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A non-anthropic origin for a small cosmological constant*

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Abstract. An impressive variety of recent observations which include luminosity evolutions of high red shift supernovae strongly suggest that the cosmological constant (Λ) is not zero. Even though the Λ–term may dominate cosmic dynamics at the present epoch, such a value for the vacuum energy is actually unnaturally small. The difficulties in finding a suitable explanation (based on fundamental physics) for such a small residual value for the cosmological term has led several authors to resort to an anthropic explanation for its existence. Here I present a few examples some based on phase transitions in the early universe involving strong or electro weak interactions and other on gravitational spin interactions to show how the cosmical term of the correct observed magnitude can arise from fundamental physics involving gravity.

There is an impressive variety of several recent observations which seems to indicate that the cosmological constant (Λ) is not zero (Ostriker & Steinhardt, 1995). A primary motivation for invoking such a constant, on and off, on many earlier occasions is the existence of stars more than 15 billion years old (Bolte and Hogan, 1995) in a universe which is apparently significantly younger (especially if closed) i.e. of ~12-13 billion years.

A cosmological constant increases the age of the universe but only marginally in a flat universe. Moreover Λ has a strong effect on the deceleration parameter of the universe q_o which provides an independent test of concordance. In particular we have : $q_o = \frac{1}{2}\Omega_m - \Omega_\lambda$, Ω_m and Ω_λ being expressed in terms of the critical density ρ_0. For instance if Ω_λ is 0.7 and Ω_m is 0.3 $q_0 = -0.55$. Recently it has become fashionable to introduce a non-zero Λ particularly in the special case of a flat universe (Carrol et al. 1992). Such a model might be consistent with cosmological inflation and early results on angular structures in the microwave background (Hancock et al., 1998). Contrary to expectation there is increasing evidence that the universe is expanding at about twice escape velocity(speed to overcome gravitational pull of all matter), that is, it is clearly accelerating with a negative q_o, indicative of a residual cosmological constant (Bahcall and Fan 1998). This evidence is further corroborated very recently by the supernova cosmology project (Perlmutter 1997, 1998). This aim is to construct a new Hubble diagram, using distant type Ia supernovae whose peak luminosities are calibrated according to the shape of their light curves and spectroscopic data available. Both ground-based and Hubble space telescope observations are used and so far the team has gathered about 50 fully

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reduced events with redshifts between 0.3 and 1.0. The current best fit model excludes the popular Ω=1, Einstein-de-sitter model. One is led to the fascinating conclusion from these very recent observations that Λ could be a significant component of the cosmic dynamics and in brief one has clear evidence for cosmic acceleration (negative q_o). If this result stands up to future work it would create considerable theoretical problems. The point is such a value for the vacuum density (though it may dominate cosmic dynamics) is actually unnaturally small being some 120 orders smaller than its value set by the Planck scale i.e. Λ_{pl}. This small residual value of Λ is considered difficult to explain by any arguments involving known fundamental physics and consequently several recent solutions invoke anthropic arguments as a last resort. For instance Weinberg (1997) argues that Λ_{eff} must be small enough to allow the formation of sufficiently large gravitational condensations. Another anthropic bound follows from the requirement that the Hubble time must be comparable to lifetime of main sequence stars evolving life giving a bound on Λ/Λ_{pl}. Another anthropic argument for a small non-zero Λ is given in (Efstathion 1996) where the probability that life would evolve based on the space density of galaxies in a Λ dominated universe was estimated, the formation of galaxies being exponentially suppressed above a certain value of Λ. However, the above arguments do not involve fundamental physics and are just so arguments.

In the remainder of this essay, I shall proceed to briefly discuss several physical possibilities with examples involving physical arguments that give a small non-zero Λ of the observed value.

To invoke a modified version of the old argument of Zeldovich (Sivaram 1996), if one considers the vacuum energy as arising from the gravitational interaction of virtual particle pairs (mass m) separated by a compton length, i.e. we write for the energy density :

$$\varepsilon \sim \frac{Gm^6c^4}{\hbar^4} \tag{1}$$

so that ε scales as the sixth power of the particle masses. Now in the early universe, a number of phase - transitions corresponding to the breaking of several symmetries connected with the fundamental interactions are believed to have occurred. The last phase transitions to have occurred in the early universe, is perhaps the quark-hadron one, at which epoch, the corresponding mass-scale is $m_{QCD} \sim 160 meV$, i.e. roughly the pion mass m_{Π}. As there were no further phase transitions, the residual minimal value (which remains to the present) Λ_0 would have been :

$$\Lambda_{QCD} \simeq \Lambda_0 \simeq \frac{8\pi G^2 m_\pi^6}{\hbar^4} \simeq \Lambda_{\text{present}} \simeq 10^{-56}\ cm^{-2} \tag{2}$$

Also

$$\frac{\Lambda_o}{\Lambda_{pl}} \simeq \left(\frac{m\pi}{m_{pl}}\right)^6 \simeq 10^{-120} \tag{3}$$

So we have a reason why the present value of Λ_0 has a value 10^{-120} Λ_{pl} (even this is enough to dominate cosmic dynamics at the present epoch). For an explanation of what happened to the vacuum energy at earliest epochs see (Sivaram 1992).

As another example consider the possible transition that could occur at the electroweak scale in the early universe. The corresponding energy scale is $m_W \sim \frac{e}{\sqrt{G_F}} \sim$ 70GeV. G_F is the universal Fermi constant of weak interactions, e is the electric charge. The expansion rate driven by the weak boson condensate at temperature T_W is matched by the Hubble expansion rate. We can set up the relation as :

$$const\ n\sigma_w c \sim \frac{T_W^2}{M_{pl}}\ const \tag{4}$$

σ_w is the cross-section given by $\sim G_F/e^2$ and n is the particle number density $\sim T_W^3$.

Thus we have :

$$T_w \sim \frac{e^2}{G_F} \cdot \frac{1}{M_{pl}} \cdot const \sim \frac{m^2 w}{M_{pl}} \tag{5}$$

So the mass (energy density) of the electroweak vacuum made up of weak boson condensate is given by : (h=c=1).

$$\rho_w \sim T^3 m_w \sim \frac{m_W^7}{M_{pl}^3} \tag{6}$$

The above phenomenon is similar to what happens in superconductors where phase invariance is broken due to condensate of Cooper pairs and the photon acquire an effective mass. This is elaborated in (Sivaram 1996). Thus the residual cosmic constant term arising from the weak scale following eq. (6) is :

$$\Lambda_0 \sim \frac{8\pi\ G\ \rho w}{c^2} \sim \frac{8\pi G}{c^2} \left(\frac{m_W^4 c^3}{h^3} \right) \left(\frac{m_w}{M_{pl}} \right)^3$$

$$\sim 10^{-56} cm^{-2} \tag{7}$$

(where $m_W \sim e/\sqrt{GF}$)

Eq.(7) giving this residual value of Λ which continues to the present epoch contains the coupling constant of the electroweak interactions i.e. e and G_F. As the weak bosons have already been produced in accelerators this argument is realistic physics Eq. (2) for Λ_0 contains the strong interaction coupling through the QCD scale $M_{QCD} \sim m_\pi$. This indicates that the present small value of Λ is related to the coupling constants of the fundamental interactions through processes occurring in the early universe. A unified approach connecting eqs. (7), (2) and (3) is given in (Sivaram 1994).

As yet another example consider the present vacuum energy to effectively consist of hypothetical particles (mass m_Λ) and the whole of phase space to be filled with such fermionic particles (consisting of oppositvely charged pairs) (so that $\Lambda_0 \; \alpha \; m_\Lambda^4$

Then $\Lambda_0 \simeq m^4$

If all the mass m of these particles is generated by self interactions then renormalisation argument (based on quantum gravitational electrodynamics) would suggest

$$\delta m / m_\Lambda \sim \sim \alpha \ln \hbar c / G m^2 \tag{8}$$

(α being the fine structure constant).

Thus $G m_\Lambda^2 \simeq e^{-1/\alpha}$

or

$$\frac{\Lambda_0}{\Lambda_{pl}} \simeq e^{-2/\alpha} \tag{9}$$

giving ($\rho_\Lambda \simeq \rho_{pl} \, e^{-2/\alpha}$

As $e^{-2/\alpha} \simeq e^{-275} \simeq 10^{-122}$

We have a theoretical justification for present small value of Λ as $\sim 10^{-122} \Lambda_{pl}$. Again $m_\Lambda \simeq m_{pl} \, e^{-1/2\alpha} \simeq 10^{-4}$eV; which is comparable to the usual axion mass for closure density in CDM models. This gives a definitive basis (which is usually ad hoc) for such a value for this mass.

As a final example I point out that gravitational spin interactions in the early universe can also generate a residual cosmical term. The intrinsic spin of the matter constituents can generate a spin-torsion term dominating in the early universe (Sivaram 1998)

Quite generally it may be pointed out that any torsion like 3 index gauge field H can mimic a cosmological term (Sivaram 1998). For instance for an addition to the action such as (Sivaram 1998).

$$I \sim \int \delta^4 x \sqrt{-g} \, A^{\mu\nu\rho\sigma} \; A_{\mu\nu\rho\sigma}$$

where $A_{\mu\nu\rho\sigma} = \delta_\mu H_{\nu\rho\sigma}$, substituting the solution $\sqrt{-g} \, A^{\mu\nu\rho\sigma} = k \, \varepsilon^{\mu\nu\rho\sigma}$ (following from $D_\mu A^{\nu\rho\delta} = 0$) into the corresponding energy momentum tensor $T^{\mu\nu}(H)$ for the H field we get $T^{\mu\nu}(H) = \frac{1}{2} K^2 g_{\mu\nu}$, which is an effective Λ -term. The occurrence of such 3 index antisymmetric gauge fields are an inherent feature of superstring theories and are hence important in the early universe. As is well known, we will then have an additional spin contribution to the Friedmann equation, thus :

$$\dot{R}^2/R^2 = 8\pi G\, \rho/3 + 2/3\, \pi G^2\, \sigma^2\, r^2/c^4 \tag{10}$$

σ being the spin density. This term would occur even if the spins are randomly oriented or unpolarised. (The extra spin density source term in the Poisson equation $\nabla^2\phi \sim G^2\sigma^2/c^4$, can be easily seen to give a contribution $\phi \sim G^2r^2\sigma^2$ term (i.e. an effective cosmic term of form $\sim \Lambda r^2$ for constant spin density).

σ is given by $\sigma \sim S/r^3$, where S is the total spin. For N spinning particles, $r \sim N^{1/3}(\hbar/mc)$,

So that the additional gravitational spin interaction term (during the last phase transition) becomes :

$$\sim G^2N^2\hbar^2/c^6\, r^6 \sim G^2\hbar^2N^2/c^6N^2(\hbar/mc)^6 \tag{11}$$

$$\sim \frac{G^2m^6}{\hbar 4} \sim 10^{-56}\ \mathrm{cm}^{-2}$$

(as $S \sim \hbar$).

We see that eq. (11) is identical in form to eq. (2) thus giving the same value that is observed for the cosmical term. Another way of looking at it is that as particle masses dropped from M_{pl} at Plancks epoch, to $M_{QCD} \sim m_\pi$ at the last phase transition, Λ dropped in proportion to m^6, as given by eq. (3). Thus the Hamiltonian of the spin - torsion interaction, i.e. the term given by :

$$H \sim \beta H_{ijk}\, S^{ijk}$$

(H_{ijk} is the antisymmetric (torsion) gauge field and S^{ijk} is the spin tensor density) or equivalently the scalar. $H \sim \beta H.S$, gives rise to the effective residual cosmological constant term given by eq. (11).

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