Bull. Astr. Soc. India (2000) **28**, 63-69

The seismic sun

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Abstract. The "Seismic Sun" is a model of the Sun constructed using helioseismological deductions about the Sun. The seismic Sun is very similar to a standard solar model which incorporates gravitational settling of helium and heavy elements. The main discrepancy is just below the base of the convection zone and can be attributed to the lack of mixing in standard solar models.

Key words : Sun : Oscillations, Sun: Interior

1. Introduction

The "Seismic Sun" is a model of the Sun that is made using helioseismological deductions about the Sun. It can still be called a 'model' since it is constantly being refined as more and more data become available. In this article I shall concentrate only on Solar structure. Solar rotation is described elsewhere in this volume (cf. Antia, this volume).

Helioseismology is the study of the Sun using solar oscillation frequencies. The Sun oscillates in millions of modes. The modes are linear - the velocity amplitudes are small (5 to 10 cm/s at the surface) compared to the solar sound-speed (9 km/s at the surface). They are adiabatic to a good approximation. The time scale of the oscillations being of the order of 5 min, which is much smaller than the Kelvin-Helmholtz time scales for the Sun ($\simeq 10^7$ years), and hence, heat transfer during each oscillation period can be neglected in most of the solar interior.

Since the Sun is a spherical body (the departures from sphericity have been measured to be very small), it is most natural to describe the angular dependence of the normal-modes in terms of spherical harmonics. Each mode is described by its radial order n, which is the number of nodes in the radial direction, the degree ℓ, where $L = \sqrt{\ell(\ell+1)}$ is roughly speaking the number of wavelengths along the solar circumference, and the azimuthal order m that measures the number of nodes along the equator. The numbers n , ℓ and m describe the mode completely and determine its frequency ν (or $\omega \equiv 2\pi\nu$). In the absence of rotation or any other agent such as magnetic field to break spherical symmetry, all modes with the same n, and ℓ have the same frequency. Thus each (n, ℓ) multiple is $2\ell + 1$ fold degenerate. In the case of the Sun, rotation, magnetic field and other large scale flows and asymmetries are small, hence these can be assumed to be perturbations. Thus for the Sun, the mean frequency of an (*n*, *ℓ*) multiple is a function of solar structure alone. These can be used to probe the structure of the Sun. A brief history of the subject and a straight forward description of observing techniques can be found in the article by Hill et al. (1991).

There are a number of observational projects that are involved in determining solar oscillation frequencies. Chief among this is the ground-based, six-station network, the Global Oscillation Network Group (GONG) (cf. Harvey et al. 1996) and the GOLF, VIRGO and MDI experiments on board the Solar and Heliospheric Observatory (SOHO) (cf. Domingo, 1995).

2. The inverse problem

It is known that there is no solar model yet whose frequencies match that of the Sun (cf. Christensen - Dalsgaard et al. 1996). As a result one normally uses the inverse approach to determine the structure of the Sun. Since solar frequencies are determined by solar structure, the inverse approach involves using the solar frequencies to determine solar structure.

The equations describing linear, adiabatic stellar oscillations are known to be Hermitian (Chandrasekhar 1964). This property of the equations is used to relate the differences between the structure of the Sun and a known reference solar model to the differences in the frequencies of the Sun and the model by known kernels. Non-adiabatic effects and other errors in modeling the surface layers give rise to frequency shifts (cf., Cox and Kidman 1984) that are not accounted for by the variational principle. In the absence of a more fundamental method, these surface effects have been treated by the *ad hoc* procedure of including an arbitrary function of frequency in the variational formulation.

When the oscillation equation is linearized - under the assumption of hydrostatic equilibrium -the fractional change in the frequency can be related to the fractional changes in the squared sound-speed (c^2) and density (ρ).Thus,

$$\frac{\delta\omega_i}{\omega_i} = \int K^i_{c^2,\rho}(r)\frac{\delta c^2}{c^2}(r)dr + \int K^i_{\rho,c^2}(r)\frac{\delta\rho}{\rho}(r)dr + \frac{F_{surf}(\omega_i)}{E_i} , \qquad (1)$$

(cf., Dziembowski et al. 1990). Here $\delta\omega_i$ is the difference in the frequency ω_i of the ith mode between the solar data and a reference model. The kernels $K^i_{\rho,c}{}^2$, and $K^i_{\rho,c}{}^2$ are known functions of the reference model which relate the changes in frequency to the changes in c^2 and ρ, respectively; and E_i is the inertia of the mode, normalized by the photospheric amplitude of the displacement. The term F_{surf} results from the near-surface errors. Once $\delta c^2/c^2$ and $\delta\rho/\rho$ between a model and the Sun is known, we can easily calculate c and ρ for the Sun using the known sound speed and density profiles of the models.

A number of different methods can be used to determine $\delta c^2/c^2$ and $\delta\rho/\rho$ using Eq. 1. The most common of these are the regularized least squares (RLS; see Antia & Basu 1994 for an implementation) and the Subtractive Locally Optimized Averages (SOLA; cf. Pijpers & Thompson 1992. See Basu et al. 1996 for implementation).

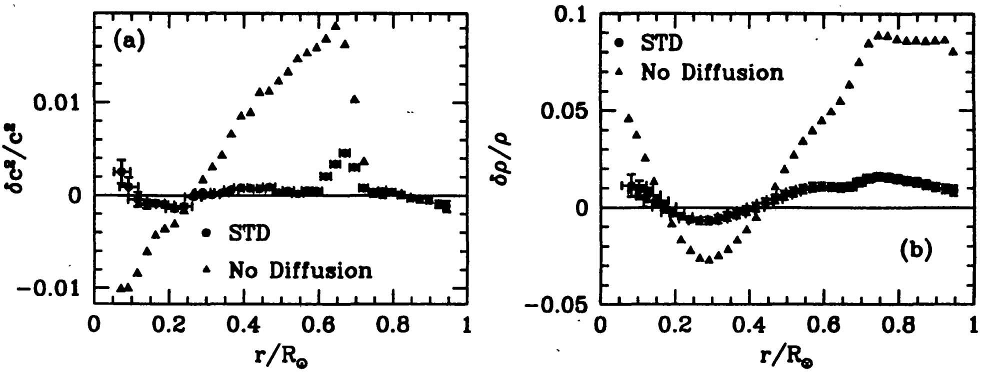

Figure 1. The relative sound-speed and density differences between two solar models and the Sun as obtained by inverting MDI data. The error-bars are 1σ errors have not been shown for the second model for the sake of clarity. Note the extremely small differences in the relative sound-speed.

3. Inversion results

Fig. 1 shows the sound-speed and density differences between two solar models and the Sun obtained by inverting solar oscillations frequencies from the MDI instrument on board SOHO (Rhodes et al. 1997).

The model STD is an up to date standard solar model. It was constructed using the latest OPAL opacities (Iglesias & Rogers 1996) and OPAL equation of state (Rogers et al. 1996) and incorporates the diffusion of helium and heavy elements below the solar convection zone. The second model is identical to STD in terms of physical inputs, but does not incorporate diffusion. It is clear from the figure that inclusion of diffusion brings the structure of the model very close to that of the Sun. Thus it is clear that diffusion of elements is an important ingredient of stellar evolution. There are a number of consequences of including diffusion. First is that the helium abundance observed in the solar envelope is less than that the helium abundance the Sun was born with. Thus corrections need to be made when estimating primordial helium abundance by extrapolating the helium abundance of stars. A second consequence is that models with diffusion have a lower hydrogen abundance in the core than models without. This implies a decrease in the main-sequence lifetime of the model which results in the lowering of estimated ages of globular clusters (cf. Chaboyer et al. 1996).

The sound-speed difference between solar models can be used to determine other properties of the Sun such as the precise depth of the convention zone (CZ) and the helium abundance in the solar envelope. The temperate gradient in the CZ is larger than the temperature gradient in the radiative zone. As a result models with a deeper convection zone will have a larger sound-speed at the CZ base than models with a shallower CZ (since sound speed $c \propto T/\mu$, T being the temperature and μ the mean molecular weight). Thus the relative sound-speed difference between two models with different CZ depths will show a jump at the CZ base. Fig. 2 shows the sound-speed difference between the Sun and five solar envelope models which differ only slightly in the depth of their convection zones. By calibrating this sound-speed difference one finds that the base of the solar convection zone is at a radius of $(0.7135 \pm 0.0005)\,R_\odot$ (cf. Basu 1998).

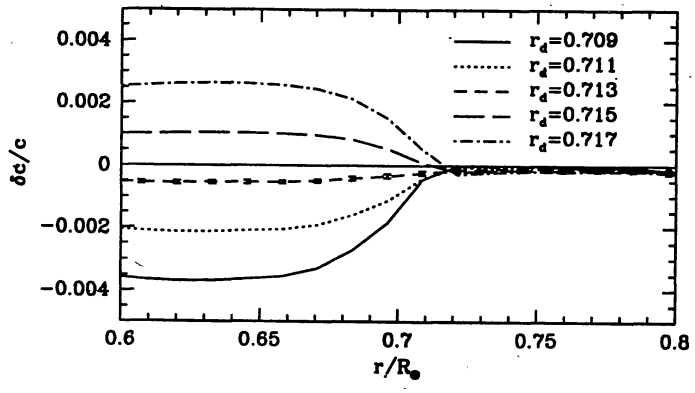

Figure 2. The relative sound-speed differences between Sun and solar models with different depths of CZ, r_d.

Similar methods have been used to determine the helium abundance in the solar envelope (cf. Basu, this volume) and details of the method can be found in Basu & Antia (1995). Helium ionizes at a radial distance of about $0.98\,R_\odot$ reducing the local sound speed. This reduction can be calibrated to determine the helium abundance. The value of the solar helium abundance obtained unfortunately depends on the equation of state used to construct the calibration models. For OPAL equation of state, one finds that the helium abundance in the solar envelope is around 0.247 (Basu, this volume; Basu 1998).

The importance of diffusion in a solar model is also brought out by the fact that only models which incorporate diffusion have envelope helium abundance and CZ depths which match solar values. Models without diffusion have very shallow CZ depths and large helium abundance.

The largest discrepancy between the sound speed of the Sun and the standard solar model occurs just below the CZ base. This can be attributed to the accumulation of helium below the CZ base in the models due to diffusion. The excess helium increases the local mean molecular weight, thereby decreasing the sound-speed. Thus it appears that although diffusion of elements in the Sun is important, some process, probably mixing keeps helium from accumulating at the solar CZ base. There is other evidence also that suggests that the helium abundance profile in the Sun is smoother than that in standard solar models (Basu & Antia 1994; Basu 1997). In fact secondary inversions, i.e. inversions of solar frequencies with additional inputs such as opacity, show that the solar helium abundance is indeed smoother than that of a standard solar model (Antia & Chitre 1998). Such a helium abundance results in a model without the large discrepancy in sound speed at the solar CZ base. Figure 3(a) shows the hydrogen abundance profile below the CZ base for the Sun as obtained by Antia & Chitre (1998), marked AC, and the standard solar model. Fig. 3(b) shows the sound-speed difference between the Sun and the model which has the same abundance profile as the Sun as well as for the standard solar model.

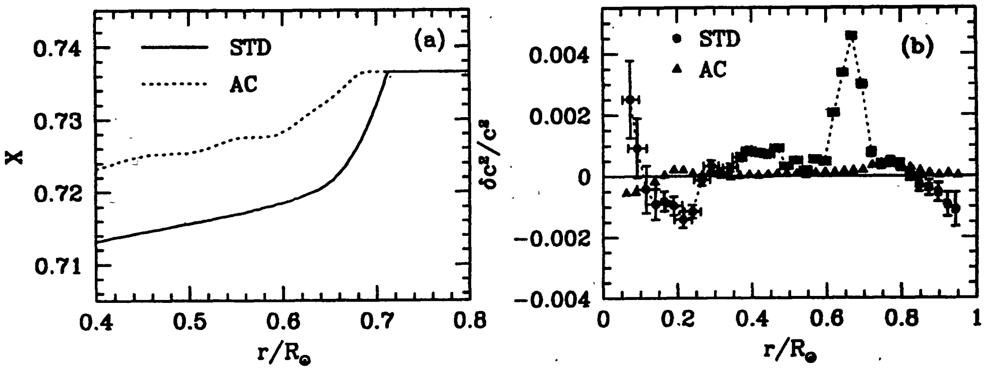

Figure 3. (a) The Hydrogen abundance profile in the Sun as obtained from inversions (marked AC) and a standard solar modelk (marked STD). (b) The relative squared sound-speed differences between the Sun and the Standard solar model (STD) and that of a model constructed with the solar hydrogen abundance profile obtained by inversion (marked AC). Note that AC does not show the sound-speed discrepancy at the CZ base. For the sake of clarity error-bars have not been shown for AC. They are similar to that for STD.

The model with the solar abundance profile is a model of the Sun today. It is still not clear what sort of mixing process operated over the lifetime of the Sun to result in such a profile. However, it appears that rotationally induced mixing is a good candidate. In fact models which incorporate mixing due to rotation (cf. Richard et al. 1996) show a smaller discrepancy in the sound speed than a standard solar model.

Helioseismic techniques have also shown that there is a very low upper limit of 0.05 H_p ($\simeq$ 2800 km) on convective overshoot below the solar convection zone (Basu & Antia 1994, Monteiro et al. 1994, Basu 1997).

Helioseismic inversions can also be used to test physical inputs into solar models, e.g. the equation of state. Solar oscillation frequencies are affected by the equation of state of solar material through the adiabatic index Γ_1. The relative difference in Γ_1 between the Sun and any model can be inverted to demonstrate that the equation of state used in the solar model is not the same as that of solar material. unfortunately, Γ_1 is not determined by the equation of state alone and it is also affected by structure and composition. However, recently Basu & Christensen-Dalsgaard (1997) showed that it is possible to separate the equation of state part of the Γ_1 difference from the structure part making it possible to test the equation of state.

In Fig. 4 we show this difference in Γ_1 between the Sun and the four models constructed with different equations of state. These are (1) The EFF equation of state (cf. Eggleton et al. 1973) (2) The Coulomb corrected EFF or CEFF equation of state (Christensen-Dalsgaard & Däppen 1992) (3) The MHD equation of state (Däppern et al. 1988) and (4) The OPAL equation of state (Rogers et al. 1996). Note that the EFF equation of state is most different from solar, particularly at the helium ionizatioin zone. The CEFF does much better, however it is not thermodynamically consistent. The two more sophisticated equations of state, MHD and OPAL do quite well. Detailed analyses however show that OPAL is better than MHD below the CZ base (Dziembowski et al. 1992, Basu & Antia 1995), though closer to the surface MHD is better (Basu et al. 1999).

4. Conclusions

Helioseismology has allowed us to study the structure of the Sun with amazing precision. The "Seismic" model that results shows that standard solar models have a structure which is fairly close to the Sun. The sound-speed difference is of a order of a few hundredths of a percent while the density difference is of the order of a few percent. The Sun however, has a smoother composition gradient below the CZ compared with the standard models. This points to mixing of elements below the CZ base. The CZ base of the Sun is at a radius of (0.7135±0.0005) $R_\odot$ and the upper limit to overshoot below the CZ base is $0.05H_p$.

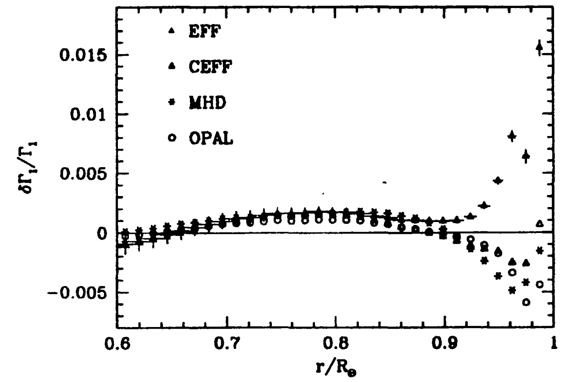

Figure 4. The relative differences in the adiabatic index Γ_1 between the Sun and four solar models constructed with different equations of state.

This work utilizes data from the SOI/MDI on the Solar and Heliospheric Observatory (SOHO). SOHO is a project of international cooperation between ESA and NASA.

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