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# Effects of the solar radiation pressure and tidal forces on the rotational motion of a satellite

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**Abstract.** Rotational motion of a satellite influenced by solar radiation pressure and tidal forces have been investigated. Melnikov's integral has been evaluated to establish the non-integrability of the equation of motion. The time series and poincare surface of section have been drawn for different ranges of values of solar radiation parameter, tidal parameter and eccentricity of the orbit to indicate the influence of these quantities.

*Keywords :* solar radiation pressure, tidal forces, Melnikov's integral and chaotic motion.

## 1. Introduction

The problem of planar oscillation of a satellite orbiting elliptically with different types of forces influencing the motion is of growing interest at present. Significant contributions have already been made to this direction by various scientists. e.g. Beletskii (1966), Singh and Demin (1973), Singh (1986), Khan et al. (1998), Pivovarov and Starostin (1966) etc. Most of these works have been incorporated with either one or more of these types of forces. In the present work we have discussed the rotational motion of a rigid satellite moving under gravity influenced by tidal effect and solar radiation parameters and eccentricity of orbits.

## 2. Equation of motion

The equation of motion of the satellite influenced by solar radiation pressure and tidal forces can be written as :

$$\frac{d^2(\phi-\alpha)}{dt^2} + \frac{\omega_0^2\,\mu}{2r^3}\,\mathrm{Sin}2\phi + \beta e\left(\frac{a}{r}\right)^6 + \varepsilon\,\mathrm{Sin}\frac{\phi}{n}(\alpha+\phi) = 0 \tag{1}$$

where

$\phi$ = orientation of the satellite's long axis
$\alpha$ = true anomaly
$\omega_o$ = mass distribution parameter
$\mu$ = GM, M is the mass of the central body and G is the gravitation constant
r = radius vector
e = eccentricity
a = major axis
n = mass distribution parameter
$\beta$ = tidal parameter
$\varepsilon$ = solar radiation parameter

## 3. Melnikov's integral

Using residue theorem the Melnikov's integral of (1) is evaluated and the Melnikov's function obtained as

$$M(V_o) = 2\pi(\pm 1 + 4)\mathrm{Sin}v_o\mathrm{Cosech}\left(\frac{\pi}{2\omega_o}\right) + 8\pi\mathrm{Sin}V_o\mathrm{Sech}\left(\frac{\pi}{2\omega_o}\right) - 8\omega_o\beta$$

$$\pm \frac{4\,\varepsilon_1\pi}{n}\ \mathrm{Sech}\left(\frac{\pi}{n}\right) \pm \frac{4\,\varepsilon_1\pi}{n}\ \mathrm{Cosech}\left(\frac{\pi}{2n}\right) \qquad (2)$$

## 4. Results and discussion

Melnikov's integral (2) for equation (1) shows the non-integrability of the equation of motion. To analyse the motion of the satellite for different ranges of values of solar radiation parameter ($\varepsilon$), tidal parameter ($\beta$) and eccentricity (e), we have carried out the following numerical studies

(i) the motion which was nearly regular at $\varepsilon = 0$, becomes more irregular and chaotic when $\varepsilon$ increased upto 0.4. Further increasing $\varepsilon$ leads towards regularity and at $\varepsilon = 0.8$, we observe again a nearly regular motion. These have been shown through surface of section, Figure 1 and also, through the time series curves, Figure 2. So, one may assume that the radiation pressure may be responsible for chaotic motion to certain range of its lower values. However, its higher values regularise the motion.

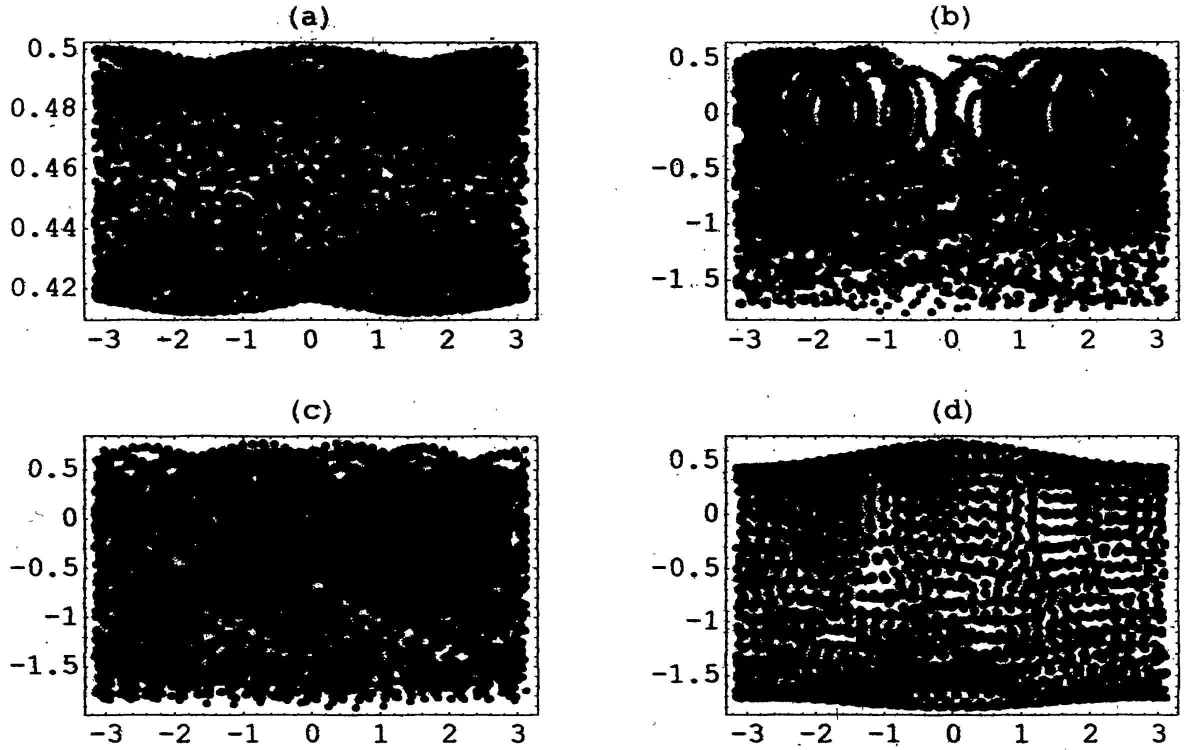


**Figure 1.** Surfaces of section for $\omega_0 = 0.89$, $\mu = 0.3$, e = 0.05, $\beta = 0$, a = 1, n = 20, and (a) $\epsilon = 0$, (b) $\epsilon = 0.3$, (c) $\epsilon = 0.4$, (d) $\epsilon = 0.8$

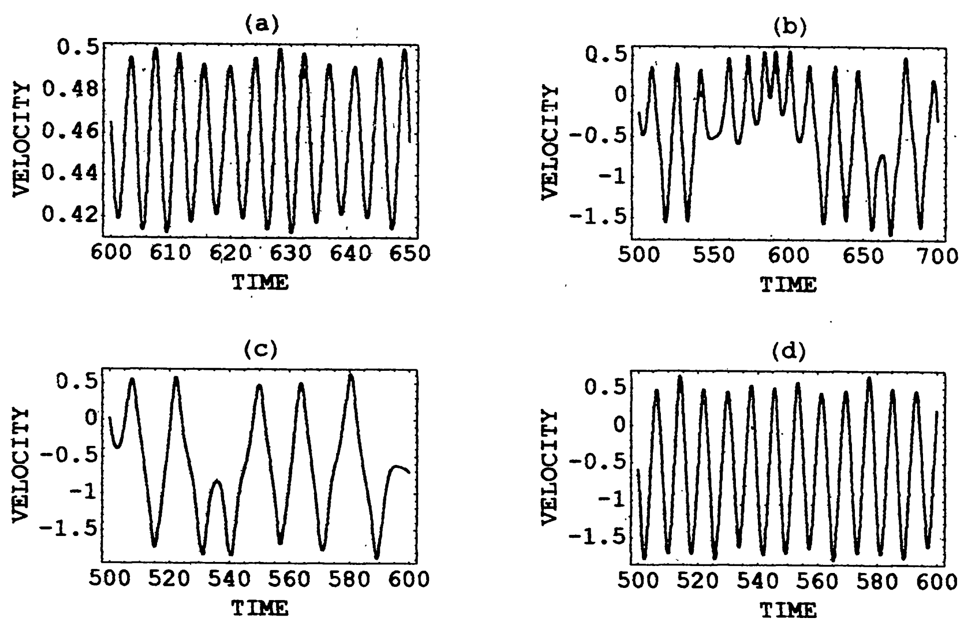


**Figure 2.** Time series graphs for $\omega_0 = 0.89$, $\mu = 0.3$, e = 0.05, $\beta = 0$, a = 1, n = 20, and (a) $\epsilon = 0$, (b) $\epsilon = 0.3$, (c) $\epsilon = 0.4$, (d) $\epsilon = 0.8$

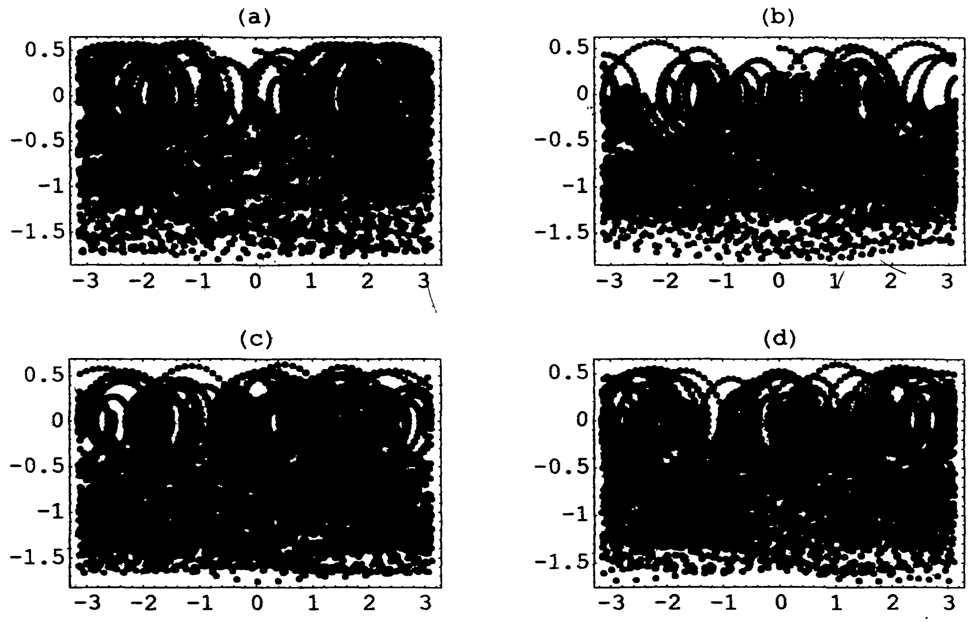


**Figure 3.** : Surfaces of section for $\omega_0 = 0.89$, $\mu = 0.3$, $e = 0.05$, $\epsilon = 0.3$, $a = 1$, $n = 20$, and (a) $\beta = 0$, (b) $\beta = 0.2$, (c) $\beta = 0.35$, (d) $\beta = 0.6$

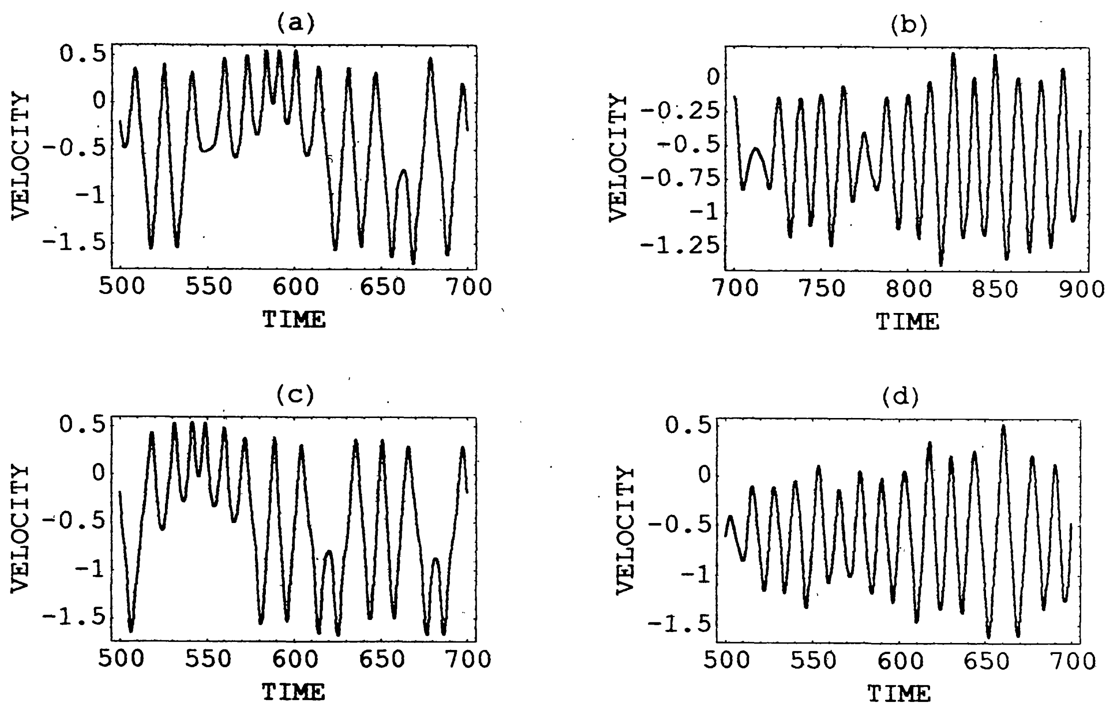


**Figure 4.** : Time series graphs for $\omega_0 = 0.89$, $\mu = 0.3$, $e = 0.05$, $\epsilon = 0.3$, $a = 1$, $n = 20$, and (a) $\beta = 0$, (b) $\beta = 0.2$, (c) $\beta = 0.35$, (d) $\beta = 0.6$

(ii) taking ε = 0.3, when the motion is fully chaotic, we made changes in tidal parameter β from 0 to 0.6. We observe that at β=0.2, the motion becomes less irregular but by increasing β to 0.35, the motion turns into highly chaotic. Again the motion tends towards regularity when β=0.6. These have been shown in the surfaces of secion, Figure 3 and time series graphs, Figure 4. We notice that the motion which was chaotic at β=0, ε=0.3 becomes regular and then chaotic with increasing values of β. In a way we may assume that the presence of tidal force in the system may play a role of chaos control with its suitable adjustment.

(iii) Next we have varied the eccentricity keeping ε and β fixed. We have started with nearly regular motion when ε=0.3 and β=0.6 by varying eccentricity from 0.01 to 0.1. We notice from the surfaces of section Figure 5 and time series curves figure 6 that by increasing e upto 0.07, the motion changes from nearly regular to highly chaotic. However, further increase in e, e=0.1, the motion becomes less chaotic. Thus, orbits eccentricity has also a major role for regular and irregular motion of the satellite.

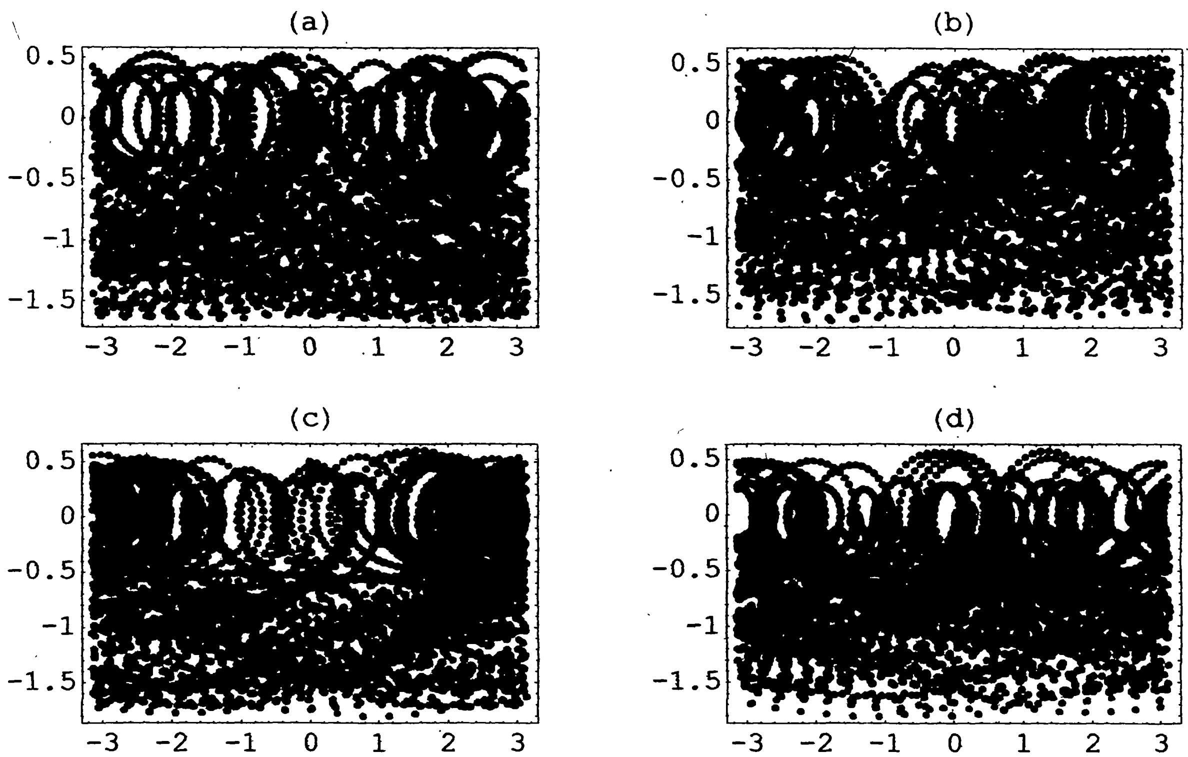


**Figure 5.** Surfaces of section for $\omega_0 = 0.89$, $\mu = 0.3$, $a = 1$, $\beta = 0.6$, n = 20, $\epsilon = 0.3$, and (a) e = 0.01, (b) e = 0.03, (c) e = 0.07, (d) e= 0.1

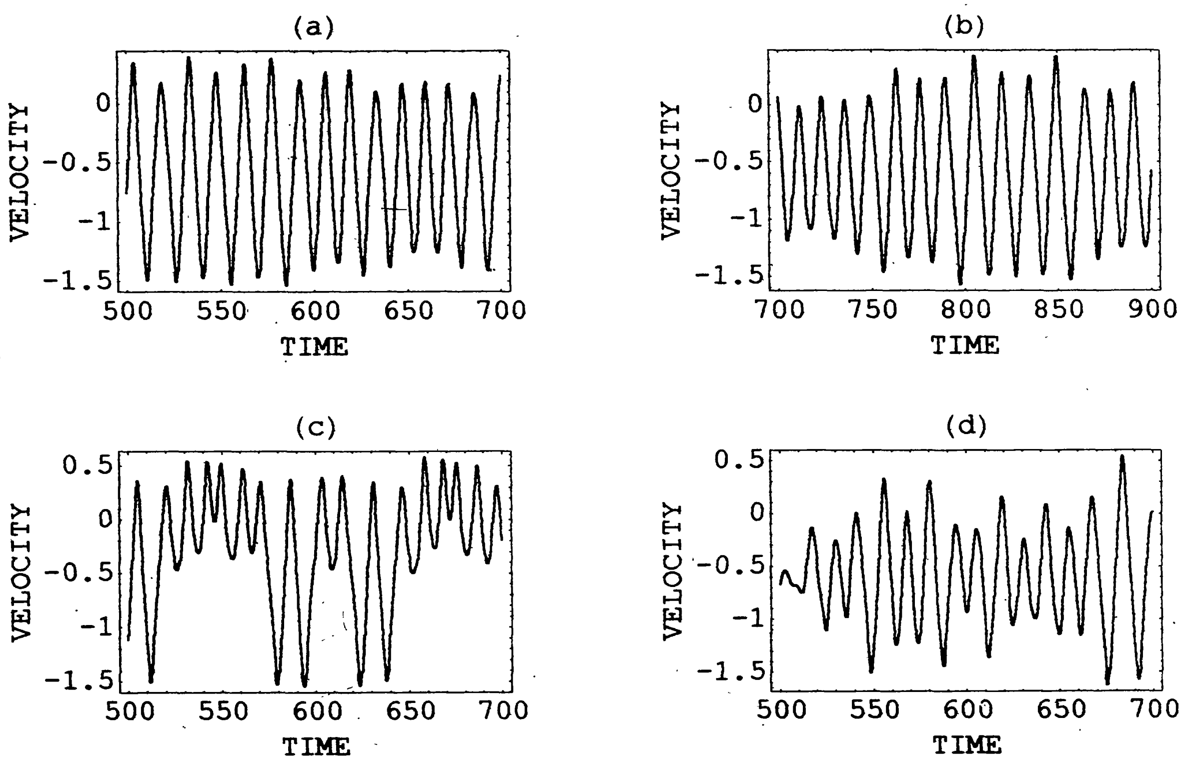


**Figure 6.** Time series graphs for $\omega_0 = 0.89$, $\mu = 0.3$, $\beta = 0.6$, $\epsilon = 0.3$, a = 1, n = 20, $\epsilon = 0.3$ and (a) e = 0.01, (b) e = 0.03, (c) e = 0.07, (d) e= 0.1

## References


Beletskii V. V., 1966, Motion of an artifical satellite about its centre of mass (Jerusalam : Israel Program Sci. Transl.)

Beletskii V. V., Pivovarov M. L., Starostin E. L., 1966, Chaos, 6, 155

Khan A., Sharma R, Saha L. M. AJ, 1998, 116, 2058

Singh R. B., 1986, In space dynamics and celestial mech., ed. K. B. Bhatnagar (Dordrecht : Reidel)

Singh R. B., Demin V. V., 1973, Cosmic Res., 11, 172