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On the accuracy of the caustic test

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1. Introduction

The Gaviola test is a standard method for evaluating and testing the surface quality of aspheric mirrors generally used in astronomical telescopes. It is alternatively called the Caustic test. It was introduced as a new method of surveying optical surfaces and systems by Paltzeck & Gaviola (J. O. S. A., 1939, 29, 484-500). Subsequently it has been routinely used for testing mirrors, especially in the field of astronomy. It has been employed even for the 200 inch Hale telescope. The test can be used to yield results with an accuracy of $\lambda/100$.

The underlying principle behind the test is that the centre of curvature for an off-axis segment of a paraboloidal mirror does not lie on the optic axis. The locus of the centres of curvature, of different off-axis elements of a parabola, lie along a curve called the caustic. Testing of a mirror at its centre of curvature will therefore imply testing it along its caustic curve. The essential steps involved in conducting the test are as follows.

Figure 1 shows a paraboloidal mirror, with radius of curvature R, and its paraxial centre of curvature at C. Consider an illuminated slit placed at C. Let the entire mirror be masked with the exception of two apertures centred at r and $-r$ (in Figure 1 we have shown only one zone at $-r$ for simplicity). It is then shown by Schroader (A. T. M. III, 1954, Ed. A. G. Ingalls, Kingsport press) that the two images of the slit are formed at H and K respectively such that the distances X and Y are given by

$$\mathrm{X} = \frac{3r^2}{R} \tag{1}$$

$$\mathrm{Y} = \frac{2\mathrm{r}^3}{R^2} \tag{2}$$

During testing the mirror, if the observed value of Y (call it Y_{obs}), is found to be different from Y, by an error given by

$$Error = Y_{obs} - Y \tag{3}$$

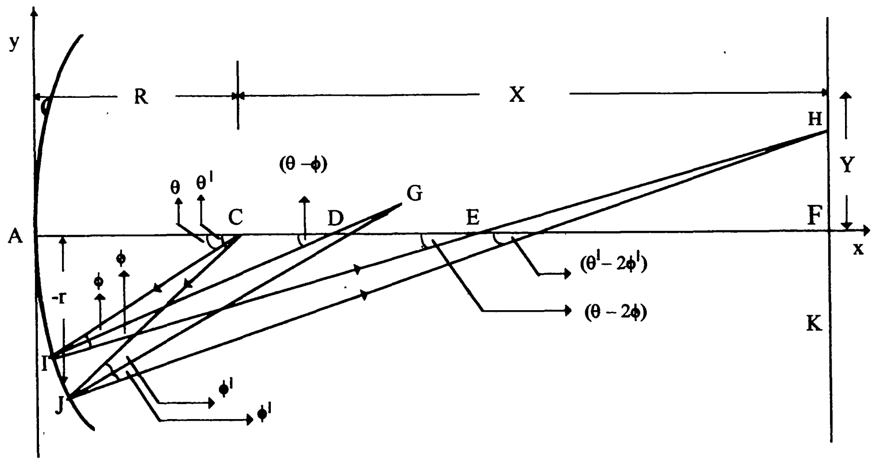

Figure 1. A schematic diagram (not to scale) of a parabolic mirror, with its pole at the origin of the coordinate system A. The centre of curvature of the mirror is at C. For a light source kept at C, the image of an off-axis element IJ (at depth - r) is formed along the caustic curve at H. GI and GJ are the normals (schematic) to the mirror at I and J.

then, the glass - height deviation at the mirror surface, indicating deviations from the necessary shape, is given by

$$deviation = k \times error \tag{4}$$

where k = (width of each zone)/4R. This deviation at the mirror surface, from the desired parabolic shape, will impair the image quality.

2. Derivation of more exact equations

The exactness of equations 1 and 2 is therefore crucial to the Caustic test. From the literature it appears that they are the standard equations used for the Caustic test. However, we wish to show below, that these equations lack sufficient precision especially when the mirror being tested is either large and / or has a small *f*-number (fast systems). By an analytical derivation, without approximations, we get more exact values for the distances X and Y of equations 1 and 2 (see figure 1). They take on new values X_N and Y_N given by :

$$X_N = \frac{3r^2}{R}\;\frac{(1+r^2/R^2+r^4/12R^4+r^6/12R^6)}{(1-r^2/2R^2-3r^4/2R^4)} \tag{5}$$

$$Y_N = \frac{2r^3}{R}\;\frac{(1+r^2/R^2)}{(1-r^2/2R^2-3r^4/2R^4)} \tag{6}$$

It may be noted that when r $\ll$ R, the derived values of X_N and Y_N tend to their corresponding values X and Y as given by Schroader (1954).

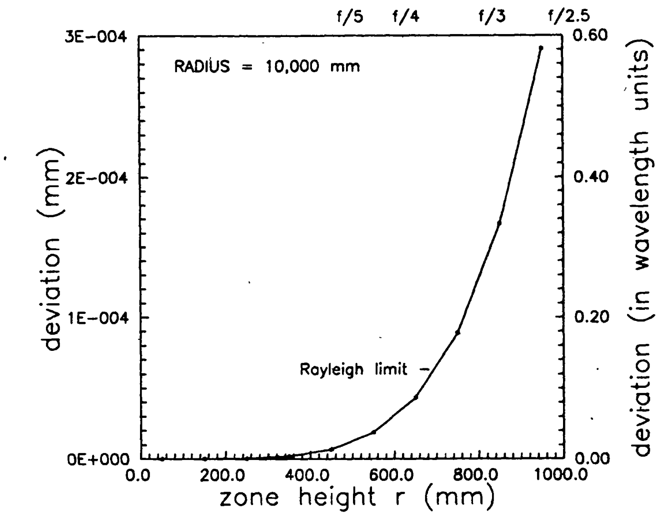

Figure 2. a graphical representation of the deviations at the mirror surface for different zone heights r (see figure 1). The deviation has been shown in milimeters and also in fractions of wavelength (l=5000 Å).

3. Analysis and discussion

It is known from Rayleigh's quarter-wave criterion that the image quality of a star begins to deteriorate if the wavefront forming it contains path differences exceeding $\lambda/4$. This would mean that the maximum allowed deviations at the mirror surface should not exceed $\lambda/8$. In figure. 2, we have shown the deviation in millimeters and also in fractions of wavelength (for λ = 5000 Å), calculated from equation 4 for different zones of the mirror (different *r* values). The zone width has been chosen to be R/100 - a typical value for the Caustic test. We have considered a mirror of radius of curvature R = 10,000 millimetres. On the upper x-axis we have indicated the *f*-ratio of the mirror, if the mirror had a radius equal to the corresponding *r* value on the lower x-axis. As may be seen, the deviation is small for an *f*/5 mirror when the approximate values of *X* and *Y* are used. But it increases rapidly at the edge of a fast mirror. For example, from figure 2, at the edge of a *f*/2.5 mirror (*R* = 10,000 millimetres; diameter = 2 meters) the surface deviation is ~ 0.6λ which exceeds the desired $\lambda/8$ limit. Similarly, for a *R* = 20,000 millimetres, *f*/2.5 mirror (diameter = 4 metres) the deviation at the edge is ~1.2λ. Since, in astronomical applications short-focal length, large diameter telescopes are the norm, the correction to the approximate form of the Gaviola test, as shown here, is of importance.

4. Conclusions

We have analysed here the Caustic test, used for testing astronomical mirrors. We have derived, in an exact way, two of the key equations used in this test. The new equations are shown to give significant improvement in the accuracy of the test. They have practical applicability in the case of all small *f*-ratio mirrors - a characteristic common to most telescope primaries.