POLARIZATION MEASURES OF COMET AREND-ROLAND (1956h) AND COMET MRKOS (1957d)

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Summary

Measurements of polarization of Comet Arend-Roland and of Comet Mrkos made photoelectrically, using interference filters and conventional *B*, *V* filters are given. The interference filters were chosen to isolate the Na emission at 5890 A and the continuum at 4800 A and 4300 A. The polarization at 4800 A in both comets is similar yielding a mean value of 20·1 per cent computed for a phase angle of 90°. This indicates that the agencies responsible for the continuum were identical in both comets. The polarization of Na emission calculated for a phase angle of 90° for Comet Mrkos was 37·4 per cent in agreement with the theoretical value for Na resonance radiation.

1. *Introduction.*—The polarization of the light of comets was first detected by Arago early in the nineteenth century. Subsequently, Secchi, Wright and others have made visual observations on many comets and have confirmed and extended the earlier results. These indicated fluctuations in amount of polarization similar to those observed in brightness and colour.

The first systematic efforts to utilize polarization data on comets towards an understanding of the mechanism responsible for the origin of the light of comets were made by Öhman (**1**). Using a polarigraph in combination with an objective prism and telescope, he examined Comet Cunningham (1940c) and Comet Paraskevopoulos (1941c). A monochromatic study of this kind furnished data separately on the amount of polarized light in the continuum and in the light of the emission bands. In Comet Cunningham the emission spectrum was very intense as compared to the continuum, and the polarization measured in the light of the Swan bands agreed with what one would expect on the assumption that a fluorescence mechanism is responsible for the origin of the bands. Comet Paraskevopoulos on the other hand had a very strong continuous spectrum with hardly any bright bands present. Polarization measures indicated the dominant role played by scattering of light by small particles in the latter comet.

The recent appearance of the two bright comets Comet Arend-Roland (1956h) and Comet Mrkos (1957d) provided a good opportunity at many observatories in the northern hemisphere for detailed studies in cometary physics. At Naini Tal we have carried out measures of the polarization of the nuclei of the two comets, as well as measuring their magnitudes and colours on a few nights. These measures were made photoelectrically, using interference filters for isolating selected portions of the spectrum in addition to the conventional filters employed in present-day photoelectric photometry. A preliminary account of some of our polarization data regarding Comet Arend-Roland was given earlier (**2**). Since the publication of these results Blackwell and Willstrop (**3**) have given an account of a similar study of Comet Arend-Roland employing photographic methods and combinations of broad passband filters to give near-monochromatic measures. For Comet

Mrkos, Hoag (4) has measured the polarization of the coma, as well as of selected portions of the tail, in integrated light, using a 1P21 photomultiplier with no filters.

2. *Observing technique.*—The 10-inch Cooke refractor of focal length 148 inches was used in conjunction with a photoelectric photometer. The light passed successively through a diaphragm in the focal plane, filters placed two inches behind it and a Fabry lens before reaching an unrefrigerated selected 1P21 photomultiplier tube. A microscope inserted behind the diaphragm, at the will of the observer, enabled the checking of proper centring on the object. The diaphragm consisted of an eccentrically located circular six-position disk containing six different circular apertures, each of which could be brought into position centred on the optical axis by rotation. Three consecutive positions of the disk presented three apertures of increasing size. Of these, the smallest opening of diameter 2·15 mm was used in the measurement of magnitudes and colours. This opening admitted all of the coma and an unavoidable portion of the tail. In all measures of magnitudes and colours the coma was centred carefully in the aperture. The remaining three positions of the disk contained three nearly identical circular apertures of diameter 2·10 mm. These openings are exactly sixty degrees apart and notches provided on the outside of the disk ensure the shifting of each opening rapidly and precisely into position. Across these three successive openings, a sheet of polaroid was cemented on to the disk. Rotation of the disk, so as to bring each of the three openings in turn on to the beam, acts as the equivalent of a single polaroid turned successively in position angle through sixty degrees. It is well known that three measures of polarized light, transmitted by an analyzer shifted through three positions sixty degrees apart, give uniquely the plane of vibration of the incident light as well as the percentage polarization, provided the orientation of the polaroid in the photometer is known.

The output of the photomultiplier is fed to a D.C. amplifier similar to the one described by Valley and Wallman (5) but with certain modifications introduced by Whitford and Johnson. The amplified signal is conveyed either to a Brown recording potentiometer or to an Esterline-Angus recorder. The former was used for Comet Arend-Roland, while the latter was used for measures on Comet Mrkos. Calibrations of coarse and fine sensitivity ratios of the amplifier were carried out frequently in combination with either of the recorders.

The polarization data obtained directly from the recorder tracings need correction before yielding final values of planes of polarization and percentage polarization. Two factors necessitate the use of such corrections. These are the non-identical areas of the three apertures behind the polaroid and the varying sensitivity of the photometer to different planes of vibration of the incident light. The first factor can be made very small, so as to be negligible. The second can be compensated for by using proper correction factors. These correction factors were obtained by measuring bright nearby stars that had no polarization. Table I gives the correction factors used for Comet Arend-Roland and also those used for Comet Mrkos, along with their probable errors. The different correction factors for the two comets became necessary since a fresh polaroid was used for measures of Comet Mrkos. The correction factors used for Comet Arend-Roland are the weighted means of 15 values obtained with six different filters. Those for Comet Mrkos were obtained from measures through two filters on two different nights.

The orientation of the polaroid in the photometer was determined by a laboratory arrangement with the aid of an artificial light source and a Nicol prism. We

TABLE I

Correction factors used for Comets Arend-Roland (1956h) and Mrkos (1957d)

Comet	Correction factors Polaroid position I	Polaroid position II	Polaroid position III
Arend-Roland	0·9714 ± 0·0028	1·0000 ± 0·0000	0·9418 ± 0·0109
Mrkos	0·9479 ± 0·0039	1·0000 ± 0·0000	1·0084 ± 0·0038

believe that the error in transferring these measures to the telescope does not exceed one degree.

During our observing run on Comet Mrkos we carried out test measures of two stars that have noticeable polarization and whose polarization has previously been determined independently by Hiltner (**6**), Smith (**7**) and Hall and Mikesell (**8**). We planned these measures to act as checks on the instrumental performance from night to night. It is unfortunate that we do not have measures on a large number of stars for which polarization data exist. As such we cannot make a distinct comparative study of the efficiency of the different methods used. The results obtained on these stars are given in Table II. Those for Rho Ophiuchi represent a mean value of the results obtained on three nights. HD 154445 was observed on only one night. The data are too meagre for any study of systematic errors, but it

TABLE II

Values of polarization of Rho Ophiuchi and HD 154445 obtained by different investigators

Investigators	HD 147888 (ρ Oph) P (per cent)	θ (degrees)	HD 154445 P (per cent)	θ (degrees)
Hiltner	3·0	53	3·4	91
Hall	3·5	51	3·0	85
Smith	2·8	52	3·5	88
Bappu and Sinvhal	4·2 ± 0·68	54	4·5	95

does seem that the percentage polarization values obtained by us are systematically higher than those obtained by others by about one per cent. The position angles of the plane of vibration agree well within the errors of measurement. The errors of observation signified by the probable errors are very reasonable. In general, for the cometary measures we estimate the probable error of a value of percentage polarization to be less than ± 1 per cent, and that of a value of plane of vibration to be within ± 5 degrees.

3. *Polarization of Comet Arend-Roland.*—For obtaining measures of polarization at selected wavelengths we have employed interference filters. These are Bausch and Lomb second order filters having a peak transmission of 35 per cent and width at half intensity of 80 A.

For Comet Arend-Roland we measured through such filters centred on 5890 A, 5000 A, 4800 A and 4300 A respectively. In an earlier publication (**2**) we mentioned having used an interference filter centred on 4700 A to isolate the C_2(1, 0) emission of the Swan system. We had depended on the manufacturer's calibration data and assumed that the filter was actually centred at 4700 A. Subsequent calibration in our laboratory indicated that the filter had its peak transmission at 4800 A. As such, much to our dismay, we have not been able to measure the polarization of any of the Swan bands in both comets. The 4800 A filter, however, transmits the continuum, without any contamination being caused by neighbouring emission. Fig. 1 shows the curve of intensity distribution in the cometary spectrum as on 1957 April 30·1. The transmission curves of two of the interference filters used (viz., those centred on 4300 A and 4800 A) for Comet Arend-Roland, drawn on an arbitrary scale for ordinates, are also given. These latter curves are meant to demonstrate only the wavelength regions isolated by the filters. The 5000 A filter picks up a portion of the C_2 (0, 0) band. Similarly the 5890 A filter isolates mostly the continuum and any sodium emission, if present.

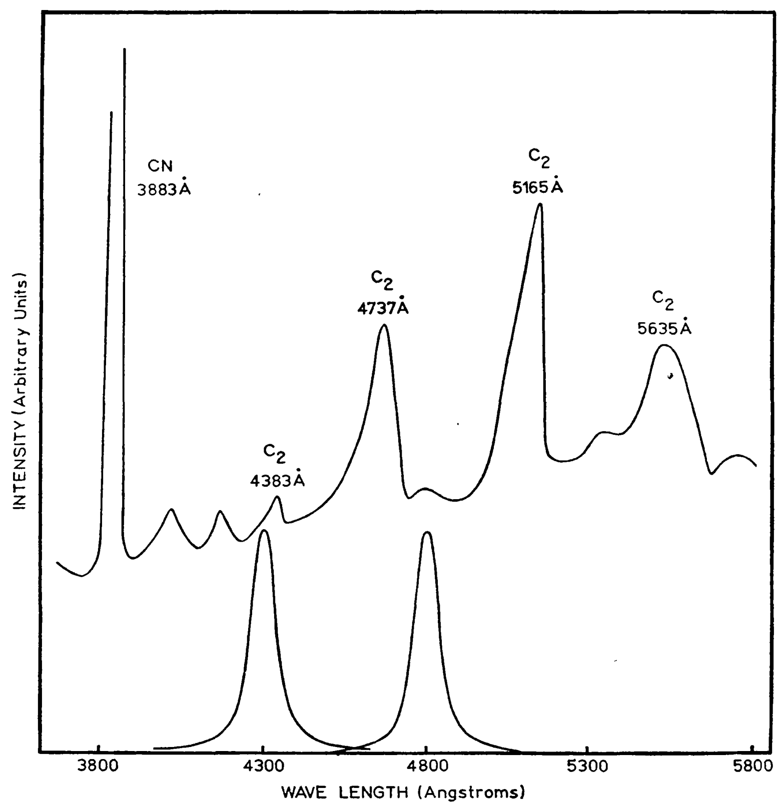

FIG. 1.—*Intensity distribution in the spectrum of Comet Arend-Roland on 1957 April 30.1. The two lower curves represent the transmission characteristics of two of the interference filters used. The ordinates of these latter curves are also on an arbitrary scale.*

In the case of Comet Arend-Roland our measures started only after the sodium emission nearly ceased to exist and hence the values obtained through this filter indicate essentially the polarization of the continuum. We had no filter readily available for the elimination of any third order contamination of the radiation transmitted by this filter and hence it is likely that a small fraction of CN emission at 3883 A has leaked through. The values of polarization obtained with the 5890 A filter compare well with those of 5000 A and 4300 A. As such the contribution of 3883 A to the measures of 5890 A must be considered to be negligible.

The values of per cent polarization and position angle of plane of vibration obtained through interference filters are given in Table III along with phase angles

11*

calculated from the ephemeris of Candy (9). By May 9 the comet had gone beyond the range of our telescope—polaroid—interference filter combination. The visual magnitude of the comet at this time was 7·3. The normal limiting magnitude for measures of precision with interference filters on the 10-inch refractor is 9·0. The absorption of light by the polaroid thus caused a loss of over one and a half magnitudes.

TABLE III

Polarization measures of Comet Arend-Roland (1956h) through interference filters

Date 1957 May	Amount of polarization Position angle of plane of vibration, α 5890 A	5000 A	4800 A	4300 A	Phase angle	Position angle of intensity equator, β	Mean α−β
4·658	17·5% 103°	...	17·9% 107°	...	76° 24′	37° 48′	67°
5·641	17·4% 116°	18·0% 112°	20·4% 114°	15·6% 115°	73° 47′	40° 01′	76°
6·648	19·7% 119°	15·0% 114°	17·3% 119°	16·6% 111°	71° 12′	42° 12′	74°
7·655	12·9% 121°	14·1 % 124°	16·8% 122°	16·9% 125°	68° 48′	44° 16′	79°
8·660	20·5% 123°	20·0% 122°	14·5% 135°	...	66° 36′	46° 13′	80°

TABLE IV

P_{90} values of Comet Arend-Roland (1956h)

Wavelength	P_{90} (mean value) per cent
5890 A	20·17 ± 2·34
5000 A	19·53 ± 2·17
4800 A	20·30 ± 0·62
4300 A	18·66 ± 0·78

TABLE V

Polarization measures of Comet Arend-Roland (1956h) through B, V filters

Date 1957 May	*V* filter: Amount of polarization (per cent)	*V* filter: Position angle of plane of vibration α	*B* filter: Amount of polarization (per cent)	*B* filter: Position angle of plane of vibration α	Phase angle	Position angle of intensity equator β	Mean α−β
4·658	18·9	104°	...	...	76° 24′	37° 48′	66°
5·641	18·3	111°	19·6	112°	73° 47′	40° 01′	71°
6·648	19·9	116°	15·9	116°	71° 12′	42° 12′	74°
7·655	17·4	118°	16·7	117°	68° 48′	44° 16′	73°
8·660	14·5	119°	16·5	122°	66° 36′	46° 13′	74°
11·656	12·8	128°	...	...	60° 41′	51° 21′	77°
14·662	14·0	147°	15·2	145°	55° 45′	55° 25′	91°
15·662	15·3	146°	16·5	145°	54° 16′	56° 32′	89°
22·647	9·6	146°	14·4	145°	45° 44′	62° 00′	84°
26·650	6·2	148°	10·7	152°	41° 54′	63° 37′	86°

Assuming that the relation between amount of polarization and phase angle is given by

$$P_\theta = \frac{P_{90} \sin^2 \theta}{1 + P_{90} \cos^2 \theta},$$

where P_θ and P_{90} are the values of percentage polarization at phase angles $\theta°$ and 90° respectively, we have calculated the mean values of P_{90} for the comet through the different filters. These are given in Table IV. The above relationship is valid essentially for fluorescence phenomena, and perhaps it is incorrect to use it for any of the wavelengths transmitted. However, it can be used as a convenient extrapolation formula. To this end, Table IV gives the most likely values of polarization at a phase angle of 90° for the different wavelengths.

For Comet Arend-Roland we have measures of polarization obtained through conventional *B*, *V* filters. These are given in Table V. We have used these filters so as to be able to follow the comet over a large range of phase angle. The broad pass-band filters, while unfit for a determination of the role of fluorescence in the origin of the emission bands, serve well in studies of the general polarization characteristics of comets. Our measures of the polarization through *B* and *V* filters serve to find a correlation between the change in position angle of the intensity equator and the position angle of the plane of vibration. The plane of vibration is nearly perpendicular to the intensity equator after May 14. Until May 14 it is tilted to the normal to the intensity equator by an angle larger than the probable error. We do not believe that moonlight has interfered with the measures because the sky measures made during the dark of the Moon period are similar to those made while the Moon was past first quarter. This indicates that the contribution by moonlight to the overall intensity was negligible. In particular, the observations of May 4·658 through interference and *B*, *V* filters show the deviation of the plane of vibration from the position angle of the normal to the intensity equator to be largest. The Moon was only four days old at the time and the sky deflections were less than 0·3 of a Brown recorder division when measured through interference filters and polaroid. This value just exceeds the errors of estimation in reading of the Brown recorder charts. As such we are confident that the large deviation between the position angle of the plane of vibration and the normal to the intensity equator observed on certain days is real and is not something caused by any residual effects of uncorrected sky polarization. On the basis of this inference regarding the plane of vibration of May 4·658, we may conclude that the deviations marked until May 14 do not arise from spurious effects due to moonlight. Using the relation mentioned previously, the P_{90} values calculated through the *B*, *V* filters are the following:

$$P_{90}(B) = 20{\cdot}71 \text{ per cent} \pm 1{\cdot}33,$$
$$P_{90}(V) = 20{\cdot}17 \text{ per cent} \pm 1{\cdot}84.$$

Within the limits of measurement these compare well with P_{90} values obtained for the continuum regions centred on 5890 A, 4800 A and 4300 A.

4. *Polarization of Comet Mrkos.*—News of the discovery of Comet Mrkos reached us during the peak of the monsoon period of cloudy weather. However, on four nights in August we had clear skies which enabled us to get some measures on the comet when its heliocentric distance was still less than 0·7 astronomical units. We had to limit ourselves to fewer filters for measurement because an

observing run on the comet on any of these four nights lasted only forty-five minutes, by virtue of the low declination of this comet. On all four nights we had observations through filters that transmit the regions centred on 5890 A, 4800 A and 4300 A. The 5890 A interference filter had along with it a Chance OY2 filter, which eliminated any possible contamination from the former's third order. The remaining two filters centred on 4800 A and 4300 A were the same interference filters used in the study of Comet Arend-Roland.

The polarization results of Comet Mrkos are given in Table VI along with values of the phase angle of the comet and the position angle of the intensity equator. The per cent polarizations at 4800 A and 4300 A are almost identical. The per cent polarizations observed at 5890 A on August 19, 20 and 22 are systematically higher than those measured at 4800 A and 4300 A. This is due to the presence of Na emission. Spectrophotometric tracings obtained by Dr William Liller of the University of Michigan indicate the presence of Na emission even on August 23·04. Judging from our polarization data, we presume that Na emission must have been fairly strong on August 19.

TABLE VI

Polarization measures of Comet Mrkos (1957*d*) *through interference filters*

Date 1957 August	Amount of polarization Position angle of plane of vibration, α 5890 A	4800 A	4300 A	Phase angle	Position angle of intensity equator, β	Mean $\alpha-\beta$
19·618	18·6% 145°	14·7% 149°	14·5% 152°	65° 21′	60° 49′	88°
20·618	16·7% 148°	14·6% 155°	13·1% 158°	64° 40′	63° 35′	90°
21·620	15·7% 154°	15·6% 156°	16·5% 163°	63° 25′	66° 05′	92°
22·618	17·1% 161°	16·4% 166°	15·5% 166°	62° 09′	68° 24′	96°

TABLE VII

P_{90} *values of Comet Mrkos* (1957*d*)

Wavelength	P_{90} (mean value) per cent
5890 A	22·02 ± 0·83
4800 A	19·78 ± 1·00
4300 A	19·21 ± 1·35

We find for Comet Mrkos small-scale fluctuations from night to night in all wavelengths, similar to those observed in the case of Comet Arend-Roland. These are, however, superimposed on the overall pattern of variation of percentage polarization with phase angle.

The values of position angle of plane of vibration, for all three wavelengths, agree remarkably well within the probable errors of their determination. These position angles are perpendicular to those of the intensity equator. For Comet Arend-Roland we had found the angle between the two position angles to differ considerably from ninety degrees for those phase angles at which Comet Mrkos was observed.

On one night we were able to carry out observations through a V filter. This was on August 22·618 and yielded a polarization of 15·6 per cent and a plane of vibration having a position angle of 162°.

The calculated mean values of P_{90} for Comet Mrkos are given in Table VII. Those for 4800 A and 4300 A are in agreement with those obtained for Comet Arend-Roland. The value of P_{90} for 5890 A is higher than those for the other two wavelengths. This is caused by Na emission. A value of P_{90} for thi wavelength has intrinsically very little significance of its own, except to indicate the role played by Na resonance radiation. This is so because the percentage polarization at 5890 A, between phase angles of ninety degrees and sixty degrees for Comet Mrkos, is the weighted mean of the polarization of the continuum and the Na radiation. The intensity of the Na radiation is very much a function of the heliocentric distance of the comet.

5. *Magnitudes and Colours.*—The comparison stars used for the two comets are given in Table VIII. The magnitudes and colours were derived by individual comparisons and by using a mean extinction coefficient for the reductions. Because of the low altitude at which the observations were made, the probable error for a magnitude determination lies around $\pm 0^m{\cdot}04$ and for a colour around $\pm 0^m{\cdot}02$. The derived magnitudes and colours are given in Table IX.

TABLE VIII

Details of comparison stars used for determining magnitudes and colours of Comets Arend-Roland (1956h) and Mrkos (1957d)

Comet	Star	R.A. (1900.0)	Dec. (1900.0)	V	$B-V$	HD spectral type
		h m	° ′	m	m	
Arend-Roland	α Cam	4 44·1	+66 10	4·45*	+0·09*	B0
	β Cam	4 54·5	+60 18	3·92*	+0·89*	G0p
	δ Aur	5 51·3	+54 17	3·69†	+0·99†	K0
	γ U Ma	11 48·6	+54 15	2·43†	+0·00†	A0
Mrkos	λ Ser	15 41·6	+07 40	4·43†	+0·60†	G0
	ρ Boo	14 27·5	+30 49	3·57†	+1·29†	K0

* Derived photoelectrically. † Taken from reference (11).

TABLE IX

Magnitudes and colours of Comets Arend-Roland (1956h) and Mrkos (1957d)

Comet	Date 1957	V	$B-V$
		m	m
Arend-Roland	May 7.699	7·20	0·84
	8.620	7·29	0·90
	11·645	6·55	0·91
	14.643	7·02	0·96
	22.628	7·87	0·90
	26·627	8·27	0·90
			Mean 0·90
Mrkos	Sept. 16.591	6·50	0·90
	18·600	6·65	0·88
			Mean 0·89

6. *Discussion.*—In this investigation we have tried to determine the following characteristics of polarization of comets :—

(1) the percentage polarization of radiation in the continuum and its variation with phase;

(2) the orientation of the planes of polarization for the different wavelengths studied and their variation with phase; and

(3) the percentage polarization of Na emission.

For a study of the first characteristic we have utilized the radiation transmitted through the 4300 A, 4800 A and 5890 A filters. The 5890 A values of polarization can be used for the continuum only when the sodium emission is totally absent or is negligible. The transmission through the 5000 A filter is partly contaminated by the Swan emission at 5165 A and hence the value of percentage polarization using this filter represents a lower limit to the value of possible continuum per cent polarization. The 4300 A and 4800 A filters yield values of the polarization of the continuum, though weak Swan emission can contaminate the result obtained through the 4300 A filter when the emission spectrum of the comet is very intense.

The second characteristic has been studied through both interference and wide-band filters, as can be seen from the data presented.

The value of the polarization of the radiation transmitted by the 4800 A filter is 20·30 per cent + 0·62 for Comet Arend-Roland and 19·78 per cent ± 1·00 for Comet Mrkos. These values are extrapolated P_{90} values. They are remarkably close and demonstrate well the conclusion that in both comets the continuous spectrum was similar. The P_{90} values of the continuum at 4300 A are 18·66 per cent ± 0·78 for Comet Arend-Roland and 19·21 per cent ± 1·35 for Comet Mrkos. The P_{90} value of radiation transmitted by the 5000 A filter for Comet Arend-Roland is 19·53 per cent ± 2·17 and is in agreement with the 4300 A values. We thus derive a value of 19·1 per cent for the mean continuum polarization at 5000 A and 4300 A for both comets when the phase angle is ninety degrees. A similar value for the mean polarization at 4800 A is 20·1 per cent. The 5000 A and 4300 A values would necessarily be lower limits to the real continuum polarization.

Our measures of a *B*, *V* colour indicate a colour excess of $0^{m}{\cdot}26$, in the coma. This value would tend to increase when the contribution of Swan emission is evaluated. The colour excess certainly originates as a result of scattering by particles in the cometary atmosphere that are larger than the wavelength of light. There would, no doubt, be a mixture of various particle sizes, and of different species of particles. It should be possible, theoretically, to derive the particle size and refractive index from the observed reddening and polarization. Liller (**10**) finds, for instance, by such a method that the particles in the tail of Comet Arend-Roland were mostly of iron with diameters around 0·6 μ. The $B-V$ value obtained by Liller in the tail is $+1^{m}{\cdot}18$, thus giving a colour excess of $0^{m}{\cdot}54$. It is, of course, possible to match the variation of polarization of the continuum with those observed by reflection from different surfaces in the laboratory, and get a qualitative idea of the nature of the reflecting surfaces in the coma. Such a procedure would perhaps give an incomplete picture of the scattering processes in the head of the comet.

To calculate the percentage polarization of Na emission we have evaluated from Liller's tracings of August 23·04, after applying necessary corrections for extinction, filter factor and photomultiplier response, the amounts of continuum and various emissions given out by Comet Mrkos in the region 5740 A–6040 A. We find that the contributions of the continuum, 5845 A emission and Na emission are 94·33 per cent, 1·83 per cent and 3·84 per cent respectively. After passing through the 5890 A filter employed by us, the composition of the light reaching the photomultiplier becomes, continuum 88·94 per cent, 5845 A emission 2·44 per cent and 5890 A emission 8·63 per cent. This is diagrammatically represented in fig. 2. Assuming the last mentioned composition to hold for our observations of August 22.618, we find on using the observed P_θ value with the 5890 A filter for the total polarization of the light passing through that filter, and

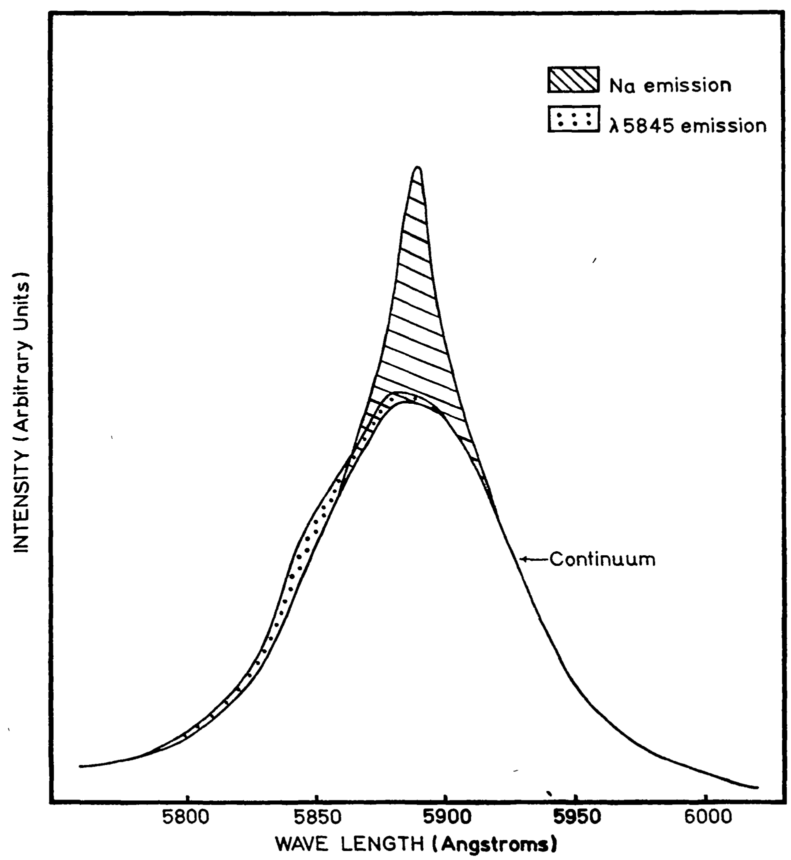

FIG. 2.—*Composition of light around* 5900 A *transmitted by the interference filter in the spectrum of Comet Mrkos on* 1957 *August* 23.04.

the P_θ value for 4800 A for the polarization of the continuum, that the P_θ value for uncontaminated Na emission is 27·0 per cent, which corresponds to a P_{90} value of 37·4 per cent. We have assumed in the above calculation the polarization of the 5845 A emission to correspond to a P_{90} value of 10 per cent. Since the intensity of Na emission relative to the continuum is small, the value of polarization derived for the emission is very sensitive to changes in the overall observed polarization of radiation transmitted by the 5890 A filter.

The sensitiveness of the P_{90} value of pure sodium emission to slight changes in the observed values of polarization, and the fact that a difference of about $0^{d}{\cdot}4$ exists between our observations and Liller's, strongly favour our conclusion that the estimated polarization of pure sodium emission is in close agreement with the theoretical value of 40 per cent for sodium resonance radiation.

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