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Some experiments in radiative transfer*

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1. Preliminary remarks

At the beginning I would like to remember and pay homage to that loving but resolute lady Smt. Sunanna Bappu, who was the mother of late Dr. M. K. Vainu Bappu. You can imagine her grief when she lost her only son at the prime age of 55 when he was at the height of his career, as he was then the President of the International Astronomical Union. Controlling her grief she decided to sell her property and utilise the proceeds for instituting an international award for Astrophysics in her son's name. Members of the Astronomical Society of India, had also resolved in the General Body Meeting at Gorakhpur to institute a similar award by collecting donations from the friends and admirers of Dr. Vainu Bappu. So we approached Smt. Sunanna Bappu to help us by her donation. But she did not accede to our request, because the office of the Astronomical Society of India was located at Osmania University and she had great antipathy towards Osmania University for not appreciating the merit of Dr. Vainu Bappu on his return to Hyderabad from USA with flying colours in 1950's. Consequently Smt. Sunanna Bappu decided to donate the money to Indian National Science Academy for instituting any International Award for Astrophysics in the name of her son. At that time some Physicists from INSA tried to tell her that Astrophysics is a restricted subject and she should broaden the scope of her award to the whole subject of Physics. But the lady was resolute and she insisted that since her son was an Astrophysicist of international repute, the award has to be in Astrophysics. As I happen to be the current beneficiary of this decision of hers, I humbly pay my homage to the memory of Smt. Sunanna Bappu.

Dr. M. K. Vainu Bappu was a good friend of mine. Although we might have differed on some occasions, we respected each other, because we realised that both of us were striving for the advancement of astronomy in India in our own different ways. My wife and I cherish many pleasant meetings with his family and friends. Particularly I remember the few days I spent together with Dr. Bappu in New Zealand in 1978, when we frankly discussed about the state of Astronomy in India.

At this time I wish to announce that my wife Smt. Shailaja and I have decided to donate the award money to the Corpus Fund of the Astronomical Society of India.

* INSA - Vainu Bappu Memorial Award (1998) lecture delivered at the Indian Institute of Science, Bangalore, on April 17, 1999.

2. Schuster problem for a moving atmosphere

Radiative transfer is a vast subject hallowed by the monumental work of S. Chandrasekhar (1960). Today I propose to recapitulate my own experiments in some areas of this broad field and put them in a proper perspective.

Berkeley Astronomy Department, where I studied for my Ph. D. Degree, had the practice of inviting a Visiting Professor during the spring semester every year. V. Kourganoff, who was the Visiting Professor during 1958-59, conducted a seminar in which I got introduced to the Schuster problem for a moving atmosphere through S. Chandrasekhar's paper on that subject. In the Schuster model of a stellar atmosphere, absorption lines are produced in a coherently scattering plane parallel reversing larger overlying the photosphere of the star. Dividing the radiation field into an outgoing and an ingoing beam Schuster showed that the atmosphere has a transmission coefficient $T=1/(1+\tau_{\nu})$, where τ_{ν} is the optical depth of the atmosphere in frequency ν. In the first order approximation of the discrete ordinate method of Chandrasekhar (1960) $T=1/(1+x_{\nu})$, where $x_{\nu} = \sqrt{3}\tau_{\nu}/2$. It is to be noted that in conservative scattering T is associated with a reflection coefficient S= (1–T).

The related problem for a moving atmosphere, in which the layers have relative velocities, was first considered by W. H. McCrea and K. K. Mitra (1936), who presented the basic equations. An analytical solution for the case, where the outward velocity increased linearly with optical depth measured from the photosphere, was given by S. Chandrasekhar (1945) for a rectangular line scattering coefficient of half width Δv. At Berkeley I tried to descritise Chandrasekhar's equations of the problem and I was able to recover the profile of the absorption line obtained by Chandrasekhar with only a few iterations. Four years later at Toronto I was able to generalise the solution for any monotonic velocity law and any isotropic line scattering coefficient of arbitrary shape (K. D. Abhyankar, 1964 a, b; 1965).

The plane parallel stratified atmosphere is divided into N layers in such a way that the mean velocity of separation between two consecutive layers is constant, say dv. In general the layers will have different optical thickness, so the transmission coefficient T will depend on I, the serial number of the layer. If the velocity of separation is expressed by the frequency interval $d\nu=\nu_0 dv/c$, where ν_0. is the frequency at the centre of the line for stationary observer, then $d\nu$ becomes a natural unit for labelling frequencies. Consequently the line scattering coefficient $\sigma(\nu)$ is divided into M strips of this width, so that beyond the last strips $\sigma/\sigma_0 < 10^{-4}$. The frequencies as observed by the stationary and moving observers have to be labelled differently, say be indices L and J, respectively. By suitably defining J we can label the frequencies for all the moving observers by the same values of J from N+1 to N+M. For this J = L + N-1 for outgoing beam and J = L + N + I – 1 for ingoing beam. The transmission coefficients will then be functions of J in addition to I, thus T(I, J) is a matrix of dimensions N × (N+ M).

Finally we write relations between the intensities at the boundaries of successive layers which can serve as an iteration formula. There is one relation for the outgoing intensity at the top of the I'th layer and another relation for the ingoing intensity at the bottom of the

I'th layer. These relations are similar to those obtained by G. Stokes (1904) for the reflection by a pile of plates; in his case all plates were identical, but here we have unidentical layers and frequency changes from one layer to another.

In addition to the above two relations we also have boundary conditions that : (i) the intensity of the outgoing beam in all frequencies is unity at the bottom of the atmosphere, and (ii) the intensity of the incoming beams at the top of the atmosphere is zero in all frequencies. Stated in this form the problem can be solved by the method of iteration. Some typical line profiles for a Gaussian scattering coefficient of half width $\Delta\nu$ and different expansion velocity laws are shown in Figure 1. for optical depths x_1=0.2, 1.0, 5.0 and 25.0. The absorption line has usually two cores one near the central frequency ν_0 and the other near the maximum frequency $\nu_{max} = V_{max}\ \nu_0/c$. The relative importance of the two components depends upon the thickness of the atmosphere and the nature of the velocity law. It was also found that the flat portion of the curve of growth is raised higher with increasing value of V_{max}, which indicates that the differential velocities within the atmosphere act like microturbulence.

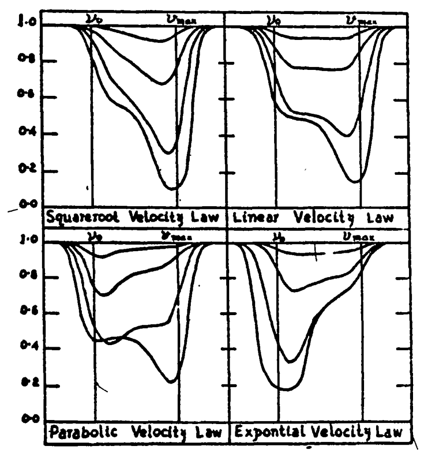


Figure 1. Line profiles for Gaussian scattering coefficient for $\sqrt{3}\ \tau_\nu/2$=0.2, 1.0, 5.0 and 25.0 with $V_{max}\ \nu_0/c = 10\sqrt{3}$ for various velocity laws.

The above method gives us the radiation field throughout the atmosphere. In particular we know the mean intensity J (ν,τ,μ) at frequency v, as judged by the stationary observer. It depends on $\mu=\cos\theta$, θ being the angle of emergence, in spite of the assumed isotropic scattering on account of the different Doppler shift of frequencies in different directions. So we can compute the emergent intensity $I(\nu,\tau,\mu)$ and the line profile $r_\nu(\mu) = I(\nu,\tau,\mu)/I(\nu,o,\mu)$ in direction θ and study the centre to limb variation of the line profile. Further by integrating over the disc we can get the line profile in the integrated flux. (See K. D. Abhyankar 1967). Figure 2. shows the results for the linear velocity law of Figure 1. with x_1=5. The upper panel shows the variation of the line profile over the disc; we see that the line is broad and shallow at the centre of the disc and narrow and deep near the limb. The lower panel shows the integrated profile for two values of limb darkening coefficient X=0 and 1.0. We note that the double humped profiles for individual points on the disc are replaced by a broad and asymmetric line profile shifted towards the higher frequencies with respect to ν_0. These results were later confirmed by E. Simonneau (1973) as well as by P. D. Nordlinger and G. B. Rybicki (1974). However they did not refer to my work, because they missed the fourth paper (K. D.Abhyankar 1967) in the series, as it was published in a book (Struve Memorial Volume) instead of a research journal. But later P. D. Nordlinger and G. B. Rybicki (1976) acknowledged the priority of my results.

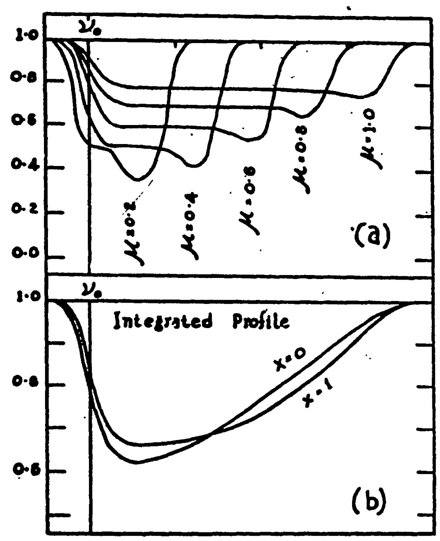


Figure 2. Top : Centre-Limb variation of line profiles for various angles of emergence $\theta = \cos^{-1}(\mu)$. Bottom : Integrated profile for two limb darkening laws X=0 and X=1.

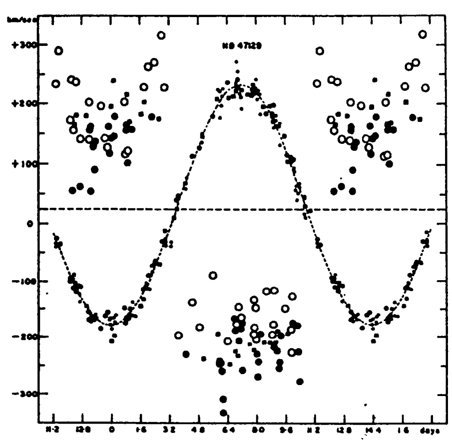
3. Stellar winds, accretion and shocks

I would now like to make a slight deviation and speak about outflow and inflow of matter in stars. It is now well established that early type O stars exhibit the phenomenon of stellar winds through the P-Cygni type profiles in their spectra with red shifted emission and violet shifted absorption. Such profiles are produced in extended expanding atmospheres. But as we have just seen we can have violet shifted absorption lines with no emission in an expanding atmosphere which is not extended enough to produce emission lines. Similarly inflow of the matter into the star by accretion or contraction of the atmosphere can produce red shifted lines. But, in the case of nonpulsating stars, such violet or red shifts cannot be distinguished from the radial velocity of the star unless it happens to be a member of a star cluster or a binary system. We shall consider two such cases.

According to Struve (1950), Trumpler had found that O stars, which happen to be members of galactic clusters, had radial velocities in excess of the cluster velocity by about 14 km / s. Taking these to be General Relativistic gravitational red shifts Trumpler found for them masses of the order of 300 to 400 M_{sun}. However, on studying some of them, which happened to be members of binary systems, they were found to have normal masses of 20 to 60 M_{sun}. Hence the observed red shift remained a mystery.

Later W. H. McCrea thought that such redshifts might indicate inflow of matter by accretion. He (W. H. McCrea, 1956) considered the production of shock waves in the steady radial motion under the gravity of the star and obtained a model of the shock. During his tenure as Visiting Professor at Berkeley in 1956-57 he suggested to me that I should look into the problem of inflow of matter into Trumpler's O stars in the light of his theory. So I tried to see whether it was possible to have a shock wave at an appropriate distance from the star by considering two boundary conditions: one specified by the atmospheric model of an O star and the other by the conditions in the interstellar space of H II region surrounding the star. I had also done some work on the outflow of material from a star on the same lines. But I was not successful and I sent my results to Prof. McCrea who might have been disappointed. However, now I feel that I should have published my work, because such failed attempts are also useful for inspiring further research. Shock waves produced by stellar winds and production of accretion discs around components of binaries are now known to be common phenomena.

My second example pertains to the Struve-Sahade effect in massive binaries, to which attention has been drawn recently by D. J. Stickland (1997), who has studied their UV spectra. It is best illustrated by the radial velocity curves of Plaskett's star HD 47129 (see Fig. 3) studied by me for my thesis (K. D. Abhyankar 1959). It pertains to the variability of the radial velocity and intensity of the secondary lines which become weak while that star is receding. Stickland has argued that these apparently non-Keplerain motions indicate gas streams produced by the mass losing primary. But we see that the secondary has a smaller value of K and more negative γ velocity, which are not explained by the above scenario. We are, therefore, of the opinion that our earlier suggestion (K. D. Abhyankar, 1960) may still be valid. We had argued that it is the secondary, which is not in equilibrium, and it is really the more massive and evolved star. It is now in expanding stage which gives it more negative γ velocity. We now suggest this is the result of a stronger stellar wind from that star which not only causes variations in the intensity of its spectral lines but also makes the star fainter.



	Primary	Secondary
Victoria (1922)	Open squares	Filled squares
McDonald (1947-1948).	Small open circles	Large filled circles
Lick and Mount Wilson (1956-1957). . .	Small filled circles	Large open circles

Figure 3. Radial velocity curves for Plaskett's star HD 47129.

That this is indeed the case can be seen from the data given in K. D. Abhyankar (1959). A shell line was seen in the Mount Wilson spectra of 1956 when the primary was receding. It had a velocity of - 900 km/s with respect to the primary and -450 km/s with respect to the secondary. The dynamics of its motion derived from a study of its spectral characteristics showed that :(i) If the secondary is a cooler and fainter star it would have no effect on the motion of the shell which would be expanding with a velocity of 150 km/s with respect to the primary (ii) If the two stars are comparable in luminosity and temperature, then the shell would be expanding with a velocity of 180 km/s with respect to the primary and of 380 km/s with respect to the secondary. Observations are in good agreement with the second alternative which shows that the shell was ejected from the secondary.

4. Inhomogeneous atmospheres

Coming back to the Schuster problem for a moving atmosphere it is seen that our method of solving it is an example of the application of the discrete space theory of radiative transfer in linear lattice presented by R. W. Preisendorfer (1965). Now, the reflection and transmission properties of a plane parallel stratified slab of optical thickness τ made of isotropic homogeneous medium are identical for radiation incident on the slab from above and from below. But R. W. Preisendorfer (1958) has shown that a slab containing inhomogeneous isotropic medium exhibits polarity in the sense that the reflection and transmission matrices $\mathbf{S}(\tau,\mu,\varphi,\mu_0,\varphi_0)$ and $\mathbf{T}(\tau,\mu,\varphi,\mu_0,\varphi_0)$, respectively for radiation incident at the top are different from the reflection and transmission matrices $\mathbf{S}^*(\tau,\mu,\varphi,\mu_0,\varphi_0)$ and $\mathbf{T}^*(\tau,\mu,\varphi,\mu_0,\varphi_0)$, respectively, for radiation incident at the bottom. So the integro-differential equations for **S** and **T** matrices of a homogeneous slab derived by S. Chandrasekhar (1960) have to be doubled to include those for **S*** and **T*** as illustrated by Z. Sekera (1963). Consequently one tries to take recourse to numerical methods for solving the problems of radiative transfer in inhomogeneous media.

The scattering properties of a medium are described by the function $\mathbf{p} = \varpi_0 \mathbf{P}$, where ϖ_0 is the albedo for single scattering and **P** is the direction dependent phase matrix of scattering. Inhomogenities can arise from a variation of both ϖ_0 and **P** with optical depth. Variation of **P** is more difficult to take into account, so one considers variation of ϖ_0 only. An approximate analytical solution for a semi infinite atmosphere in which ϖ_0 decreases slowly with optical depth was given by J. W. Chamberlain and M. B. McElroy (1965). But the only analytical solution for a more general variation of ϖ_0 with optical depth was obtained by the author in collaboration with A. L. Fymat at Jet Propulsion Laboratory. (A. L. Fymat and K. D. Abhyankar 1969, a, b and K. D. Abhyankar and A. L. Fymat 1970 a, b). Here I was guided by S. Chandrasekhar's (1960) treatment of a nongrey atmosphere as a perturbation of a grey atmosphere which enabled him to replace Rosseland mean by his Chandrasekhar mean absorption coefficient.

As we are interested in Rayleigh scattering we considered the scattering function where $\mathbf{P}(\mu,\phi;\mu_o,\phi_o)$ is the usual Rayleigh phase matrix. In this case the azimuth - dependent parts of **S,T,S*** and **T*** can be expressed in terms of the scalar $X(\mu)$, $Y(\mu)$, $X^*(\mu)$, and $Y^*(\mu)$, functions of Chandrasekhar, one set for $(\varphi-\varphi_o)$ and the other set for $2(\varphi-\varphi_o)$ terms, respectively. But in the case of the azimuth - independent part the scalar functions are to be replaced by the (2 × 2) matrices **K**(μ), **L**(μ), **K***(μ) and **L***(μ) introduced by Z. Sekera (1963). These however, can be expressed as linear combinations of two other sets of X and Y functions. In the case of the semi-infinite atmosphere, for which only **S** is relevant, the X(μ) function is replaced by Chandrasekhar's H(μ) function and **K** (μ) matrix by **N**(μ) matrix.

We considered the problem of an inhomogeneous atmosphere in which ϖ_0 varies arbitrarily with optical depth τ and represented it as a perturbation in the form $\varpi_0=\Omega_0[1+\omega(\tau)]$. Now the integral equations for X,X*, H, **K** and **N** contain only the very same functions, i.e. they are independent of other functions in the set. So we put $X=X_0[1+x(\tau)]$,$X^*=X_0^*[1+x^*(\tau)]$ $H_0=H[1+h]$, $\mathbf{K}=\mathbf{K}_0[\mathbf{E}+\mathbf{k}(\tau)]$ and $\mathbf{N}=\mathbf{N}_0[\mathbf{E}+\mathbf{n}]$ where $x(\tau)<.1$, $x^*(\tau)<1$, $h<1$, **E** is unit (2 × 2) matrix and elements of **k**(τ) and **n** are all less than unity. We then obtained linearised equations for x(τ), x*(τ), h, **k***(τ) and **n** by neglecting the second order terms as negligible peturbations and put them in operator form. A formal solution of the operator equation can be written in the form of a series involving iteration of the operation process. According to the definition of E. Hopf (1934) it will represent the unique N-solution of the relevant equation if the series is found to be convergent.

Convergence for the series solution for x(τ), and x*(τ) was tested for isotropic scattering. It was found that there is convergence for all values of Ω_0 and optical depth when $\omega(\tau) < -0.238$, In other cases the domain of convergence decreases with increasing optical depth. In the case of Rayleigh phase matrix the series solution for **k**(τ) converges when $\omega(\tau) <-0.291$. In the case of h(μ) it is found that the best choice of Ω_0 is halfway between the maximum and minimum values of ϖ_0. Further the linearised solution for H(μ) obtained by our perturbation method can serve as the starting point for getting the exact solution by iteration of the original equation of H(μ). The linearised solution for **n**(μ) is found to be convergent for all values of Ω_0, < 0.84 and $\varpi_0(\max) < 1$. This linearised solution can be used for obtaining the exact solution for **N**(μ) by iteration.

No further work in this direction was attempted, beacause computers can now take care of all kinds of complications introduced by the variation of ϖ_0 and mixing of Rayleigh, Mie and other phase matrices. But our approach provided us with a clue to treat the practical problem of nonconservative or imperfect Rayleigh scattering.

5. Imperfect or nonconservative Rayleigh scattering

Extinction of radiation in a coherently scattering gaseous medium is caused partly by scattering and partly by true absorption, which give rise to an albedo for single scattering ϖ_0 less than unity. The classical Rayleigh Scattering (see K. D. Abhyankar 1996) is supposed to be conservative, so ϖ_0 is taken to be one. But in reality, there is absorption in the gas, so $\varpi_0<1$ as found by D. Harris (1961) for explaining the phase curve of Venus. Harris used nonconservative isotropic and Rocord scattering laws, because the only solution for Rayleigh scattering available was that of S. Chandrasekhar (1960) for conservative scattering. It was therefore necessary to obtain the solution for nonconservative (imperfect) Rayleigh scattering for application to the phase curves and spectral lines of Venus and other planets. We have considered this case for both semi-infinite and finite atmospheres.

(a) Semi-infinite atmosphere : K. D. Abhyankar and A. L. Fymat (1970 c) have considered a semi-infinite atmosphere exhibiting nonconservative Rayleigh Scattering with scatterring function $\mathbf{P} = \varpi_o \mathbf{P}_R$, with $\varpi_o<1$, illuminated from direction $(\mu_0\phi_0)$. The emergent intensity vector $\mathbf{I}(\mu, \phi\mu_o,\phi_o) = (I_l, I_r, U, V)$ in direction (μ, ϕ) can be expanded as a Fourier series in argument $(\phi - \phi_o)$. The terms involving $(\phi - \phi_o)$ can be expressed in $H(\mu)$ furnctions with characteristic functions $\psi^1(\mu)$ and $\psi_v{}^1(\mu)$, where

$$\psi^1(\mu) = 3/8\varpi_o(1-\mu^2)(1+2\mu^2) \text{ and } \psi_v{}^1(\mu) = 3/8\ \varpi_0(1-\mu^2).$$

Similarly the terms involving $2(\phi - \phi_o)$ can be expressed in an $H(\mu)$ function with $\psi^2(\mu) = 3/16\varpi_o(1+\mu^2)$. In the azimuth independent term there is no U component, and the V component can be expressed in terms of an $H(\mu)$ function with $\psi_v{}^o(\mu) = 3/8\ \varpi_o\mu^2$.

There is no difficulty in evaluating these functions for $\varpi_o<1$ by the standard procedures. But the aximuth independent terms in I_l and I_μ components involve elements of the $N(\mu)$ matrix given by

$$\mathbf{N}(\mu)=\mathbf{M}(\mu) + 1/2\ \mu\mathbf{N}(\mu) \int_0^1 \mathbf{N}(\mu')\ \mathbf{M}(\mu')\ d\mu'/(\mu+\mu') \quad (1)$$

where

$$\mathbf{M}(\mu)= \sqrt{3/2} \begin{pmatrix} \mu^2 & \sqrt{2}\ (1-\mu^2) \\ 1 & 0 \end{pmatrix} \quad (2)$$

For $\varpi_o=1$, S. Chandrasekhar (1960) has shown that it is possible to obtain the elements of $\mathbf{N}(\mu)$ in terms of two $H(\mu)$ functions corresponding to the characteristic functions $\psi_l(\mu) = 3/4(1-\mu^2)$ and $\psi_r(\mu)=3/8(1-\mu^2)$. But we failed to do so for $\varpi_o<1$. Consequently we calculated the elements of $\mathbf{N}(\mu)$ by iterative numerical integration of Equation (1) starting from two different initial functions : (i) From $\mathbf{N}(\mu) =\mathbf{M}(\mu)$ and (ii) From $\mathbf{N}(\mu)$ obtained from the first approximaion of our perturbation method derived from Chandrasekhar's solution for conservative scattering. Both procedures converged rapidly to the same values of $\mathbf{N}(\mu)$. The results are given in K. D. Abhyankar and A. L. Fymat (1971). They were used for computing the diffusely reflected radiation with various values of $\varpi_o < 1$. On comparing our results with those for conservative scattering given by S. Chandrasekhar (1960) it was found that (see Fig. (4)) :

(i) The degree of polarisation increases with decreasing value of ϖ_o due to the reduction in the effect of multiple scattering.

(ii) Babinet and Brewster neutral points approach the sun with decreasing value of ϖ_o and merge with the Sun when ϖ_o approaches zero.

(iii) Limb brightening for low zenith angle of the Sun in the Sun's vertical and in the perpendicular direction first increases with decreasing value of ϖ_o and then drops again when ϖ_o becomes smaller than 0.25.

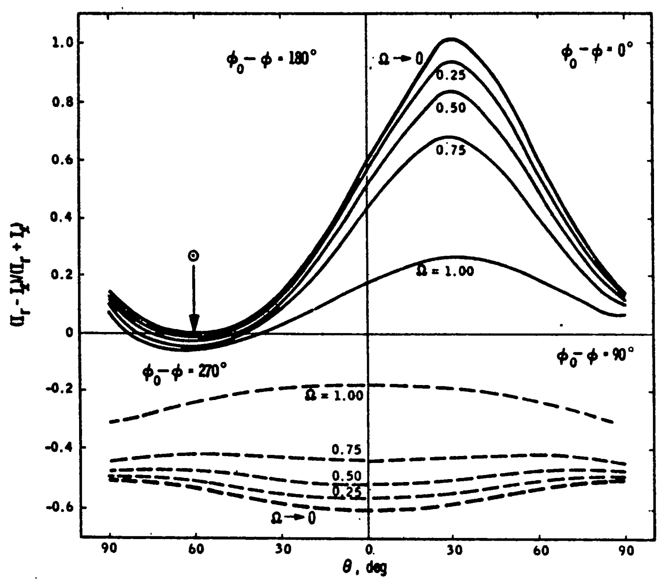


Figure 4. The degree of polarisation of reflected intensity for μ_0 = 0.5 with various albedoes for single scattering.

(b) Finite atmospheres : The case of a finite atmosphere was considered during my tenure of UGC Emeritus Professorship in collaboration with K. E. Rangarajan and D. Mohan Rao of Indian Institute of Astrophysics, Bangalore. In this case one has to evaluate **K**(μ) and **L** (μ) matrices for various optical depths. But, since we are interested only in the emergent radiation K. E. Rangarajan, D. Mohan Rao and K. D. Abhyankar (1994) took recourse to the discrete space theory technique of I. P. Grant and G. E Hunt (1969) and A. Peraiah (1978). Calculations were made for various values of ϖ_0<1 and optical depths $\tau \leq 2$. Here also it was found that in both transmitted and reflected radiation the Babinet and Brewster Neutral points approached the Sun and Arago neutral point moves towards the antisolar point with decreasing value of ϖ_0. In this respect nonconservative scattering behaves like turbidity of the atmosphere. Secondly the degree of polarisation increases with decreasing values of ϖ_0 and τ due to less number of multiple scatterings.

6. Study of Venus

Scattering plays a fundamental role in deciphering the information content of the emergent radiation from a planetary atmosphere (see R. K. Bhatia and K. D. Abhyankar, 1981). In this respect study of Venus has attracted maximum attention, because it is a bright object which exhibits all phases during its revolution round the sun and whose spectrum can be observed with high resolution that is adequate for showing the details of its molecular spectrum. For practical purposes the atmosphere of Venus can be considered semi-infinite in extent.

(a) Study of the phase curve : Phase curve of Venus, i.e. variation of its brightness as a function of phase angle α, has been observed by several authors. It was found by D. Harris (1961) that it agreed with the predictions of a modified Rayleigh phase function with ϖ_0=0.975 upto α = 120°. But there is 2 to 3 percent excess raidation at larger phase angles, which was qualitatively attributed to the radiation diffused around spherical atmosphere of Venus. As an analytical solution for Rayleigh phase matrix or phase function for a spherical atmosphere was not available the author (K. D. Abhyankar, 1968) made approximate calculations by making use of the scattering functions for a plane parallel finite Rayleigh scattering atmosphere of very large optical depth published by Z. Sekera and B. Kahale (1968). This was done by dividing the illuminated disc of Venus into four overlapping regions which could be treated as parts of a thick finite atmosphere that are shown by designations R, 1, 2 and 3 in Figure 5. But the calculated amount of extra scattered light could account for only about one third of the observed excess radiation. Hence it was concluded that the atmosphere of Venus contains 0.1 to 1.0 micron size aerosols which scatter about three times strongly in the forward direction. They are now identified with drops of concentrated sulphuric acid (J. E. Hansen and J. W. Hovenier, 1974), which also explain the peculiar variation of polarisation of Venus with phase angle.

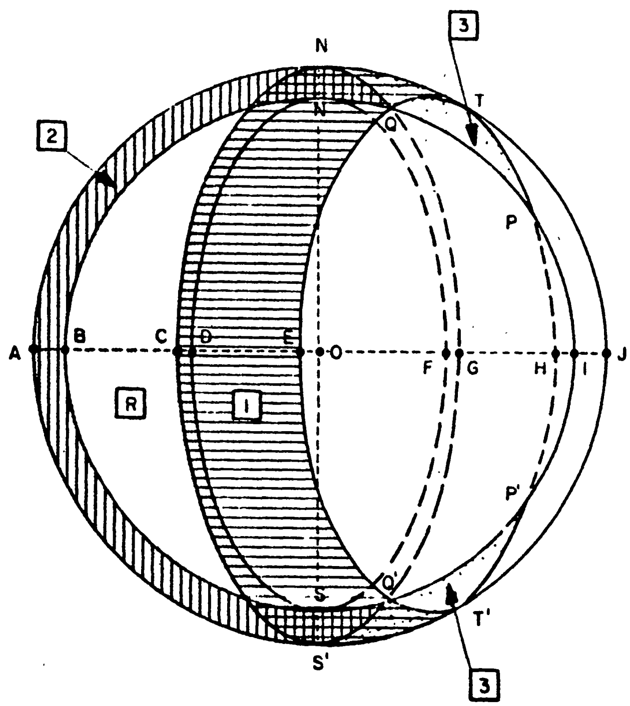


Figure 5. Appearance of Venus in the sky at phase $\alpha > 90°$.

(b) Study of the spectrum : The auxiliary functions for a semi-infinite atmosphere scattering according to Rayleigh phase matrix were computed by us (K. D. Abhyankar and A. L. Fymat, 1971) for studying the absorption lines in the spectrum of Venus. This task was accomplished after my return to India with the help of my student R. K. Bhatia. We (R. K. Bhatia and K.D. Abhyankar 1982, 1983 a, b) made detailed calculations of the intensity and polarisation profiles in both the integrated light and at various points on the disc of Venus along the equator and on the mirror meridian over the whole range of phase angles from 0 to 180°. Here we shall discuss only the results for the integrated flux.

It was found that the equivalent width of F^e_r component with polarisation perpendicular to equator, which is scattered isotropically, decreases monotonically with increasing phase angle in agreement with the results for isotropic scattering by J. W. Chamberlain (1976). But as shown in Fig. 6 the equivalent width of F^e_l component with polarisation parallel to equator, as also that of the combined profile first increases with phase, reaches a maximum at phase angle α=90° and then decreases again with increasing phase angle. This so called inverse phase effect becomes more pronounced as we go from stronger to weaker lines. This is in agreement with the observations of Venus as reported by various authors (L. D. G. Young 1972, E. S. Barber and W. W. Macy(Jr) 1977, W. W. Macy (Jr), and L. Traften 1972). Thus Rayleigh scattering explains quite well the phase variation of the absorption lines in the spectrum of Venus. But as noted earlier the observed variation of both brightness and polarisation of the continuum can be accounted for by Mie scattering of aerosols. Hence we have to conclude that the lines are produced in the upper layers where Rayleigh scattering is important, while the continuum is produced in deeper layers where Mie scattering predominates.

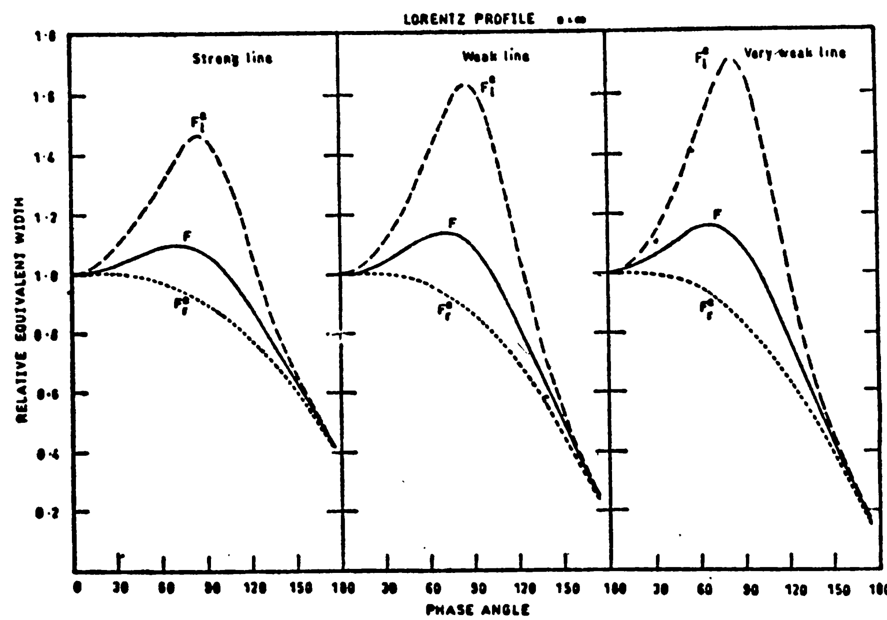


Figure 6. Inverse phase effect for spectral lines of Venus; Strength of the line decreases to the right.

Anecdote : At JPL I was a visiting scientist, so I was attached to one of the regular staff who happened to be Alain Fymat. He was a fresh Ph.D. from UCLA who had worked with Z. Sekera as his student. We hit off quite well as is evident from the 15 odd papers we wrote together in 2½ years. However there was a slight hitch at the beginning. Fymat was an expatriate from one of the north African countries which had just won their independence from France. So he had a slight colonial streak in his make up, which came to light as follows. In addition to working with Fymat, in the beginning I was also pursuing independent research. And my paper on Venus was completed within a few months after joining JPL. When I handed it over to the group leader C. B. Farmer for onward transmission for publication Fymat became furious. He threatened me that he would put his foot down if I ventured to do so in future without his consent. Anyway the matter was patched up and thereafter we often used to walk in the JPL campus arguing about the sequence of the names of the authors in our various joint papers. But I also decided that I will do future work on Venus only after returning to India. And this was delayed by a decade as it took that much time to find a student of R. K. Bhatia's calibre to do the job.

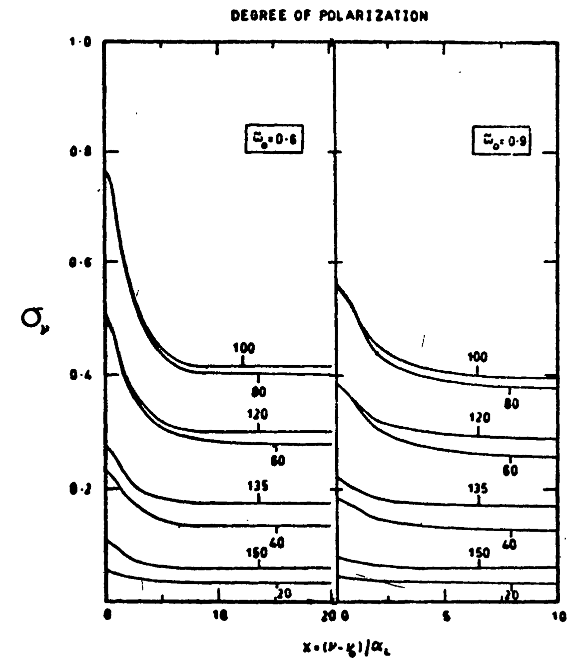


Figure 7. Polarisation profiles of lines in Venusian spectrum

Calculations also show (see Figure 7) that the degree of polarisation increases with line strength and it is maximum at the line centre. Both effects are caused by the reduced number of scatterings. Polarisation also increases with phase angle upto α=90° and then decreases with further increase of phase angle. These are no observations to verify these results.

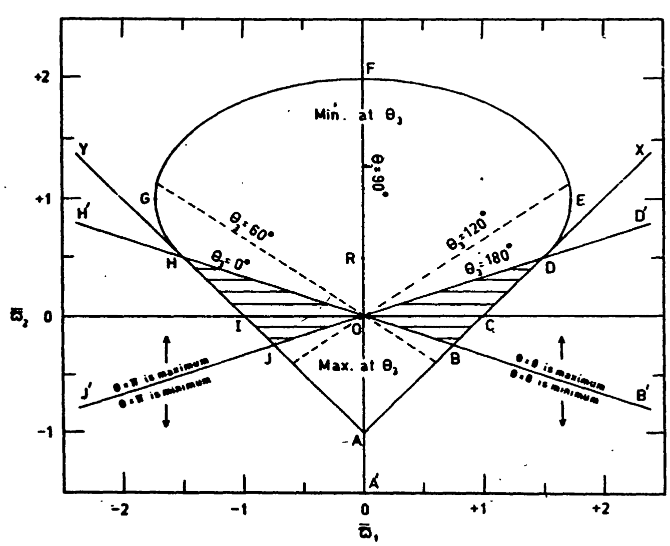


Figure 8. Region of $\bar{\varpi}_1$- $\bar{\varpi}_2$ plane where p(cos θ) > 0.

(c) Venus and Earth : While Venus has a thick almost semi-infinite atmosphere the terrestrial atmosphere is finite in depth. So D. Mohan Rao, K. E. Rangarajan and K. D. Abhyankar, (1995) used the discrete space theoretical technique of L. P. Grant and G. I. Hunt (1969) and A. Peraiah (1978) for studying the polarisation in the lines of the telluric bands of O_2 and H_2O. It was found that here also the polarisation was maximum at the line centre and more in the reflected light. Further it increases with line strength and decreases with increasing optical depth.Thus the polarisation in the centres of the lines of the strong bands - 13500 A band of H_2O and 7600 A band of O_2, which have large optical depth, is found to be the same as that at the centres of lines of the weaker bands- 9000A and 11000A of H_2O and 6400A and 12000A of O_2, which have smaller optical depth. We see that the reduction caused by increased optical depth is compensated by the enhanced strength of the line. Again there are no observations to verify these results.

7. Scattering functions of the type $p(\cos\theta) = \varpi_0+\varpi_1 P_1(\cos\theta) + \varpi_2 P_2(\cos\theta)$

In order to consider the effect of different scattering functions on the phase variation of the equivalent widths we (R. K. Bhatia and K. D. Abhyankar; 1983) have considered scattering functions involving two Legendre Polynomials viz., $p(\cos\theta) =\varpi_0+\varpi_1 P_1(\cos\theta) + \varpi_2 P_2(\cos\theta)$ which can represent a wide variety of phase functions. Solution for such scattering functions in the case of a semi-infinite atmosphere was given by H. G. Horak and S. Chandrasekhar (1961).

First we note that $p(\cos\theta)$ is positive for only certain combinations of $\bar{\varpi}_1=\varpi_1/\varpi_0$ and $\bar{\varpi}_2=\varpi_2/\varpi_0$ as shown by the top shaped figure ADEFGHA of Fig 8. We made calculations for 37 combinations of $\bar{\varpi}_2$ and $\bar{\varpi}_2$, the scattering diagrams for 21 of them are shown in Fig. 9. Each of the 37 pairs was combined with 15 values of ϖ_0 thus making a total of 555 cases. In each case one has to compute three $H(\mu)$ functions and five auxiliary functions χ,ψ,ϕ,σ and θ; a sample for the 15 cases with $\varpi_1=0$ and $\varpi_2=0.95$ is given by R. K. Bhatia and K. D. Abhyankar (1989).

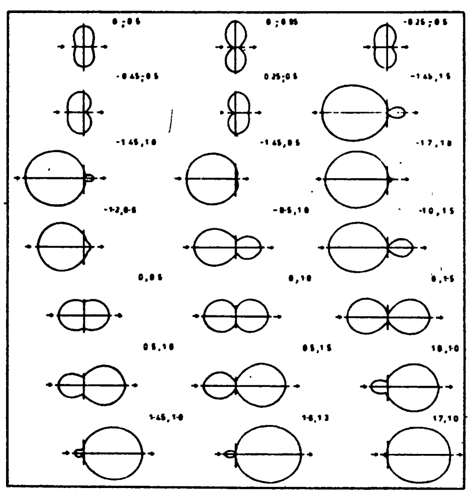


Figure 9. Scattering diagrams for various combinations of $\bar{\varpi}_1$ and $\bar{\varpi}_2$.

Using these functions for studying the variation of the equivalent widths of spectral lines with phase angle it was found that :

(i) Absolute values of equivalent widths at any phase angle are inversely proportional to the value of the scattering function at that angle. For example for α=0, W/2 α_L, where α_L is the Lorentz half width for the line, decreases from 23 at p(o) = 0 to 8 at p(o) = 40.

(ii) As shown in Figure 10, the usual inverse phase effect occurs whenever the scattering function has a maximum at $\theta = 0$ and a dip somewhere between $\theta = 0$ and $\theta = 180°$.

(iii) Whenever the scattering function has minima at $\theta = 0$ and $\theta = 180°$ one obtains an incipient inverse phase effect at a large phase angle.

(iv) As found for Rayleigh phase matrix the total variation is larger for weaker lines.

In conclusion we can say that any phase curve of relative equivalent widths can be reproduced by appropriately choosing the scattering function. As such a function can be produced by different combination of Rayleigh and Mie phase matrices it is not safe to draw conclusions about the structure of the atmosphere from such phase curves alone, alternate cheques have to be employed. Anyway all such theoretical investigations have lost their importance with launching of the direct space probes into the atmospheres of the planets.

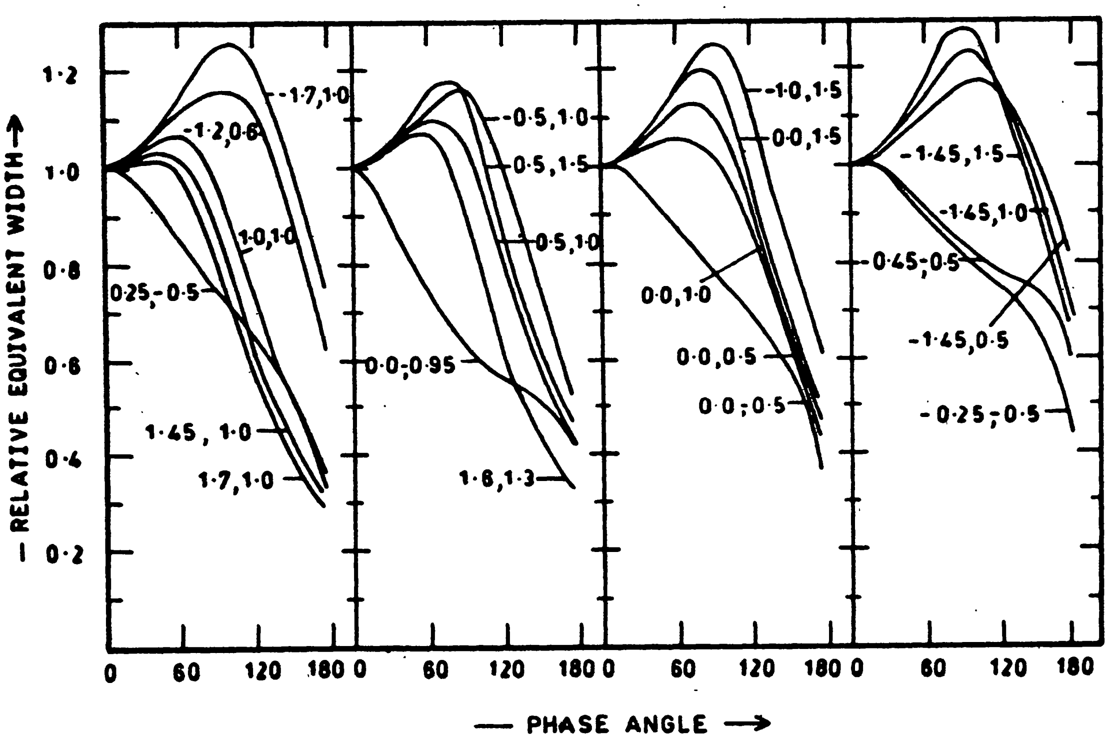


Figure 10. Variation of relative equivalent width for line strength $\varpi_0 = 0.1$ for various scattering functions specified by pairs of $\bar{\varpi}_1$ and $\bar{\varpi}_2$.

8. Concluding remarks

From this review of my work done during 1959 to 1995 it would be clear that Prof. S. Chandrasekhar has been my guiding star. So although I was not his student like S. K. Trehan and Bimla Buti I claim to be one a la Ekalavya. In fact when I met Prof. Chandrasekhar at IUCAA in December 1992 he recognised me as a person who has worked in his favourite field. I would like to end this talk with that happy memory.

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