Effect of a magnetic field on solar oscillation frequencies

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The measured rotational splittings of solar oscillation frequencies have been effectively employed to infer the rotation rate in the solar interior. Rotation affects, to first order, only the splitting coefficients which represent odd terms in the azimuthal order m of the eigenmodes, while even terms arise largely from second order effects contributed by rotation and magnetic field. The second order contributions estimated from the rotation profile inferred from the odd splitting coefficients can be subtracted from the observed even coefficients to estimate the magnetic field strength.

We adopt the formulation due to Gough & Thompson (1990), but our analysis includes perturbation in the gravitational potential and also assumes differential rotation in the interior, though the symmetry axis of magnetic field is taken to coincide with the rotation axis. Further, we use only the toroidal magnetic field.

We adopt the rotation rate inferred from using the GONG (Global Oscillation Network Group) data for the months 4–14 (Antia et al. 1998) to estimate the splitting coefficients a_2 and a_4. This contributioin should be subtracted from the observed splitting coefficients for obtaining the residual contribution which could be due to magnetic field, other velocity fields or asphericity in solar structure. Since the errors in individual splitting coefficients are too large to give significant differences, we average over 30 neighbouring modes in $w = \eta\mu/(\ell + \frac{1}{2})$.

There have been earlier suggestions that a significant toroidal magnetic field may be concentrated in a layer located around the base of the convection zone (Dziembowski & Goode 1992). We, therefore, investigate splittings that are expected from such a field by assuming the magnetic field to be given by

$$\mathbf{B} = \left(0,\ 0, a\,(r)\frac{dP_k}{d\theta} \right), \qquad a\,(r) = \begin{cases} b_0\,(1 - (\frac{r - r_0}{d})^2) & \text{if } |r - r_0| \le d \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

Here $P_k(\cos\theta)$ is the Legendre polynomial of degree k while b_0, r_0 and d are constants defining respectively, the maximum field strength, mean position and thickness of the layer where the field is concentrated. The coefficients a_2 and a_4 from such a magnetic field have a characteristic

signature which could be used to detect this signal in the observed splittings if the magnetic field in the layer is strong enough. We again take averages over neighbouring modes and compare the residual after removing the contribution due to rotation from the expected splitting due to the magnetic field and the results are shown in Fig. 1 There is no clear signature of any feature near the base of the convection zone and hence we can only set an upper limit on the strength of the magnetic field in this layer. This limit roughly increases as $1/\sqrt{d}$ and for a half-thickness of $0.05R_\odot$ the upper limit on magnetic field is about 200 kG, a value close to that obtained by Basu (1997) using a similar technique and is also consistent with the value independently inferred by D'Silva & Choudhuri (1993). Note that the tachocline has a much smaller thickness of $0.01R_\odot$ (Basu 1997; Antia et al. 1998) and adopting this thickness, the
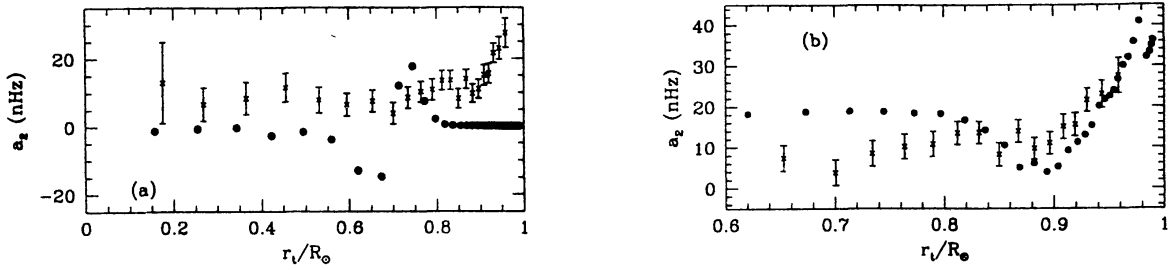

Figure 1. The splitting coefficients from a toroidal magnetic field plotted as a function of the lower turning point of the modes. Each point represents an average over 30 neighbouring modes. The estimated contribution from rotation has been subtracted from the observed splittings marked by crosses. For panel (a) the magnetic field is given by Eq. (1) with $k = 2$, $b_0 = 4 \times 10^5$ G, $d = 0.05R_\odot$, $r_0 = 0.713R_\odot$, while for panel (b) the field is given by Eq. (2) with $\beta_0 = 10^{-4}$, $r_0 = 0.96R_\odot$, $d = 0.1R_\odot$, $k = 2$.

upper limit on the magnetic field would naturally be doubled.

The residual in coefficients a_2 and a_4 appears to peak around $r_t = 0.96\ R_\odot$, and indeed, if this is assumed to be solely due to magnetic field, the field may be distributed around this depth (≈ 28000 km). Note that this is approximately the depth to which the shear layer seen in rotation profile extends (Antia et al. 1998). We now attempt to estimate splittings due to the field concentrated in this region. For this purpose we assume the field to be given by the form,

$$a(r) = \begin{cases} \sqrt{8\pi p_0 \beta_0}\ (1 - (\dfrac{r - r_0}{d})^2) & \text{if } |r - r_0| \le d \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

where p_0 is the pressure and β_0 is a constant which gives the ratio of magnetic to gas pressure at the centre of the region where field is concentrated. Fig. 1b shows the splittings due to a magnetic field of this form. A comparison of these with the observed splittings indicates that there may be an azimuthal magnetic field with $\beta < 10^{-4}$ (i.e., B ≈ 20000 G), with peak around $r = 0.96\ R_\odot$. Evidently, more work is required for checking the field configurations that are

consistent with observations.

References

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