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The merger time of binary globular clusters

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Abstract. In a binary globular cluster the tidal forces increase the internal energy of the clusters and may lead to either merger or disruption of a cluster depending upon whether the two clusters are !of equal mass or vary much in mass. Since the physical process of merger is the same for both galaxy and cluster pairs, we have estimated the merger time and disruption time for two globular clusters by using the simple formulae used for merger time of binary spherical galaxies derived by integrating the tidal force over one period of the binary. The results thus obtained are compared with those of n-body simulations and the agreement is found to be fairly good.

1. Introduction

Physical pairs of globular clusters do exist in Magellanic Clouds as was shown by Bhatia and Hatzidimitriou (1988), Bica and Schmitt (1995) and others. A double cluster has also been identified in M31 by Holland et al (1995) and a possible triplet globular cluster system in LMC by Hilker et al (1995). S. van den Bergh (1996) has suggested that globular clusters with composite colour-magnitude diagrams, such as NGC 1851, NGC 2808 and Fornax 3 may have been formed by mergers. This discovery of physical pairs of globular clusters has motivated extensive study of their dynamical evolution through gravitational interaction effects.

The main tool for such studies has been n-body simulations of stellar system encounters. In stellar system encounters, mutual tidal forces increase the total internal energy of each member of the pair. This may lead to either merger or disruption of a cluster depending upon whether both the members of the pair are of equal mass or vary much in mass. It has been noted that the simple formulae obtained by integrating the tidal force over time for fixed positions of stars (impulsive approximation) give values of merger and disruption times for a pair of galaxies which are in reasonable agreement with those obtained by n-body simulations (Alladin et al 1998). Since the physical process of merger and disruption is the same for both the galaxy pair and the cluster pair, the difference being only in scales of mass and size, it is our aim to test analytical formulae obtained for galaxies also give reasonable agreement with the results of n-body simulations for globular clusters.

2. Basic equations

Consider an encounter between two globular clusters of masses M_0 and M_1 moving in elliptic orbits. The merger time (Alladin et al 1988) t_m is given by :

$$t_m = \frac{a^{7/2} (1-e^2)^3 (M_0 + M_1)^{1/2}}{\pi G^{1/2} [M_0 (R_{rms})_1{}^2 + M_1 (R_{rms})_0{}^2]} \tag{1}$$

and the disruption time t_d of M_0 is given by :

$$t_d = \frac{a^{9/2} (1-e^2)^3 M_0 (M_0 + M_1)^{1/2}}{2\pi G^{1/2} M_1{}^2 (R_{rms})_0{}^3} \tag{2}$$

where a is the semi-major axis, e is the eccentricity of the orbit, p is the distance of closest approach and R_{ms} is the root mean square radius of the cluster. These could be compared with the orbital period t_{orb} of the clusters given by :

$$t_{orb} = 2 \pi a^{3/2} / [G (M_0 + M_1)]^{1/2} \tag{3}$$

A Plummer model describes well the mass distribution in a globular cluster but it is of infinite radius; hence $R_{rms} = \propto$. We adopt a Plummer model with a cut-off radius containing about 93% of the total mass. For this model R_{rms} = R, the dynamical radius.

3. Discussion and results

(a) Merger time and disruption time

Sugimoto and Makino (1989) have carried out n-body simulations to obtain the merger time of two identical globular clusters, using King's model to start with, for two initial orbital parameters defined by a=10, e-0 (Case A) and a=6, e=0.5 (Case B) which in effect correspond to roughly the same apocentre distance. They choose units so that the mass of a cluster M = 1, dynamical radius R = 1 and the gravitational constant G = 1. They find that the merger time t_m for the two cases is about 350 and 40 time units. We shall use the same units and estimate t_m from eq.(1) which for two identical clusters of mass M reduces to :

$$t_m = a^{7/2} (1 - e^2)^3 / [2^{1/2} \pi (R_{rms})^2] \tag{4}$$

Assuming R_{rms} = R the above relation gives merger times equal to 700 and 50 time units for the Cases A and B respectively. Thus our value for Case A is greater by a factor of 2 as compared to n-body results, while the value for Case B is comparable.

de Oliveira et al (1998) have studied the disruption of globular clusters through n-body simulations using Plummer models for both the clusters. For their model E9BD10 (mass ratio equal to 10) they find disruption time to be greater than 91.5 Myr. For the same model E9BD10, our eq. (2) gives about 120 Myr for disruption which is comparable to their value.

(b) Critical separation for merger and disruption

Sugimoto and Makino (1989) found that the tidal effects bring about a circularization of the orbit of the clusters and a synchronization of the spin of each cluster with orbital revolution. The loss of orbital angular momentum ultimately leads to a quick merger when a critical separation is reached. They do not however specify the exact time within which the merger will take place at this critical separation.

We estimate this critical separation from eqs. (1) and (3) by imposing the condition that cluster pair should merge within one orbital period. Since the orbit gets circularized before merger we put e=0. This gives for the critical separation.

$$a_{crit} = 2^{1/2}\,\pi = 4.4 \qquad (5)$$

This is about twice the sum of the dynamical radii. The value of a_{crit} =3.3 given by Sugimoto and Makino correspond to the merger of the cluster pair within half the orbital period. Similarly we can find the critical separation for disruption by imposing the condition that the less massive member should disrupt within one orbital period. Eqs.(2) and (3) then give :

$$p = \frac{(2\pi)^{2/3}}{(1+e)}\, R_{rms} \left(\frac{(M_1)^{1/3}}{M_0}\right) \left(1+\frac{M_0}{M_1}\right)^{-1/3} \qquad (6)$$

Since for the disruption of M_0, M_1 should be much more massive than M_0 the above expression may be approximated to :

$$p = \frac{4.4}{(1+e)}\, R_h \left(\frac{M_0}{M_1}\right)^{1/3} \qquad (7)$$

Where we have used R_{rms} = 1.3 R_h, R_h being the half mass radius.

de Oliveira et al (1998) who have carried out n-body simulations of unequal mass pairs moving in elliptic orbits with eccentricities ranging from 0.6 to 0.9, give the following expression for the distance of closest approach for disruption to take place.

$$p = 2\, R_h\, (M_1 / M_0)^{1/3} \qquad (8)$$

It may be noted that the expressions (7) and (8) for p are in good agreement for the range of e = 0.6 to 0.9.

4. Conclusion

The analytical results for merger and disruption times and the critical distances for the case of gobular clusters are in reasonable agreement with the results obtained by lengthy n-body simulations. The agreement is quite impressive considering the simplicity of the analystic formulae and the range in mass and size to which they may be applied. The formulae can be used to make quick order of magnitude estimates against which the results of detailed numerical work can be compared. They also provide valuable insight into the dependence of the tidal effects on the parameters of the orbit.

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