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Rotation rate and flows in the solar interior

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Abstract. The rotation rate in the solar interior can be inferred from measured splittings of solar oscillation frequencies using various inversion techniques. While other large scale flows can be studied using local techniques, like ring diagrams or time-distance helioseismology. These techniques can be used to study variation in flow velocities in the three spatial dimensions as well as the temporal variations.

Key words : Sun : Oscillations, Sun : Interior, Sun : Rotation

1. Introduction

The Sun oscillates simultaneously in many millions of modes, which are characterized by three quantum numbers (n, l, m), where l and m are respectively, the degree and azimuthal order of a spherical harmonic and n is the radial order, which for most modes is the number of nodes in the eigenfunction. In the absence of rotation and magnetic field the frequencies $\nu_{n,l,m}$ of solar oscillations are independent of the azimuthal order m. Rotation lifts the degeneracy and introduces rotational splittings. Since the centrifugal force due to rotation is about five orders of magnitude smaller than the gravitational forces and the rotation rate is also much smaller than the typical frequencies of acoustic modes, rotation can be treated as a small perturbation to the non-rotating model. The first order contribution from rotation arises from the Coriolis term and the resulting splitting varies linearly with the rotation rate in solar interior. The magnitude of splitting depends on the rotation rate in the solar interior and hence the measured splittings can be used to infer the rotation rate as a function of both radial distance and latitude. Further, it turns out that the splittings are sensitive only to the north-south symmetric component of the rotation rate and hence only this component can be inferred from the measured splittings of the global modes.

It is found that for the high degree modes that are trapped in the immediate subsurface layers, the life-time is smaller than the travel time for sound waves around the Sun. As a result, these modes may not be considered to be global. These high degree modes have been employed in local techniques to study large scale flows inside the Sun. These techniques allow us to determine the flow velocities as a function of latitude, longitude, depth and time.

2. Rotation rate in solar interior

To the leading order the rotational splitting in a slowly rotating star can be expressed as (Pijpers 1997)

$$D_{n,l,m} = \frac{\nu_{n,l,m} - \nu_{n,l,-m}}{2m} = \int_0^{R_\odot} dr \int_{-1}^{1} d\cos\theta \, K_{n,l,m}\ (r,\theta)\Omega(r,\theta) \quad (1)$$

where θ is the colatitude, $\Omega(r, \theta)$ is the rotation rate in solar interior and $K_{n,l,m}$ (r,θ) are mode kernels which are determined by the mean spherically symmetric structure and the eigenfunctions for the corresponding mode. The kernels are symmetric in θ about the equator and hence the splittings are sensitive only to the north-south symmetric component of the rotation rate. Further, the kernels and hence the splittings are odd functions of m, since they depend on the direction of propagation of mode in longitude. On the other hand, departures from spherical symmetry in solar structure as well as magnetic field cause frequency splittings that are even functions of m. These even terms are filtered out by taking the difference between modes with $\pm m$ in Eq. (1), which can be used to infer the rotation rate from known frequency splittings.

The frequencies of individual modes $\nu_{n,l,m}$ are not available from all helioseismic observations. More often we get only the splitting coefficients for each n, l, defined by

$$\nu_{n,l,m} = \nu_{n,l} + \sum_{j=1}^{J_{\max}} c_j^{n,l} \mathcal{P}_j^l(m) \quad (2)$$

where $\mathcal{P}_j^l(m)$ are orthogonal polynomials of degree j in m. In this expansion $J_{\max}$ is generally much less than $2l$, thus reducing the number of data points that are available. The odd coefficients c_1, c_3, ... are determined by the rotation rate, while the even coefficients are contributed by second order effect of rotation, magnetic field and any other aspherical perturbation to structure. We can write an equation similar to Eq. (1) connecting the splitting coefficients to the rotation rate in solar interior, which can be used for inversion. These inversions are referred to as 2D inversions and have the advantage that both radial and latitudinal dependence of rotation rate are determined together. However, 2D inversions in general require large computing resources.

Efficiency of inversion programs can be improved by exploiting the form of kernels to obtain a 1D × 1D inversion technique (Pijpers & Thompson 1996). Alternately, for suitable choice of orthogonal polynomials $\mathcal{P}_j^l(m)$, it is possible to separate out the r and θ dependence (Ritzwoller & Lavely 1991) by expanding the rotation velocity in terms of Legendre polynomials. With this choice each component can be determined independently of the rest and the rotation rate can be determined by performing a sequence of 1D inversion for each splitting coefficient C_{2j+1}. This technique is referred to as 1.5 D inversion.

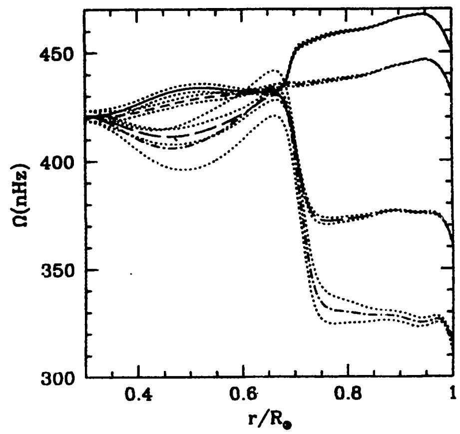

Figure 1. Rotation rate in solar interior as a function of radial distance at latitudes of 0° (continuous line), 30° (short-dashed line), 60° (long-dased line) and 90° (dot dashed line) as inferred using 1.5D inversion of GONG months 4-14 data. The dotted lines mark the error limits for each of these.

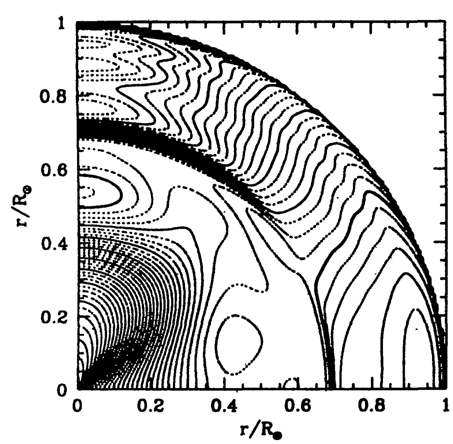

Figure 2. A contour diagram of the solar rotation rate as obtained by 2D inversion of GONG months 4-14 data. The dotted contours have been drawn at intervals of 5 nHz., and the continuous ones at intervals of 20 nHz. The thick continuous line is the contour at a level of 440 nHz.

For 2D or 1.5D inversions one can use either a Regularized Least Squares (RLS) technique or a variant of Subtractive Optimally Localized Averages (SOLA) technique to perform the inversion. All inversion techniques have been tested through a Hare and Hound exercise (Schou et al. 1998), where two sets of artificial data were used for inversion. These data were produced by the Hare using some artificial rotation rate to calculate the splitting coefficients, to which random errors with same distribution as those in real MDI (Michelson Doppler Imager) data were added. The splittings so obtained were supplied to the teams of Hounds, who tried to infer the rotation rate which was unknown to them and then the results were compared against the actual rotation profile used in constructing the data. All the methods succeeded in recovering the rotation rate in the solar interior, except in regions at high latitude and the solar core.

Having tested these inversion techniques we can use them with the real data from GONG (Thompson et al. 1996) or MDI (Schou et al. 1998) to infer the rotation rate in solar interior and the results are shown in Figures 1 & 2. These results show a distinct shear layer just underneath the solar surface where the rotation rate increases with depth. The rotation rate at solar surface inferred from helioseismic inversions agrees well with that obtained from Doppler measurements, but is slightly less than that inferred from tracking of magnetic features. This is probably due to the fact that magnetic features are anchored at a layer below the solar surface where the rotation rate is larger. The surface differential rotation persists through the solar convection zone, while below the base of convection zone the rotation rate appears to be relatively independent of latitude. The transition region near the base of the convection zone is referred to as the tachocline and it is centered at a radial distance of $(0.7050 \pm 0.0027)R_\odot$ with a half width of $(0.0098 \pm 0.0026)R_\odot$ (Basu 1997). There is no definitive evidence for latitudinal variation in the position or width of the tachocline (Antia et al. 1998). The origin of shear layer in the tachocline is not clear and may have some bearing on the theory of solar dynamo. In the radiative interior rotation is slower than that at equatorial region near solar surface.

3. Large scale flows in solar interior

Local helioseismic techniques have been developed for the study of large scale flows. Time-distance helioseismology is based on the travel time of acoustic waves between different points on the solar surface. This travel time is determined by using cross-correlation functions (Duvall et al. 1993). Travel time also depends on the flow velocities in the region being studied and from the analysis of the observed time-distance diagram it is possible to infer the flow velocities in solar interior (Duvall et al. 1997). Using this technique the flow pattern in a supergranule has been determined to extend below the solar surface to a depth of 2000 km. Similarly, meridional flow with a maximum value of about 25 m/s has been detected to extend up to a depth of at least 25000 km (Giles et al. 1997).

Instead of time-distance we can also use the ring diagram technique (Hill 1988; Patron et al. 1997) to study large scale flows. Although this technique has lower spatial resolution, it is much simpler to implement. Ring diagram analysis is based on the study of 3d power spectra of solar p-modes on a part of the solar surface. The frequencies of these modes are affected by horizontal flow fields suitably averaged over the region under consideration, hence an accurate measurement of these frequencies will contain the signature of large scale flows and can be used to study these flows. The measured frequency shifts for different modes can be inverted to obtain the horizontal flow velocities as a function of depth. The local nature of these modes allow us to study different regions on the solar surface, thus giving a three dimensional information about the horizontal flows. Since the high degree modes used in these studies are trapped in the outermost layers of the Sun, such analysis gives information about the conditions in the outer 2-3% of the solar radius. This study also shows a meridional flow from equator polewards with a maximum amplitude of about 30 m/s near the surface (Basu et al. 1999). The dominant component of meridional flow has the form $\sin(2\theta)$, but higher order components are also significant at intermediate depths. The outer shear layer can be studied in greater detail using the ring diagram technique.

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