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MAGSAT AND GEODYNAMO

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Abstract

Accurate, uniform and global vector magnetic field data obtained by MAGSAT provided a high resolution model of the earth's magnetic field It became possible to separate the core and crustal parts in spherical harmonic representation Such a separation also permitted a reliable projection of the field and its secular variation at the core mantle boundary (CMB) The results show that quadrupole and higher order multipoles originate from currents at the CMB The dipole term is about 80% from currents involving a part or whole of the core and the remaining 20% from the surface currents The vertical component (B_r) and its secular variation at CMB were used to estimate the field velocity under the assumption of frozen field core (FFC) approximation Purely toroidal flow does not fit the observed data as satisfactorily as the combined poloidal and toroidal flows Poloidal motion represents upwelling and downwelling hence vertical motions This indicate that either the top of the core is not stably stratified or that such stratification is insufficient to stop vertical motion at depth The fluid flow has a bulk westward drift with superposed jets and vortices The ratio of westward velocity and the radial derivative of vertical velocity supports gravitationally and/or thermally powered dynamo The FFC approximation itself is found to be violated over patches at the CMB Features like sun spots are seen under southern Africa and South America Postulates for occurrence of such spots are discussed and it is argued that it may now be desirable to study solar and geo dynamo jointly

1 Introduction

It was Sir Joseph Larmor who suggested in 1919 that the magnetic field of the sun might be sustained by a mechanism analogous to that of self regenerative dynamo However, no one has yet succeeded in building a fluid dynamo in laboratory and hence our best chance to study this fundamental physical phenomena remain through observations made on the closest available dynamo the geodynamo This dynamo operates in the liquid outer core of the earth and manifests itself over the earth' surface through the main magnetic field that we observe Studies of this process through theoretical approaches are no less easy because of the non linear and probably chaotic nature of the problem MAGSAT (magnetic field satellite) by making vector measurements of the earth s magnetic field from a low altitude has provided a global data of uniform quality to study the geodynamo This data set has been used to develop a good model of the main field and the knowledge gained has subsequently been used to analyse in a better way the past observatory and survey data The new and interesting information on the geodynamo that these studies brought out forms the subject matter of this article Before taking that, it will be relevant to point out the basic differences of solar and geodynamo Firstly whereas the latter is deeply buried inside a non conducting container the mantle, for the former this is not the case Secondly the balance of forces seem to be quite different in the two cases, so the sun and earth probe different part of the parameter space of the fluid dynamos For example, the magnetic Reynold's number which is about 300 for the earth's core has perhaps a value of 10^6 for the sun However, basic similarity is there between the two, since both obey the same magnetohydrodynamic equations

Structurally, the article first describes the MAGSAT mission and the representation of the main field Then we give a brief outline of the geodynamo Subsequently, the derivation of the fluid velocity at the core mantle boundary (CMB) i summarised We shall also discuss the new information on the geodynamo that has emerged from the patterns observed in the fluid flow The article will conclude by showing that the geo dynamo is still a complex problem and the MAGSAT has provided some boundary conditions to numerically handle the problem Processes like sunspot eem to occur on CMB. The frozen flux approximation that make fluid dynamos a workable problem, appears to fail in the case of geodynamo These are results of special significance for olar and planetary studies

2. The MAGSAT Mission

One way of studying the geodynamo is through the field it produce on or near earth's surface called the main field For an adequate repre entation of the main field it is necessary that it be measured over the whole globe Satellites have made tremendous impact in this regard no region of earth i inaccessible to them The data collected by them is of uniform quality and free from secular drift as they cover the entire earth' surface in a short time Such measurements have another advantage in the sense that hort wavelength crustal features get automatically uppressed Thus, in the absence of crustal contribution, with such a data set a better model could be developed for the main field

Earlier to MAGSAT, satellite data were collected with POGO and COSMOS series of satellites These did considerably improve the representation of main field but limita tions were encountered because the measurements were scalar in nature In this context, MAGSAT has been a great improvement by making measurements of both the magnitude and direction of the field As the primary aim of the mission wa to provide data for crustal studies, the altitude was kept low This led to a short life time of the mis ion October 30, 1979 to June 11, 1980 Even then, it did provide adequate data to deduce a reliable model of the main field On the other hand, for estimating the rate of change of the model parameters (i e the secular change) the time span was not enough For the latter, needs remained to combine MAGSAT data with earlier measurements This naturally has limited the accuracy of the secular variation estimates

The characteristics of MAGSAT mission are described in Langel et al (1982) The satellite altitude varied between 350 and 560 km with an inclination of 96 8° The altitude was chosen to provide a compromise between better resolution of cru tal features and sufficient coverage of the globe The objective of vector field accuracy to 0 01%, required the orientation of the sensors be known to 15 arc seconds Further, the altitude of the satellite must also be accurately known The small Astronomical Satellite SAS 3 met these requirements It is a small spacecraft capable of being launched by an inexpen sive scout rocket Precise determination of the orbit is possible with the Doppler tracking system and attitude with two star cameras to an accuracy of 10 arc seconds But the system had a 2 kgm of essential magnetic shielding and thus the magnetic sensors had to be put 6 meters away The boom made for the purpose turned out to be mechani cally unstable within 5 arc seconds A system called attitude transfer system (ATS) was then added to provide the orientation of the sensors through an optical technique With these arrangements the orientation of the sensors were determined correctly to 10 to 20 arc seconds and position of the satellite to better than 60 meters radial and 300 meters horizontal The final error budget for the measured values (Langel et al, 1982) came to

Error Source	Scalar (nT)	Vector (nT)
Instrument	1 5	3 0
Position and time errors	1 0	1 0
Digitisation noise	0 5	0 5
Attitude error (20' @ 50000 nT)		4 8
Space craft fields	0 5	0 5
r s s	1 96	5 8

Other special features of MAGSAT were a sun synchronous dawn dusk orbit to reduce ionospheric contribution and coils for controlling the attitude of the satellite In terms of achieving the scientific objectives the mission has been highly successful Its data has given a good representation of the main field The secular variation estimates had also improved The coefficients of the international geomagnetic reference field (IGRF) can reproduce the field at the earth surface to an accuracy of 20 nT a great improvement over earlier model One could model the magnetic field within an error limit of 0 05 per cent

3 Main Field Representation

Models developed to represent the main field are based on the assumption that the region is source free Under these conditions the magnetic potential V satisfies Laplace equation

$$\nabla^2 V = 0, \quad (1)$$

and the magnetic induction $\vec{B}$ of the field is given by

$$\vec{B} = \nabla V \quad (2)$$

In spherical coordinate system under the condition that the source of the field is inside the earth, the expression for V written as

$$V = a \sum_{n=1}^{\infty} \sum_{m=0}^{n} \left(\frac{a}{r}\right)^{n+1} \{ g_n^m \cos m\lambda + h_n^m \sin m\lambda\} P_n^m(\cos\theta) \quad (3)$$

where a = 6371 km is the radius of the earth and r is the radial distance of the point of observation $P_n^m(\cos\theta)$ is the Schmidt quasi normalized function θ and λ are the co latitude and longitude of the point of observation g_n^m and h_n^m's are called the Gauss coefficients The set of these coefficients is the model parameter and once this is given the field can be calculated for any given r, θ, λ In practice, the series (Eq 3) must be terminated and different representations vary in their choice (n_{max}) of the maximum value of the degree n One would expect the coefficients g_n^m and h_n^m to remain independent of n_{max} as the base is orthogonal However, this is seldom true, because of the summation that replaces intigeration in actual calculation Such an approximation makes g_n^m and h_n^m depend on n_{max} Thus the model parameters are n_{max}, g_n^m s and h_n^m 's

The main field of the earth being variable with time makes the Gauss coefficients time dependent The variation is rather large to the extent that coefficients change significantly even over a period of one year Such a situation requires constant revision of the coefficients As their rate of change is highly non linear, models are continuously updated at gaps of five years We now have published and accepted models starting from year 1965 Each of these has a validity period of 5 years and is called international geomagnetic reference field (IGRF) for the epoch Models available thus are IGRF 1965 0, IGRF 1970 0, IGRF 1975 0 and IGRF 1980 0 The year with the model refers to the beginning epoch of the models Models contain values of g_n^m, h_n^m, $\dot{g}_n^m$, $\dot{h}_n^m$ and sometime also $\ddot{g}_n^m$, $\ddot{h}_n^m$ It is ensured that continuity of field values is maintained at the end of the IGRF and beginning of the next one

The Gauss coefficients and their time derivatives are estimated in such a way that the model values fit the measured fields in the sense of least squares The model values of the field are estimated through the equations

$$X = \partial V/\partial r \quad (4)$$

$$Y = \frac{1}{r}\frac{\partial V}{\partial \theta};$$

and
$$Z = \frac{1}{r \sin\theta}\frac{\partial V}{\partial \lambda}$$

The total field B is determined from

$$B = (X^2 + Y^2 + Z^2)^{\frac{1}{2}} , \quad (5)$$

Each of X Y Z or B are functions of g_n^m and h_n^m and one can use them to get an estimate of the Gauss coefficients Before starting to calculate the coefficients, a choice has to be made on maximum degree (n_{max}) of terms to be retained in equation 3 One would like to take a large n_{max} to improve the fit The choice hould take into account the resolving power of data vis a vis spectral features of core and the crust We must be aw ire that the number of coefficients in the series, increa es exponentially with n_{max} e g , the e are respectively 80, 120 and 195 for n_{max} = 8, 10 and 13 Further, when the data to be fitted is not for a particular epoch, rather i spread over a time span time, dependence of g_n^m and h_n^m has to be incorporated in formulation Even if we consider only g_n^m and $\dot{g}_n^m$ terms, the number of parameters to be estimated gets doubled The scale length of features accounted by n_{max} also limits its choice The horizontal wavelength of degree n on earth s surface being [4πa/(2n+1)], the smallest wavelength included in the expansion will be 4000 km with n_{max} = 10 and 3000 km when n_{max} = 14 We know that isomagnetic charts contain features of size 1000 km or even less, terms with n > 14 are then needed to properly represent the observed field Before that it must be ascertained what origina tes in the core and what in the crust In this context, the quality of MAGSAT data has provided useful informations We will show later that core is considered to contribute mainly to $n \leq 12$ terms, the crust to $n \geq 16$ terms and both contribute in the band $13 \leq n \leq 15$ This will be apparent from the nature of the spatial power spectrum (Meyer et al, 1983)

The spatial power spectrum can be expressed as the sum of squares of the (2n+1) coefficients, i e

$$W(n) = (n+1) \sum_{m=0}^{n} [(g_n^m)^2 + (h_n^m)^2] \quad (6)$$

Physically W(n) defines the energy density of the constituent field averaged over the whole earth, except for the factor ($\frac{1}{2}\mu_0$) (Lowes 1966) Its variation as a function of n, shown at the top of Fig 1, represents the changes of mean energy density The figure is taken from Meyer et al (1983) and is based on Gauss coefficients estimated by J C Cain of US Geological Survey The model used MAGSAT data selected for quiet periods with n_{max} = 29 The variation of W(n) splits into two quasi linear sections with change in slope taking place around n = 15 The right portion from n = 16 onwards resembles a white noise and seems to originate from crustal sources The left portion decreases linearly with n upto n = 12 and must surely be attributed to deeper sourc s, i e the core dynamo Because, for a random source in the crust $\ln < |B_n|^2 >$ would be approximately independent of n, we attribute terms $n \geq 16$ to have crustal origin We may thus conclude that at the surface of the earth the core contributes mainly for $1 \leq n \leq 12$, crust mainly for $n \geq 16$ and both for $13 \leq n \leq 15$ The dipole term (n = 1) clearly stands above the general trend of the figure We shall see later that it has a different source mechanism

Equation (3) permits a possibility of analytically continuing the field downwards inside the earth, if we assume the mantle to be source free This assumption is not unjustified in view of the fact that the mantle is both a poor conductor and has a permea bility close to that of vacuum Continuation downwards beyond mantle will be incorrect since equation (3) will not apply in core (the core is not source free) In the situation that exists inside earth, if we keep plotting $\ln < |B_n|^2 >$ against n for different depth levels, a stage will be reached where this parameter will become independent of 'n' The depth will mean the location of the boundary of the source causing the field In the lower portion of Fig 1 we see that such a situation occurs at a radius of 3471 km The plotted values were obtained on multiplying each of the W(n) by a factor

$$\left[\frac{6371}{3471}\right]^{2(n+2)}$$

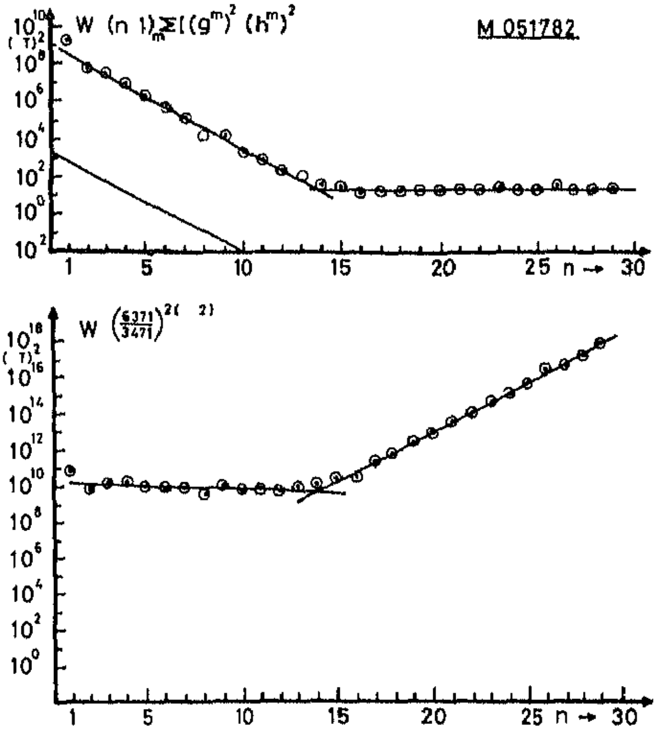

Fig. 1 Observed spatial energy density spectrum W(n) of the earth's magnetic field W(n) represents the total mean square contribution from the harmonic of degree n The bottom diagram shows the variation of W(n) at the core mantle boundary (From Meyer et al 1983)

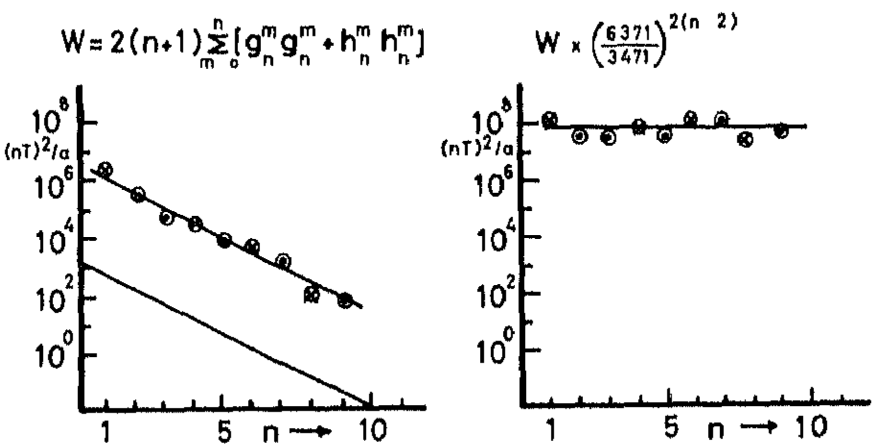

Fig. 2 Observed spatial energy density spectrum of the secular variation W(n) for the earth's surface on the left and for the core mantle boundary on the right (From Meyer et al 1983)

The equation is equivalent to calculating ln $<|B_n|^2>$ at a radial distance of 3471 km This estimate compares favourably with seismic radius of the core 3485 km In fact, in the plot there is a slight but perceptible decrease of reduced W(n) with n from n = 2 to 12 which corresponds to an effective source layer depth of 162 km beneath the reference level The steep increase for n > 15 is more of formal character arising from the situation that the source causing the field is located outside in the crust The line in the lower left corner of the upper diagram shows the theoretical slope of a white spectrum at the surface of the earth s core i e 2 ln(6371/3471) = 0 5275 This slope is very close to the slope of the n = 2 to 12 line

A similar analysis has also been made by Meyer et al (1983) for energy density of the secular variation, i e , for the time derivative of the spectral function W(n)

$$W(n) = 2(n+1) \sum_{m=0}^{n} [g_n^m g_n^m + h_n^m h_n^m] \tag{7}$$

In this case too ln $<W_n>$ akes the shape of a white spectrum at a radial distance of 3471 km (Fig 2) The regression line indicates an effective source depth of 66 km beneath the surface of the core An important point to note is that the dipole term now does not deviate so significantly from the regression line We may conclude that though the dipole term involves currents in the deeper layers or whole of the core, its secular variation is predominantly from the part originating in relatively thin layer of the core The latter component appears to constitute 15 20% of the observed dipole field

The encircled dots in Fig 2 denote positive rate of change (i e increase in field enerlgy) and crosses indicate negative rate of change or decrease of energy at the CMB. The energy seems to be well balanced Secular variation simply represents a change in the structure of the field and redistribution of energy This does not contradict the decrease in field strength that we are presently observing on the earth' surface By the time the changes reach the surface, different degree (n) terms attenuate differently Over the earth, the observed contribution is mostly from lower degree terms and the decrease that we see is dominantly due to the decreasing dipole field

The analysis of MAGSAT data, in summary, showed that the geomagnetic field has a relatively stable dipole part originating probably in the deeper core or in the core as a whole The secular variation originates from changes in a thin surface layer of the core Secular variation thus becomes a useful parameter to estimate the fluid flow at CMB.

4. Geodynamo

Paleomagnetic studies show that the earth s magnetic field has existed since 2×10^2 years In this period, the direction has undergone many reversals, but the magnitude has never differed significantly from the present value The source of the field is now under stood to be the electric current in the liquid core But, in a bounded and stationary system, an electric current will decay with a time constant proportional to $\sigma\ell^2$ (σ is the electrical conductivity and ℓ characteristic length of the field) For the earth's liquid core, this number turns out to be $\sim10^5$ years Under this situation, the magnetic field cannot be a relic of the past and a mechanism has to be found that regenerates the current in the core and consequently the magnetic field produced by it We believe that the decay is stopped by the motions of the interaction that takes place between the liquid core and the magneti field present there Such an interaction generates electic current that regenerates the magnetic field and the cycle goes on The process works like a dynamo and we call the system geodynamo The major problem is to find a mechanism that reinforces the original magnetic field so that we can have a self excited dynamo The existence of such a dynamo depends primarily upon the relative strength of motional induction and ohmic dissipation terms in the magnetic induction equation (Jacob 1975 Merrill and McElhinny 1983)

$$\frac{\partial \vec{B}}{\partial t} = \frac{1}{4\pi\sigma} \nabla^2 \vec{B} + \nabla \times (\vec{v} \times \vec{B}) \tag{8}$$

The first term on the right hand side is called the Ohmic dissipation term and the second motional induction term If $\vec{v} = 0$, the $\nabla^2 B$ term will cause the field to decay The equation in this case reduces to vector diffusion equation and the field will leak through the material The decay is prevented by the second term $\nabla \times (\vec{v} \times \vec{B})$ which represents interaction between the velocity and the magnetic field The ratio of the second to first term gives an estimate of the rate of build up to decay

$$R_m = \frac{\nabla \times (\vec{v} \times \vec{B})}{(\frac{1}{4}\pi\sigma)\nabla^2\vec{B}} \sim \frac{vL}{\frac{1}{}\pi\sigma} = 4\pi\sigma LV$$

For the earth's liquid cofre, we can take $L = 10^8$ cm, $V = 10^{-1}$cm/sec and σ 10^{-6} emu, which gives $R_m = 100$ The constant R_m is called magnetic Reynold's number and must be greater than unity for the dynamo to be self sustaining This condition is rarely satisfied in the laboratory, but in cosmic masses it is easily satisfied because of large L

An understanding of the working of the geodynamo is still incomplete We know that the magnetic Reynold's number is large enough for the dynamo to work, but we must explain how the dipolar field is generated back by the core The earth's field outside the core is mainly dipolar The liquid motion inside contains some differential rotation which winds up the dipole field into a toroidal field This part of mechanism is simple and well understood By suitable fluid motion the entering field can be amplified to an infinite limit But for the dynamo to work, the dipole field must be produced back from the toroidal field It is this stage of cycle that is difficult to account for Parker (19ננ) suggested that cyclonic or twisting motions are created within the convection cells through the action of corolios force, similar to the circular motions in the atmospheric weather system The resulting convecting lifting and rotational twisting in the cells are considered to turn the toroidal lines of force into a loop of flux This loop includes a new poloidal field regenerating back the dipole field and completing the cycle

The key element making the geodynamo work is the Lorentz force, the $\vec{v} \times \vec{B}$ term or also called the motional induction term This term generates, from mechanical energy, electric currents needed to prevent the decay of magnetic field The greatest limitation in involving the kinematics of the geodynamo has been the paucity of information on this term Its constituents are not known for the core MAGSAT in this regard has made a significant headway by providing their estimates at the core mantle boundary (CMB) We can now expect that the new informations will guide the development of theoretical studies related to geodynamo The procedures adopted to estimate $\vec{v}$ and $\vec{B}$ at CMB are discussed in the next section

5 $\vec{B}$ and $\vec{v}$ at the CMB

It has been mentioned earlier that the secular variation of the magnetic field is mostly associated with currents at CMB The variation comes from the advection of lines of force arising from fluid flow at CMB The magnetic Reynold's number for the liquid core being 100 or more, we can neglect the diffusion term $(1/4\pi\sigma)\nabla^2 B$ in equation (8) and write

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B}) \tag{9}$$

If $\vec{B}$ and $\partial\vec{B}/\partial t$ are known, the equation can be solved for $\vec{v}$ A global consistent solution is not easy to obtain, however, approximate results have been arrived by introducing certain approximations

We are considering a picture of the earth where a spherical invisiced and perfectly conducting liquid core is surrounded by a rigid electrically insulating mantle of vacuum magnetic permeability This simplifies the problem in two days First under thi condition we can neglect the diffusion term of equation (8) and work with equation (9) This is equivalent to a frozen flux approximation where the main field is rooted in the core and the secular variation arises from the fluid flow at the top of the core Secondly, the conditions also permit us to take a source free mantle and continue field downdards to CMB using equation (3) Our formulation is thus based on two approximations source free mantle (SFM) and frozen flux core (FFC) We must first test their validity The appropriateness of SFM is well founded The electromagnetic decay time of the mantle has been estimated to be less than four years, a time span which is rather small compared with characteristic scale of secular variations We may take that the effect of current induced in mantle on relationship between the field at surface and CMB is in ignificant Magnetisation sources are unlikely because the mantle temperature is above Curie iso therm On the other hand, the concept of FFC is not well established Some believe it to be true while there are others who think that over some patches on CMB FFC fails We shall first discuss the results obtained under the FFC approximation and then indicate the finding that prove the failure of this approximation

Fluid flow under SFM FFC approximation has been calculated by Voorhies (1986) The presentation follows his paper He solves for $\vec{v}$ through equation (9) taking the radial component (B_r) of the field B_r was selected because it is the only component of B that is guaranteed to be continuous acros CMB Horizontal component may jump across the diffusive CMB Further, uncertainty arises from unkown contribution of the toroidal component of the core field The r component of equation (9) can be written as (Roberts and Scott 1965)

$$\frac{\partial B_r}{\partial t} = v_r \left[\frac{1}{r \sin\theta} \frac{\partial}{\partial\theta}(B_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial B\phi}{\partial\phi}\right]$$

$$B_r \left[\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(v_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial v\phi}{\partial\phi}\right] +$$

$$\left[\frac{B\theta}{r}\frac{\partial}{\partial\theta} + \frac{B\phi}{r\sin\theta}\frac{\partial}{\partial\phi}\right] v_r \quad \left[\frac{v\theta}{r}\frac{\partial}{\partial\theta} + \frac{v\phi}{r\sin\theta}\frac{\partial}{\partial\phi}\right] B_r \tag{10}$$

We note that the equation

$$\text{div } \vec{B} = 0$$

ensures that B_r does not change in crossing the CMB Similarly because of incompressi bility of the fluid, v_r should remain unchanged across the CMB Since at the outer edge of this boundary the fluid is in contact with the rigid mantle $v_r = 0$ at the CMB From this it follows that

$$\frac{\partial v_r}{\partial\theta} = \frac{\partial v_r}{\partial\phi} = 0$$

After making these substitutions in equation (10), we get for the boundary layer

$$\frac{\partial B_r}{\partial t} = B_r \left[\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(v_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial v\phi}{\partial\phi}\right] \left(\frac{v\theta}{r}\frac{\partial}{\partial\theta} + \frac{v\phi}{r\sin\theta}\frac{\partial}{\partial\phi}\right) B_r \tag{11}$$

Equation (11) contains only the horizontal components of the velocity and their horizontal derivatives To handle this equation, the velocity at CMB (say r = b) is expressed in terms of effective stream function T ($\vec{b}$) and effective velocity potential U ($\vec{b}$)

$$\vec{v}\,(b) = \nabla_s T \times \hat{r} + \nabla_s U \tag{12}$$

The first term on the right hand side is divergence free and is called the rotational part of the velocity field, the second term which is curl free is called the irrotational part of the field r represents unit vector in direction of r and ∇_s represents surface gradient In actual calculation T ($\vec{b}$) and U ($\vec{b}$) are expanded in a spherical harmonic form as

$$T(b\ \theta\ \phi) = \sum_{n=1}^{N_T} \sum_{m=0}^{n} [a_n^m \cos m\phi + b_n^m \sin m\phi]\ P_n^m(\cos\theta),$$

$$U(b,\theta\ \phi) = \sum_{n=1}^{N_U} \sum_{m=0}^{n} [C_n^m \cos m\phi + d_n^m \sin m\phi]\ P_n^m(\cos\theta) \qquad (13)$$

N_T AND N_U are truncation level of the expansions The computational work comprises determination of a_n^m b_n^m, c_n^m and d_n^m such that $v(\vec{b})$ calculated with them best fits the observed$(\partial B_r/\partial t)$ in sense of least squares The choice of N_T and N_U is made by examining the stability of the solutions

The physical condition for validity of equation (11) are (i) that $v_r = 0$ at the top of the core and (ii) that magnetic flux diffusion in the core be negligible over a period of few decades and over length scales of megameters For the second condition to be fulfilled the core should have high conductivity and the geomagnetic field considered should have large lateral wavelengths Backus (1968) has shown that the solutions of equation (11) are not unique To a given solution $\vec{v}_0$ say, an arbitrary purely toroidal flow can be added without affecting B_r A purely toroidal flow, which is everywhere tangential, does not cause B_r to change An alternative way of looking at this ambiguity is that at most point on CMB equation (11) is but one equation is too unknowns v_θ and v_ϕ and to reach a unique solution some additional information is required Voorhies and Backus (1985) suggest that this limitation can be circumvented by supposing the flow at CMB to be steady over time scales of a decade or two They show that under such an assumption the toroidal ambiguity can be resolved $v(\vec{b})$ can be determined uniquely In essence, uniqueness in solution is introduced by considering simultaneously a set of equations in velocity field each pertaining to a different time The solution, of course, is the one that satisfies all of them in the sense of least squares

Before describing their results, it is relevant to first test the validity of the SFM and FFC approximations Voorhies (1986) argues for the effectiveness of FFC on three consideration The radial distance of CMB estimated through frozen flux concept is very close to the value determined from seismic methods No significant change in the absolute flux linking the core has been found over the last 50 years if we consider terms upto degree eight in equation (3) and over the last 20 years if terms upto degree ten are considered Thirdly, lthe magnetic flux linking each patch bounded by null flux contours on CMB are fairly constant for several of the recent geomagnetic field models The last point is being questioned and we shall take it again in a later section SFM approximation seems justified because the mantle conductivity does not seem to affect secular variations of period greater than four years or so

Solutions for $\vec{v}(\vec{b})$ were obtained by changing the degree N_B (i e n_{max} of equation 3) and by varying N_T and N_U Results were divided into two groups: one for steady purely toroidal flows for which $N_U = 0$ and another for combined toroidal poloidal flows i e for which both N_T and N_U are non zero Main field data used for the period 1960 80 i e we assume $\vec{v}(\vec{b})$ to remain steady for this period of 20 years Flow pattern of the combined poloidal toroidal solution is given in Fig 3 A bulk westward drift of about 0 107°/yr relative to the base of the mantle is noticed This is equivalent to a speed of 6 51 km/yr at the geographic equator The westward drift is complicated by superimposed jets and gyres plus fluid upwelling and downwelling The mean kinetic energy per unit volume at the top of the core is

$$\tfrac{1}{2}\ \rho_c (v_{rms})^2 = 1\ 5 \times 10^{3}\ J/m^3$$

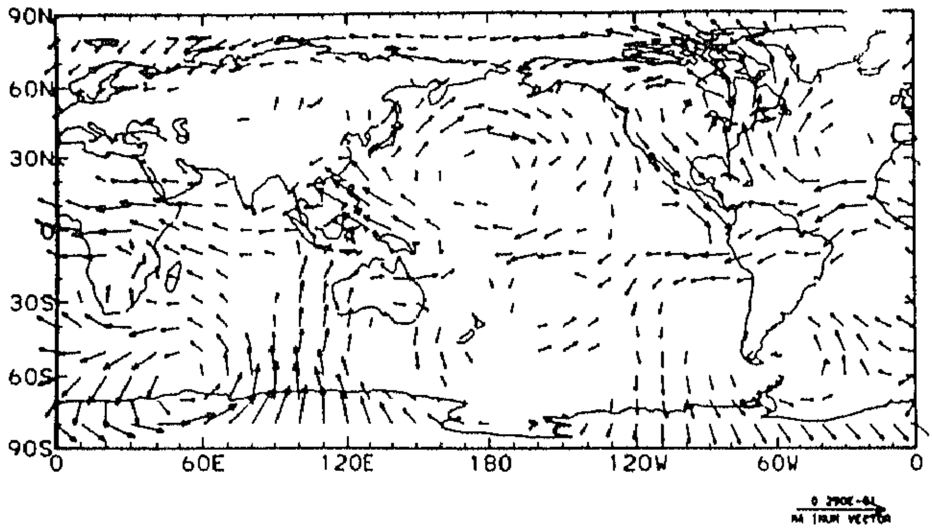

Fig.3 A teady combined flow at the top of the earth's liquid core calculated with $N_T = N_U = 8$ The reference vector is 87 125 km/year (From Voorhies 1986)

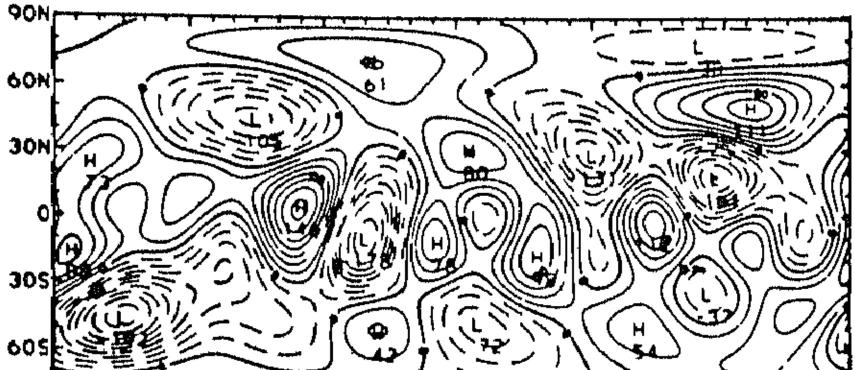

Fig.4. Steady downwelling at the top of the earth s liquid core calculated with $N_T = N_U = 8$ Labels are in units of 10^{-4} yrs^{-1} and contour interval is 2×10^{-3} yrs^{-1} (From Voorhies 1986)

for $P_c = 9.9 \times 10^3 kgm/m^3$. If this value is typical of the flow throughout the entire outer core, the total K.E. of the core motions comes to 2.5×10^{17} joules or about 2 hours worth of geothermal flux across the earth's surface.

Another important finding of Voorhies has been that none of the purely toroidal flows give a good fit to the secular variation data. Fit improves when both toroidal and poloidal flow are considered. A poloidal part indicates surface divergence or convergence of the fluid motion. Under the assumption that the fluid is incompressible, surface divergence (or convergence) implies upwelling (or downwelling) hence vertical motion at depth. The downwelling or surface convergence $\partial v_r/\partial r$ derived with $N_T = N_U = 8$ are shown in Fig 4. The contour interval is 2×10^{-3} yr^{-1} and the extrema are labelled in units of 10^{-4} yr^{-1}. Maximum upwelling is found near 45°S 30°E with magnitude 1.92×10^{-2} yr^{-1} and maximum downwelling near 5°N 110°E with magnitude 1.46×10^{-2} yr^{-1}. There are about five centres of strong upwelling, three centres for strong downwelling and large region of downwelling under the central Pacific and Africa. It seems upwelling is more concentrated (plume like) than is downwelling.

Root mean square (rms) value of upwelling is $(7.33 \pm 0.93) \times 10^{-3}$ yr^{-1} or $(2.32 \pm 0.29) \times 10^{-10}$ sec^{-1}. The ratio of rms velocity to rms value of gradient of the vertical velocity provides an apparent scale height of vertical motions. This ratio comes out as 2340 ± 300 km which includes the 2260 km thickness of the outer core. This result supports a gravitationally and/or thermally powered dynamo.

Presence of upwelling near the CMB shows that either the top of the core is not stably stratified or that such stratification is insufficient to prevent vertical motion at depth. Representative value of the flow speed at CMB is 17 km/yr, or upwelling 7.3×10^{-3} yr^{-1} and of mean westward drift 4.9 km/yr. The typical value of surficial rigid body rotation rate is 0.11° yr^{-1}. This proceeds in a westward sense about an axis inclined to an angle of about 22° to the axis of geodetic coordinate system.

6. The Frozen Flux Approximation

The fluid velocity at the CMB that we discussed in the last section was determined under the frozen flux hypothesis. The calculation neglected magnetic diffusion over time scales of few decades and length scales of few thousand kilometers. Bloxham and Gubbins (1985) mention that this may not be a valid supposition since the early attempts of testing the hypothesis paid insufficient attention to the problem of non uniqueness. They argue that the process of calculating the field at CMB by downward continuing equation (3) truncating the series at n = 8 or 10 is totally arbitrary. While at or near the earth's surface the core contribution decreases with n and becomes insignificant for $n \geq 16$, at CMB the situation is different. We have seen in Fig 2 that all terms contribute equally to W(n) at CMB and may be terms with $n \geq 12$ are important there which the earlier methods have not been able to resolve. Bloxham and Gubbins have tried to remote this limitation by estimating g_n^m and h_n^m through the technique of stochastic inversion. Some prior information on g_n^m and h_n^m is introduced so that the estimates are biased towards the core contributions. The procedure is detailed in Gubbins and Bloxham (1985) and is based on Bayesian formalism. The Bayesian interpretation of probability takes a mathematical expression for our belief in some particular proposition against the restrictive frequency distribution of sampling theory. Through Baye's theorem, a posterior distribution of the values of the Gauss coefficients (g_n^m, h_n^m) is prepared taking into account our belief of their permissible range.

In practical application, we must realize that the fit to the data will be best without restrictive assumptions. Prior informations in general reduce the quality of fit, still they are sometimes necessary for they drag the solution to the real "physical world". A parameter (λ) called damping factor is introduced to control the bias. Large values of λ place more emphasis on prior information and smaller values of λ place more emphasis upon fit to the data. A trade off between misfit and norm is made by varying λ. Gubbins and Bloxham supposed that variance in the radial field at CMB is independent of

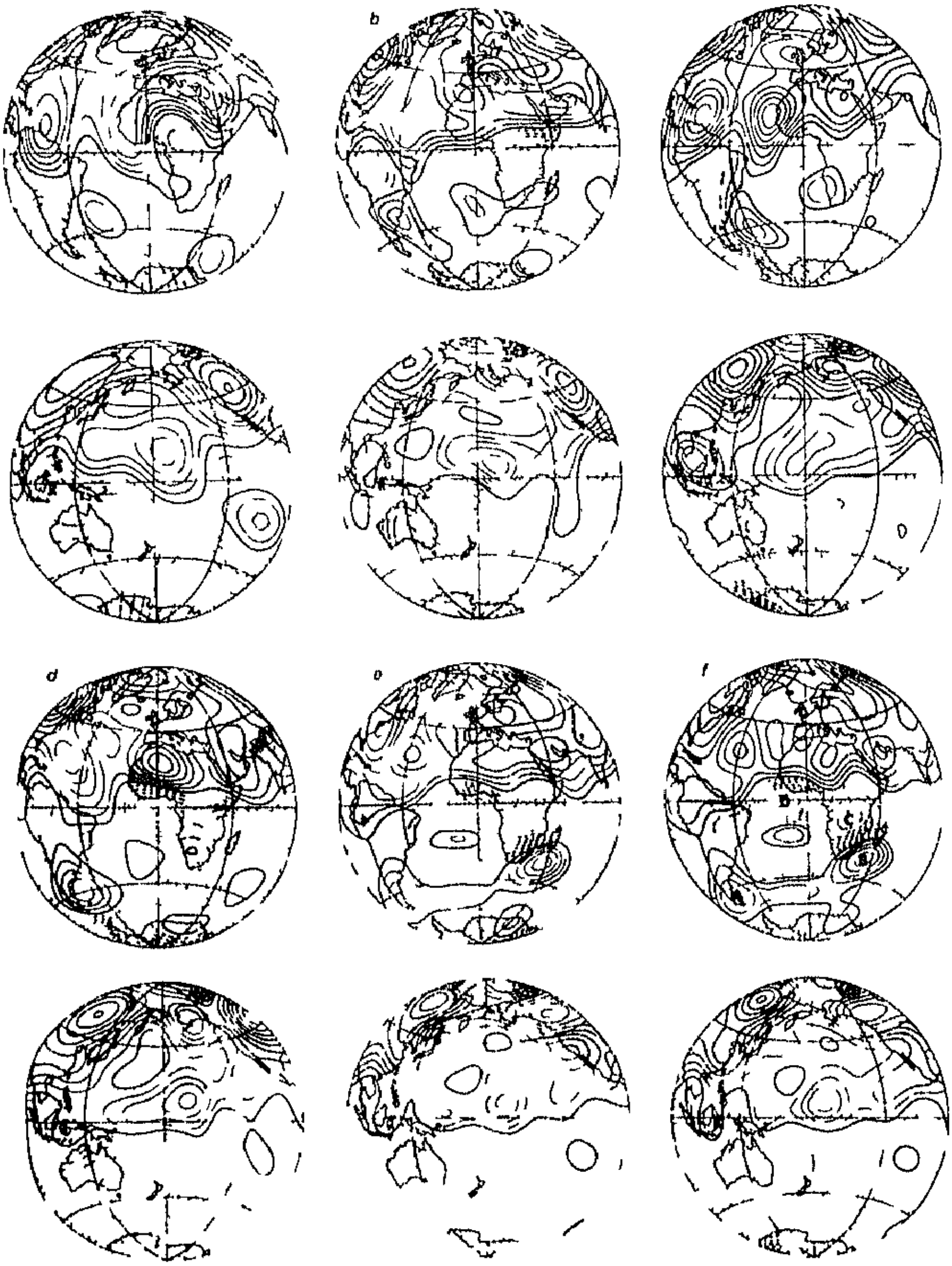

Fig.5 Contour plots of the radial field (B_r) at the core mantle boundary for 1715 0 (a); 1777 5 (b), 1842 5 (c), 1905 5 (d); 1969 5 (e) and 1980 0 (f) The contour interval is 100 μT Solid contours correspond to field into the core and broken ones to field out of the core Zero radial field contours are darkened (From Bloxham and Gubbins 1985)

the degree of the pherical harmonic term This follows from the physical situation that the magnetic field originates in the core The constraint is a legitimate supposition for any dependence on the degree (n) would imply a departure from spherical symmetry, which is not to be expected Also in Fig 2 we find that W(n) is independent of n at CMB

They find that core does contribute to terms greater than degree 12 Above n= 14 the prior information takes over and only harmonics greater than 20 are effectively zero Consequently, in their re ults, Bloxham and Gubbins (1985) see features of shorter wavelength Contour plots of the radial field at CMB for epochs 1715 0, 1777 5, 1842 5, 190 , 1969 5 and 1980 5 are given in Fig The features are more complex than hitherto a sumed The flux integral over the northern hemisphere indicates a fall in magnitude, mainly due to the decay of the dipole term The dip equator under the Indian region appear to have migrated southwards in recent years Westward drift occurs only over certain well defined regions of the core

Backus (1968) mentions that under the frozen flux hypothesis the radial flux through patches of CMB bounded by contour of zero radial field must be conserved It becomes immediately obvious from Fig 5 that the hypothesis is not true over the CMB A apid intensification of patch unde Southern Africa is an obvious violation of the FFC concept According to Bloxham (1986) the situation seems to arise from the intraction of fluid upwelling and toroidal field The upwelling convects the toroidal field towards the CMB as illustrated in Fig 6 Such a process results in two adjacent regions of intense flux with radial fields of oppo ite sign as seen under South Africa A similar pair is also observed near South America The two differ in the ense that while the Southern African loop i presently expanding as a result of upflow, the South American loop is contracting as a result of downflow

These two are unique regions in the sense that a special phenomenon is seen there We must examine what makes these two regions to unique that underneath them we have both intense vertical motion and a strong toroidal field There is similarity here with solar physics as the phenomenon appears to show features of sunspot activity

7 Conclusion

So far, the solar and geomagnetic field studies have remained as totally independent disciplines This has been so because of the different physical conditions over sun and the earth The geodynamo is deeply buried inside a non conducting mantle which is not the case for the sun The solar magnetic field should be compared with the magnetic field that we see on the CMB Now that the latter is being mapped, the two disciplines could complement each other There will remain some obvious limitation which will be difficult to eliminate Most obvious amongst these is the condition that unlike sun the plasma can not flow radially outwards at CMB due to the presence of the mantle Now that magnetic and velocity fields are available, one should attempt to see whether the analogue of short period fluctuations that occur in solar plasma are present in the earth's liquid core Perhaps, the boundary condition may not permit the same Even if short period changes have been occurring at CMB, they won t be recorded at the earth's surface for the mantle acts as a low pass filter and permits signals of periods greater than 4 years only to reach the earth's surface

The refinements in the model of the geomagnetic field that followed MAGSAT data and the eventual estimation of fluid velocity ($\vec{v}$) and the magnetic field ($\vec{B}$) at CMB provides informtion for direct comparison of the solar and geomagnetic fields One interesting finding is equivalent of "Sun spot" under Southern Africa The process occurring there seems to be identical to what causes spots on the sun The physical parameters like magnetic Reynold's number, etc and the dimension of the two plasmas being different, comparison of observations will need caution; but this difference can also be a good diagnostic tool An interesting area of research immediately could be to follow in detail the development and decay of two "CMB-spots" Future satellite missions will definitely improve the geomagnetic field models and will be providing data base to see the spatio

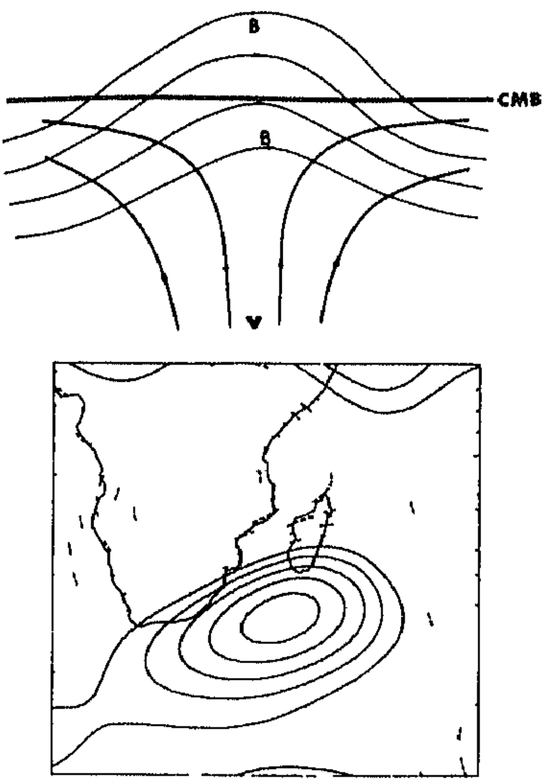

Fig.6. Top Schematic illustration of expulsion of toroidal field from core to the mantle by an upwelling motion of the fluid in the core
Bottom Contour plots of the radial field (B_r) at CMB for the epoch 1969 5 Solid contours correspond to field into the core and broken ones to field out of the core (From Bloxham 1986)

temporal developments of the two spots We then will have a real opportunity to integrate solar and geo dynamo The validity of frozen flux hypothesis needs to be further checked with the oncoming missions If proved untrue, the fluid velocity given in Figs 3 and 4 will become questionable We will then need a new thinking on the solution of magnetic induction equation Here solar physics can give clues Another major finding has been physical evidence of presence of strong toroidal field in the core We have also learnt that the geodynamo is either gravitationally or thermally powered A time has now come to study the solar and geo dynamo jointly for that may bring out a better under standing of one of the fundamental processes of nature the "co mic dynamo"

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