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Scattering in a parabolic encounter of a single star with a circular binary

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Abstract. Huang and Valtonen (1987) obtained an analytic expression for the energy transfer in a parabolic encounter of a single star with a circular binary keeping terms of the first order in a/q where a is the radius of the circular orbit and q is the distance of closest approach of the single star with respect to the centre of mass of the binary. They also obtained numerically the energy transfer and change in eccentricity of the single star for the case of mass of the binary equal to the mass of the single star. We extend their analytic treatment to obtain angular momentum transfer and change in eccentricity up to first order a/q and compare the change in eccentricity obtained by us with theirs. We find that our first order analysis is reasonable for $Q=q/a > 2.5$ for direct orbit and $Q > 1.125$ for retrograde orbit.

1. Introduction

The study of the effects of stellar encounter on a binary star has been a subject of extensive study (Hut(1984), Heggie (1988) and Valtonen (1988)). An important recent work, Heggie and Rasio (1996), studies the effect of stellar encounters on the eccentricity of binaries.

In this paper we study the special case of a parabolic encounter of a star with a circular binary. An approximate solution to the energy change in this case was obtained by Huang and Valtonen (1987). We extend their work to obtain an analytic expression for the changes in angular momentum and eccentricity of the binary.

2. Theory

The masses m_1 and m_2 move in a circular orbit about their common centre of mass. A third body of mass m_3 moves in a parabolic orbit in plane inclined at an angle i to the orbital plane of the binary. The node line is used as the direction of reference. The angle subtended by the line joining m_1 and m_2 with the node line is denoted by Ω and the pericentre direction of m_3 is denoted by ω.

We reckon time from the instant m_3 is at the pericentre. At t = 0, om_1 is taken as the OX-axis. Further, let $\vec{r}$ and $\vec{R}$ denote the position vectors of m_2 relative m_1 and of m_3 relative to the centre of mass of the binary, respectively.

The expression for the change in energy E has been obtained by Huang and Valtonen (1987). We proceed in the same manner to obtain the expression for the change in angular momentum. Taking $K = \frac{n}{n3}$, the relation between K and Q is given by :

$$K = \left[\frac{2(m_1 + m_2)Q^3}{(m_1 + m_2 + m_3)} \right]^{1/2} \tag{1}$$

The expressions for change in energy obtained by Huang and Valtonen is given by :

$$\frac{\Delta E}{Eo} = - \frac{3m_3}{(m_1 + m_2 + m_3)} \frac{f_1(K)}{K} \sin 2\Phi \tag{2}$$

where

$$f_1(K) = \left(\frac{I_1}{2} + \frac{I_2}{2} - I_3\right) \tag{3}$$

$$I_1 = 4 \int_{-\infty}^{\infty} \cos\left[\frac{n}{n_3} \left(\sigma + \frac{1}{3} \sigma^3\right)\right] \frac{(1-\sigma^2)^2}{(1+\sigma^2)^4} d\sigma \tag{4}$$

$$I_2 = 16 \int_{-\infty}^{\infty} \cos\left[\frac{n}{n_3} \left(\sigma + \frac{1}{3} \sigma^3\right)\right] \frac{\sigma^2}{(1+\sigma^2)^4} d\sigma \tag{5}$$

$$I_3 = 8 \int_{-\infty}^{\infty} \sin\left[\frac{n}{n_3} \left(\sigma + \frac{1}{3} \sigma^3\right)\right] \frac{\sigma(1-\sigma^2)^2}{(1+\sigma^2)^4} d\sigma \tag{6}$$

and $\sigma = \tan f_3/2$ (7)

We obtain for the change in angular momentum for the direct case i-0° :

$$\frac{\Delta \vec{h}}{ho} = \frac{3m_3}{(m_1 + m_2 + m_3)} \frac{[f_2(K) + \cos(2\Phi + f_3(K))]}{K} \hat{h} \tag{8}$$

where

$f_2(K) = (2I_6 + (I_7)$ and $f_3(K) - (2I_4 + I_5)$ (9), (10)

and

$$I_4 = 4 \int_{-\infty}^{\infty} \sin[\frac{n}{n_3} (\sigma + \frac{1}{3} \sigma^3)] \frac{(1-\sigma^2)^2}{(1+\sigma^2)^4} d\sigma \quad (11)$$

$$I_5 = 16 \int_{-\infty}^{\infty} \sin[\frac{n}{n_3} (\sigma + \frac{1}{3} \sigma^3)] \frac{\sigma^2}{(1+\sigma^2)^4} d\sigma \quad (12)$$

$$I_6 = 16 \int_{-\infty}^{\infty} \cos^2[\frac{n}{n_3} (\sigma + \frac{1}{3} \sigma^3)] \frac{\sigma(1-\sigma^2)^2}{(1+\sigma^2)^4} d\sigma \quad (13)$$

$$I_7 = 16 \int_{-\infty}^{\infty} \sin^2[\frac{n}{n_3} (\sigma + \frac{1}{3} \sigma^3)] \frac{\sigma(1-\sigma^2)}{(1+\sigma^2)^4} d\sigma \quad (14)$$

The expression for the change in angular momentum for the retrograde case $i = 180^0$ is given by :

$$\frac{\Delta \vec{h}}{ho} = \frac{3m_3}{(m_1 + m_2 + m_3)} \frac{[f_2(K) + \cos(2\Phi + f_5(K))]}{K} \hat{h} \quad (15)$$

where

$f_4\,(K) = (2I_8 + I_9)$ and $f_5(K) = (2I_{10} + I_{11})$ (16), (17)

$$I_8 = \int_{-\infty}^{\infty} \sin[\frac{n}{n_3} (\sigma - \frac{1}{3} \sigma^3)] \frac{(1-\sigma^2)^2}{(1+\sigma^2)^4} d\sigma \quad (18)$$

$$I_9 = 4 \int_{-\infty}^{\infty} \sin[\frac{n}{n_3} (\sigma - \frac{1}{3} \sigma^3)] \frac{\sigma^2}{(1+\sigma^2)^4} d\sigma \quad (19)$$

$$I_{10} = 16 \int_{-\infty}^{\infty} \cos^2[\frac{n}{n_3} (\sigma + \frac{1}{3} \sigma^3)] \frac{\sigma(1-\sigma^2)^2}{(1+\sigma^2)^4} d\sigma \quad (20)$$

$$I_{11} = 16 \int_{-\infty}^{\infty} \sin^2[\frac{n}{n_3} (\sigma + \frac{1}{3} \sigma^3)] \frac{\sigma(1-\sigma^2)}{(1+\sigma^2)^4} d\sigma \tag{21}$$

Using the relation between eccentricity, energy and angular momentum given in Goldstein (1950).

$$e = \sqrt{1 + \frac{2El^2}{mk^2}} \tag{22}$$

$$\text{with } E = - \frac{Gm_1m_2}{2a}, \; l = \frac{m_1 m_2}{m_1 + m_2} \sqrt{G(m_1 + m_2)a(1-e^2)} \tag{23}$$

the expression for change in eccentricity takes the form :

$$\Delta e = \sqrt{\frac{7}{2}} \frac{3m_3}{(m_1 + m_2 + m_3)} (\frac{f_1(K)}{K} \sin 2\Phi - \frac{[f_2(K) + \cos(2\Phi + f_3(K))]}{2K}) \tag{24}$$

3. Results and discussion

Let $m_1=m_2=0.5$ and $m_3=1$, radius of the binary be equal to 1 and G=1. In figure 1 we compare the change in eccentricity obtained by us theoretically using equation (24) with the change in eccentricity obtained by Huang and Valtonen numerically for the first order term. It may be noted that the agreement is good for Q>2.5 in the case of direct orbit and Q>1.125 in the case of retrograde orbit. We therefore conclude that the change in angular momentum due to the first order term derived by us is also valid for the same range. For still lesser values of Q the second order term neglected by us becomes important.

In figure 2 we plot change in energy, angular momentum and eccentricity against the phase angle for direct and retrograde orbits for the value of Q=2.5, 3 and 4. It may be noted that the phase dependence is nearly the same for all the three quantities. Positive values of $\frac{\Delta E}{E_O}$ in the graphs denote tidal capture of m_3 by the binary since its initial orbit is a parabola. In the range of Q studied, $\frac{\Delta E}{E_O}$ is always less than unity. This implies that the disruption of the binary does not occur for Q>2.5 in the case of direct orbit and for Q>1.125 in the case of retrograde orbit. Also we note that the change in the case of direct orbit is an order of magnitude greater than that for the retrograde orbit. This can be understood as due to the difference in interaction time which is greater for the direct orbit.

If for m_1,m_2 and m_3 we put the masses of the Earth, the Moon and the Sun and let K=12, we obtain the changes in energy and angular momentum for the hypothetical situation in which the Sun encounters the Earth-Moon system in an orbit which is parabolic rather than circular with the same angular velocity ratio at closest approach as that in the Sun, Earth Moon system. It is of interest to compare the secular change in the semi-major axis in this case with the periodic change predicted by an important perturbation term called the "variation" in lunar theory given in Danby (1962).

For the variation term in the lunar theory, the first order perturbation term gives :

$$\left(\frac{\Delta a}{a_0}\right)_{max} = \frac{3}{2K_2\,(1-\frac{1}{K})} = 0.011$$

For the parabolic motion of the Sun the first order term gives

$$\left(\frac{\Delta a}{a_0}\right)_{max} = \left(\frac{\Delta E}{E_0}\right)_{max} = \frac{3f_1(K)}{K} = 0.015$$

The value in the parabolic case is thus slightly larger than that for the circular case. This is quite reasonable. It shows that the analytical expressions obtained can be used over a wide range of masses m_1,m_2 and m_3.

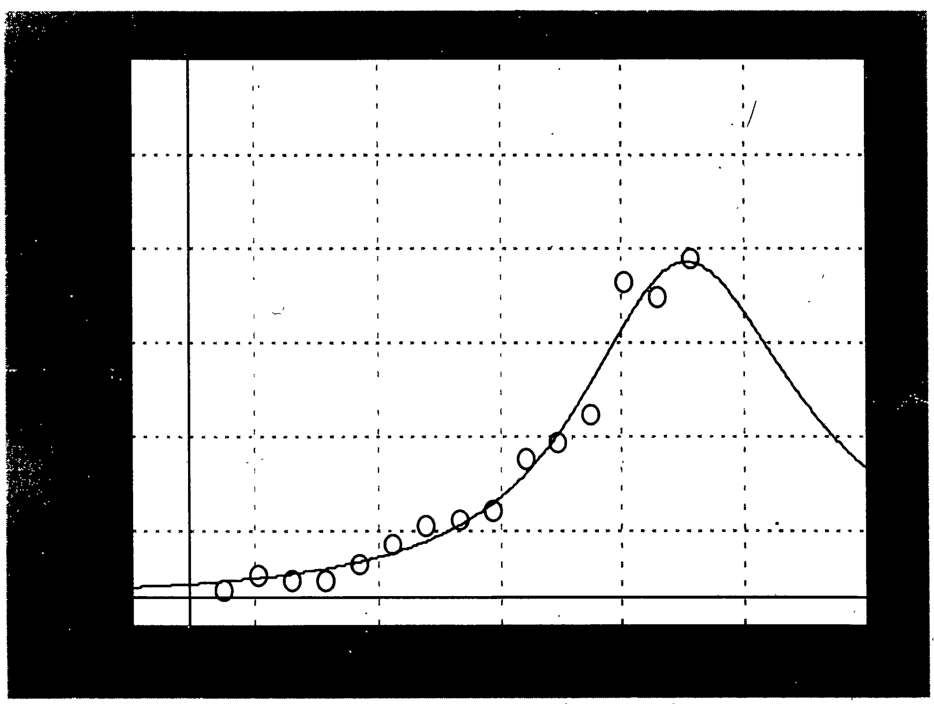

Figure 1. Dependence of change in eccentricity on Q for i=0^{0} and Ω=72^{0} and ω=17^{0}. Theoretical first order represented as circles and Huang and Valtonen numerical values are represented as the continuous line.

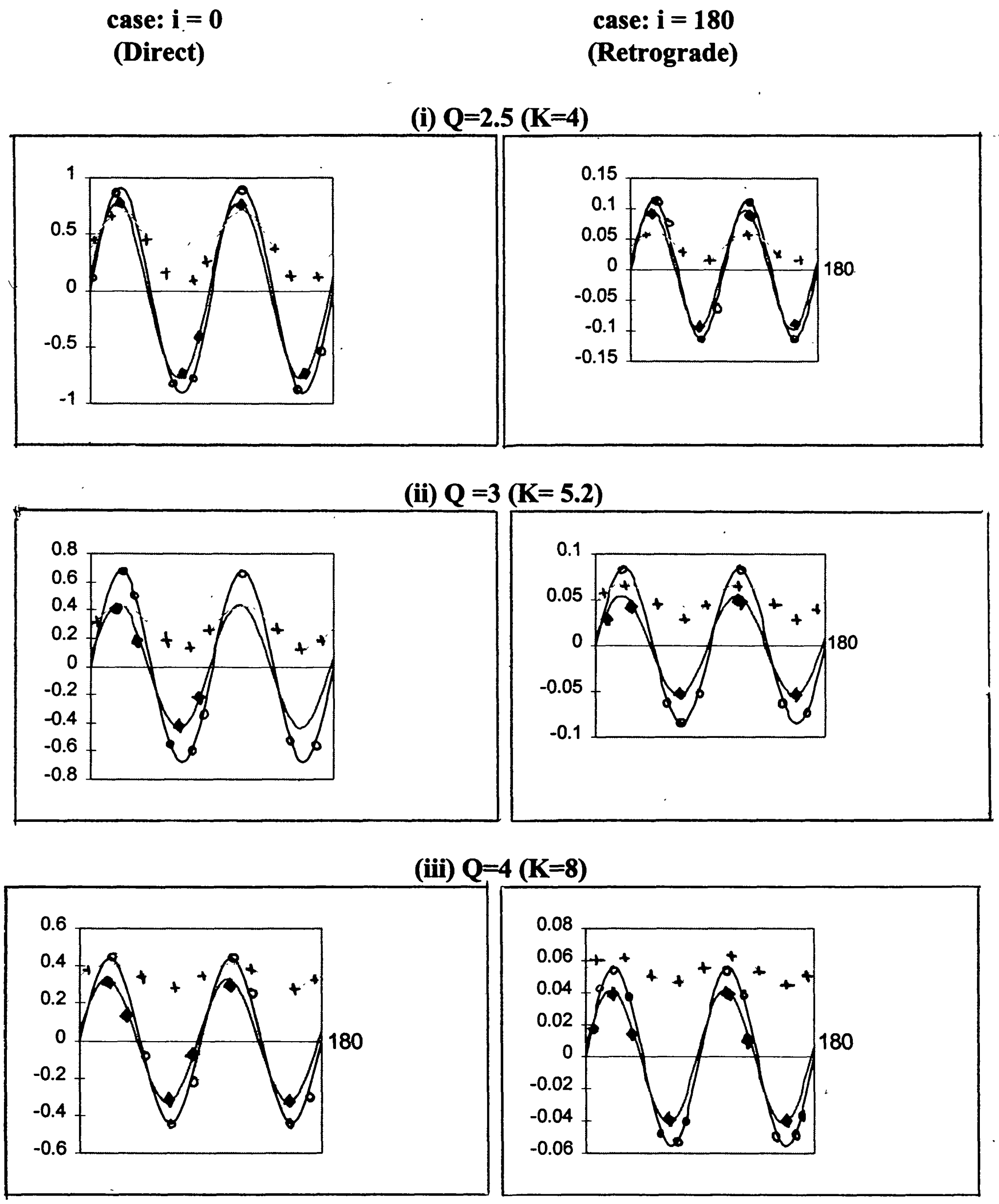

Figure 2. Dependence of change in energy, angular momentum and eccentricity on the phase angle. In the graphs o indicate energy, ♦angular momentum and + eccentricity.

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