizing.

4.4. *The unstable layer*

Flux tube models that include energy balance of the tube with its surroundings exhibit drastic changes in the internal and external temperature structure in a rather shallow layer close to $\tau_c = 1$ where the energy transport changes its character from convective to radiative (e.g. Hasan 1988; Steiner 1990; see also Steiner & Stenflo 1990; Hasan & Kalkofen 1993). As a consequence the gas pressure drops rapidly with height, forces the magnetic

Table 1. Thin tube models with α = 0.2 and β = 0.5; first line: Hasan (1988; open circles in Fig. 2), second line: Hasan & Kalkofen (1993; filled circles in Fig. 2)

Φ [Mx]	$R(0)$ [km]	z_W [km]	z_{crit} [km]	v_B [km/s]	$\Lambda(z_{crit})$ [km]	unstable layer [km]	
$3.0 \cdot 10^{16}$	25	−40	−17	0.4	95	−265	+45
		−29	−5	0.4	94	−260	+50
$1.2 \cdot 10^{17}$	50	−50	−17	0.9	112	−240	+40
		−29	−5	0.9	89	−260	+50
$4.8 \cdot 10^{17}$	100	−60	−17	1.9	155	−125	+35
		−33	−5	1.8	102	−255	+50
$1.9 \cdot 10^{18}$	200	−77	−17	3.9	234	−130	+25
		−41	−5	3.5	133	−245	+35
$4.3 \cdot 10^{18}$	300	−95	−21	5.8	267	−160	+15
		−47	−9	4.8	142	−215	+25
$7.6 \cdot 10^{18}$	400	−105	−21	7.5	282	−180	+5
		−51	−13	5.6	144	−170	+20
$9.6 \cdot 10^{18}$	450	−98	−25	8.1	263	−115	+5
		−52	−13	5.8	148	−150	+20

field to expand, and thereby leads to the intrinsic instability of the small magnetic flux concentrations. Hence, in those layers where the interchange instability is of importance, we also find the largest horizontal temperature differences and temperature changes (cf. Fig. 1).

The results are summarized in a more quantitative form in Table 1 for tubes with α = 0.2 and β = 0.5. Listed are the magnetic flux Φ of a tube, its radius $R(0)$ at $\tau_c = 1$ (in the external atmosphere), the height, z_W, of the $\tau_c = 1$ level inside the tube (the Wilson depression), the height z_{crit} where the maximum critical whirl velocity v_B is found when scanning the tube surface, the photon mean free path, Λ, inside the tubes at $z = z_{crit}$, and lower and upper boundaries of the region where the surface is unstable (the region where the critical $v_B > 0$, cf. Fig. 1).

In all cases listed here, the tube radius $R \approx R(0)$ at the height of the most unstable surface point is of the order of the internal photon mean free path, $\Lambda(z_{crit})$. This implies an efficient lateral influx of radiation, leading to a strong heating of the interior of the tube with respect to the surroundings (see Fig. 1). This general behaviour is of importance for the stability of the surface: as mentioned in the previous section, it is not the temperature contrast (or equivalently $H_i - H_e$) alone that leads to a destabilization of the surface, but also the related differences in the temperature gradients. This is contained in the linear treatment Eq. (4.1) and also found in the exact result: note that the peak velocities in Fig. 1 do not occur where the temperature differences are largest, but rather where $T_i - T_e$ increases most drastically.

It must be borne in mind, however, that in the model tubes of Hasan & Kalkofen (1993) the downward increase of the internal temperature below the external $\tau_c = 1$ level is much steeper than suggested by recent semi-empirical models (Solanki & Brigljević 1992). In reality, therefore, the most unstable region might lie slightly deeper in the atmosphere than derived for the present theoretical models.

5. Discussion and conclusions

We have investigated the principle effects of convective and radiative energy transport on the stability of magnetic flux tubes using the thin tube approximation. The magnitude of stabilizing external whirl flows surrounding the tubes has been used as a measure of the instability. From the study of a wide range of thin flux tube models, we can draw the following conclusions:

1. The layer susceptible to the instability extends over 200 – 300 km just below $\tau_c = 1$ in the external atmosphere, in agreement with earlier results (Meyer et al. 1977; see also Paper I).
2. The stability of the tubes is independent of the efficiency of internal convective transport. This is a direct consequence of the insensitivity of the thermodynamic state of the flux tube to changes in α (Deinzer et al. 1984; Hasan 1988).
3. The magnitude of the whirl velocities drops slightly with increasing plasma beta due to the reduced magnetic field strengths and related tension effects. However, differences between the internal and external temperature structure can still lead to destabilization.
4. For highly evacuated flux tubes, the internal atmosphere affects the stability of the surface by [cf. Eq. (4.1)] (a) reducing the gas pressure jump, (b) introducing a difference in temperature[1], and (c) in temperature[1] gradients with respect to the surroundings. The relative importance of the latter two effects with respect to the former are determined by the radius R of the tube and the properties of the external atmosphere [Eqs. (4.2)].
5. The influence of the internal atmosphere depends upon the size of the tube. For small tubes ($R \lesssim 200$ km), the reduced gas pressure jump (relative to the evacuated case) balances possible destabilizing effects due to differences in the temperature structures. The resulting stability properties are close to those of completely evacuated tubes. For larger tubes, the lateral radiative energy influx cannot compensate the losses through the top of the tubes. The resulting increased temperature contrast (of up to −620 K for the HK models) slightly destabilizes the tubes, so that whirl flows 0.3 km/s larger than in the corresponding evacuated cases are needed. For the earlier models of Hasan (1988), with maximum ΔT as high as −2800 K, the effect is more pronounced, yielding v_B values 2.6 km/s higher than in the evacuated case.

[1] more precisely the pressure scale height $H_{i,e} \propto T_{i,e}/\bar{\mu}_{i,e}$, where $\bar{\mu}_{i,e}$ are the internal and external mean molecular weights, respectively

6. Tubes with fluxes above 5×10^{17} Mx require whirl flows of 5.8 km/s, well in excess of what can be expected on the sun (≈ 2 km/s). Due to numerical problems in computing tubes with fluxes $\Phi \gtrsim 10^{19}$ Mx, the critical flux, $\Phi_{\rm crit}$, above which all tubes are stabilized by buoyancy, could not be determined. However, comparison of the stability curve with other models in Fig. 2 suggests a value of $3 \times 10^{19} \lesssim \Phi_{\rm crit} \lesssim 4 \times 10^{20}$ Mx.

From the results in the highly evacuated limit it is apparent that temperature differences between the interior and exterior of a tube are likely to further destabilize its surface. Thermal effects which lead to an appropriate adjustment of the internal temperature structure have been suggested by Parker (1975) for the stabilization of larger flux tubes. However, as pointed out by Meyer et al. (1977), such a mechanism – even if it could prevent fluting at the tube surface – would render the structures unstable to symmetric radial contractions. Also for small structures it seems questionable whether purely thermal readjustment of the tube interior can stabilize the surface. As pointed out in Sect. 4.2 the stability properties of the smaller structures are very close to those of completely evacuated tubes, indicating that it is the external rather than the internal atmosphere which mainly determines their stability. Hence, the stability problem for small magnetic flux tubes will be even more severe in a 2D calculation, in which cooling of the immediate surroundings of the tubes is taken into account. This cooling would steepen the vertical gradient of ΔT, thereby strongly destabilizing the tube surface.

Finally we want to point out once more that the absolute numbers for the magnitude of the stabilizing whirl flows and the location of the unstable layer have been derived from theoretical models which contain various approximations and will therefore be subject to revisions when more sophisticated models are available in the future. On the one hand, a thin tube analysis overestimates the whirl velocities needed for stabilization by up to 60 %, due to the omission of magnetic tension forces (cf. Paper I). This shifts the lower and upper boundaries of the unstable flux interval to slightly higher and lower values, respectively. On the other hand, deviations of parts of the internal temperature structures from semi-empirical models (Solanki & Brigljević 1992) might shift the region of instability somewhat further down (Sect. 4.4).

In summary, the stability problem of small magnetic flux tubes remains, and its final answer will have to await time-dependent three-dimensional models with a spatial resolution that allows the boundary layer between magnetic field and fluid motion to be resolved. This layer is a vortex sheet and hence might be liable to the Kelvin-Helmholtz instability (Schüssler 1979; Tsinganos 1980). As discussed in Paper I the onset of the latter depends on the detailed structure of the surface layer, at the smallest scales where the various dissipative processes operate.

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