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# Contribution of turbulence to Tully - Fisher Relation

R.D. Prabhu [a, b, 2] and Vinod Krishan[a, 1]

[a] *Indian Institute of Astrophysics, Bangalore 560 034, India*

[b] *Indian Institute of Science, Bangalore 560 012, India*

**Abstract.** We have proposed a model for the flat rotation curves of spiral galaxies in an earlier paper of ours (Prabhu & Krishan 1994). We could resolve the galactic velocity field for the outer regions into a turbulent and a gravity component. Since the Tully-Fisher relationship highlights a tight correlation between the galactic velocity and its luminosity, we think it is worthwhile to study the individual correlation between the luminosity of a galaxy and its turbulent and gravity components of velocity. Towards this end we have modelled the velocity fields of 76 galaxies and the individual correlations were studied in the U, B, V, I and $I_{23.5}$ bands. This sample is severely limited by the fact that the overlap between the set of galaxies which have been photometrically observed in all the related bands and the set consisting of galaxies whose rotation curves are available is very small. Nevertheless, the study revealed an interesting feature viz. the turbulent component of the velocity correlates better than the gravity component, for the U, B and V bands, whereas the gravity component correlates better with the luminosity in the I bands. This is expected since the long wavelength luninosity of a galaxy is very sensitive to its mass. The significant correlation between the short wavelength luninosity and the turbulent component of velocity indicates that our velocity model is doing well as it reproduces the expected correlations. Thus from the statistical significance of our results, we conclude that turbulence - apart from contributing to the spectral line widths in the form of random motions - also gives rise to ordered motions, which manifest through rotation curves.

## 1. The model and the observations

Nature exhibits a tendency to self-organise on various length scales and time scales. This feature of self-organisation is also evident in the form of cyclones and storms which may be viewed as typical coherent structures emerging from a turbulent system (the atmosphere in this case) (see Fig. 1). The theory of formation of coherent structures is a relatively recent

[1] E-mail:vinod@iiap.ernet.in

[2] E-mail:prabhu@iiap.ernet.in

proposal (Levich 1985; Sulem *et al.* 1989). Their work has revealed the possibility of formation of large scale structures in three dimensional turbulence as a consequence of 'inverse - cascade of energy'. Sulem *et al.* (1989) have shown that there exists an instability, in 3D turbulence, called Anisotropic Kinetic Alpha (AKA) effect, which is analogous to the Alpha-effect observed in the generation of large scale magnetic fields. Moreover the similarity between the magnetic induction equation and the vorticity equation in fluid dynamics is what inspires the analogy! It is this AKA effect which has a feature of 'negative-viscosity', thus facilitating the flow of energy to the mean flow (large scales) from the random flow (small scales). Alternatively Levich and his coworkers have approached the problem from the point of view of invariants in hydrodynamics. They have identified that fluctuations of the helicity - density (which they call the 'I - invariant') plays the role of a new invariant along with energy. It may be noted that enstrophy (vorticity squared) which is the invariant accompanying energy invariance in 2D turbulence (and thus facilitating an inverse cascade of energy, and subsequent formation of large structures) is no longer an invariant in the 3D case. Thus this identification of the new invariant plays an important role in the evolution of a turbulent fluid. Arguing in the Kolmogorovic style, one can show that this invariance implies that there could be different inertial - subranges within which the invariants could decay on inclusion of dissipation. This is prompted by the fact that each of the invariants has a different power law dependence on the scales in question, and since they cannot be satisfied in the same regime of length scales it is obvious that different domains of inertial - ranges exist. It is also a well studied fact that upon inclusion of dissipation the slowly decaying invariant cascades towards the larger scales and the faster decaying one cascades towards the smaller scales. Based on this it is found that the I invariant cascades towards the larger scales in 3D turbulence. The exact power law dependencies can be worked out in the way we have shown in the figure 2. (please refer to Prabhu & Krishan 1994 for a detailed explanation).

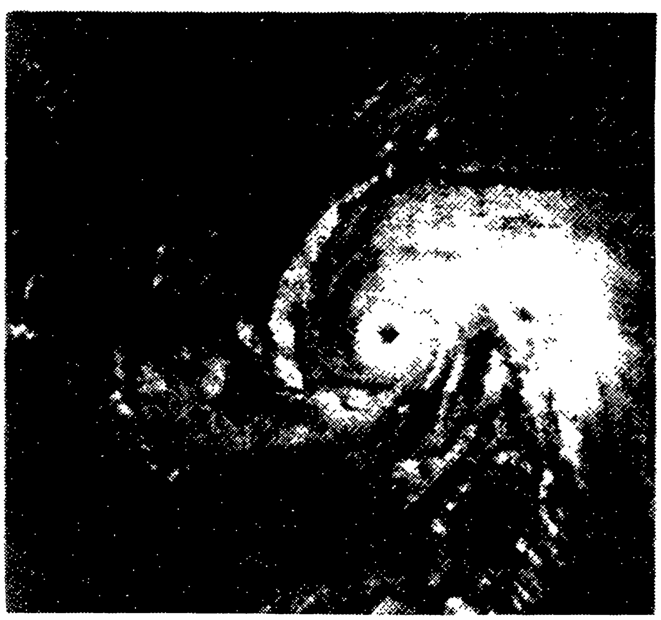

**Figure 1.** Typical satellite photograph of a northern - hemisphere tropical cyclone with a horizontal extent of about 1000 Km.

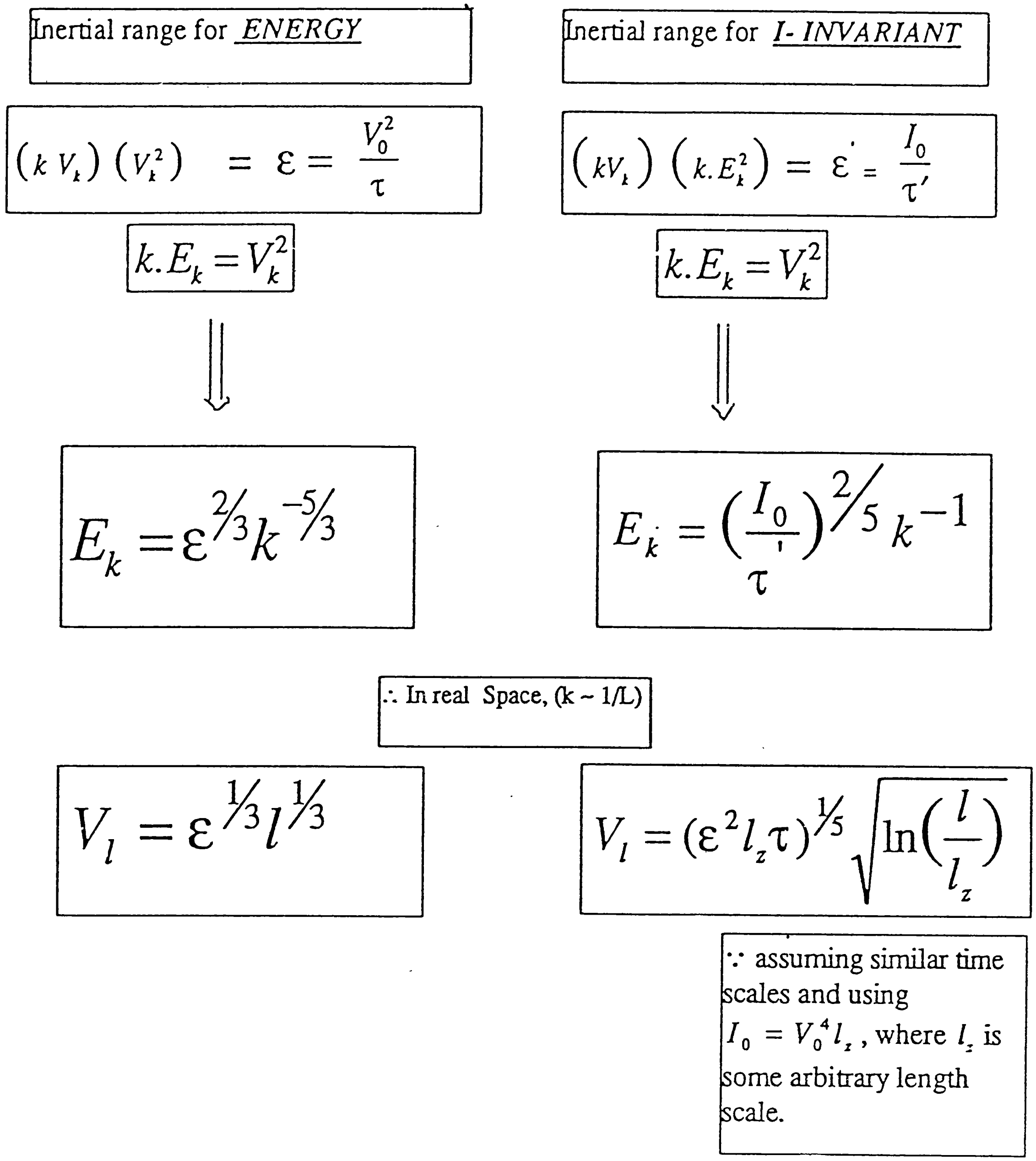


**Figure 2.** The Kolmogorovic way of studying cascades producing different intertial ranges. $V_k$ is the velocity in fourier space, K being the wave number. The boxes on top show the rate of cascade of each invariant. The I-invariant is the helicity-helicity correlation; helicity (h) being the projection of vorticity in the direction of velocity viz., $h = \vec{V}.\vec{w}$

It can be inferred from the above analysis that the real space velocity fields could be reflecting such a feature of cascading with the power-laws deduced above.

We tested such an idea by fitting the observed velocity fields of galaxies, to the above power laws on different scales. We proposed a simple velocity law which combines the above power laws of the inertial subranges with the velocity induced by the gravity of the galaxy (See figure 3).

We obtain very good fits between the model and the observations. We reproduce few samples of the fit here (See figure 4).

The time scales derived from such fits were comparable with the galactic time scales viz. the age of a galaxy, which is quite an encouraging result. This opens up the possibility of considering the galaxy as a coherent structure. We may visualise the galaxy as a fluid system (mainly in the formative stages i.e., the protogalactic stage, when the major component is gas and star formation has just begun). These signatures of inverse cascade may then be thought of as one persisting for a long time even after star formation becomes active, especially in the outer gas dominated regions of a galaxy.

## 2. The Sample and the Tully - Fisher Relation

The necessary data for studying the Tully - Fisher (TF) relation consists of apparent magnitudes, (usually corrected for Galactic and internal extinction,) and some measure of rotation velocities, corrected for projection effects, for a sample of galaxies. Usually, rotation veloity is obtained via the Doppler broadening of the HI 21 cm line, although Fabry - Perot imaging and long slit rotation curves (both obtained via H $\alpha$) are useful as well.

The TF relation has been studied with samples drawn from the set of galaxies which are sufficiently closeby. This was done presumably to get rid of the environment effects. The relation has been studied in different bands also.

Hitherto the rotational velocity was obtained either by finding out the maximum of the rotation curve $V_{max}$ or the rotational velocity at a suitably chosen radius (Holmbergh radius) – corresponding to a suitable aperture magnitude definition. Estimates of $V_{max}$ using the line profile measured at 20% of the peak, have also been used. We use a different way to characterize the galactic velocity field. Since our model gives a good fit for the velocity field we use the flat portion of the curve to estimate the average velocity in that regime. We use our proposed law to do this averaging numerically. As for the photometric properties of our sample, we obtained the data from the NASA extragalactic database, and the RC3 catalouge.

Our sample of galaxies were drawn from different clusters and field galaxies as observed by Rubin *et al.* (1980, 1982) and Amram *et al.* (1992, 1994). Our primary interest was to get a sufficiently large set of galaxies, (irrespective of their distance, environment, mass, radius, or luminosity) for which the photometry (U, B, V, I & $I_{23.5}$) was done and velocity fields mapped. All in all our sample consisted of :

Using the above two velocity relations we modeled the velocity fields of a number of galaxies by combining the effects of rigid - rotation, gravity, and turbulence! We proposed the following simple velocity relations for modeling the observed rotation -curve of a galaxy :

$$V_l = A.l + B.l^{1/3} \quad \text{for } l \leq l_z$$

$$V_l = C.l^{-1/2} + D.\sqrt{\ln(l/l_z)} \quad \text{for } l \geq l_z$$

Where,

$A = \omega$, the angular velocity of the galaxy.

$B = \varepsilon^{1/3}$, $\varepsilon$ is the rate of transfer of energy between the various scales involved.

$C = \sqrt{G.M}$, $M$ being mass of the galaxy. $G$, is the gravitational constant.

$D = (\varepsilon^2 l_z \tau)^{1/5}$, $\tau$, is the time scale over which the energy transfer takes place.

Thus, obtaining the coefficients from the fits ..we can derive the galaxy parameters like ..$\omega$ and M, and its turbulence parameters like ...$\varepsilon$ and $\tau$.

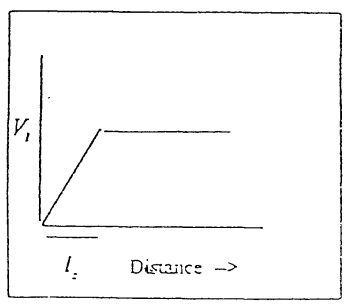


**Figure 3.** The velocity law and the procedure adopted in fitting it to the observations.

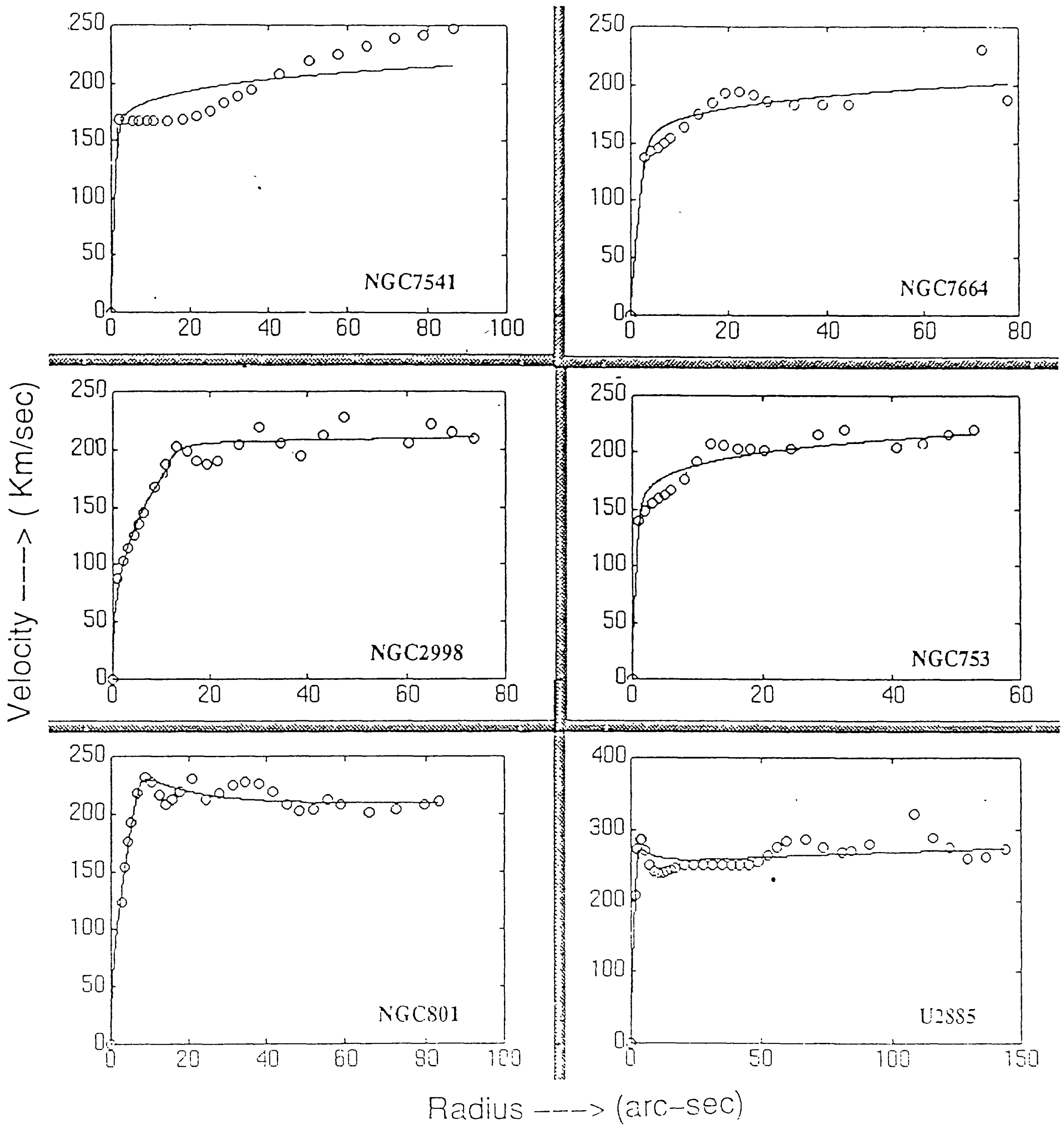


**Figure 4.** Rotation curve fits for the set of galaxies observed by Rubin *et al.* (1980).

20 Sb galaxies – observed by Rubin *et al.* (1980).
20 Sc galaxies – observed by Rubin *et al.* (1982).
35 other galaxies – observed by Amram *et al.* (1992, 1994).
(drawn from different cluster environments viz. the Coma, Pegasus, Abel, Hercules, etc.)

## 3. Discussion

We observed an interesting trend in our statistical analysis. (see figure 5 and Table 1.)

**Table 1.** Table summarizing the statistics of the sample.

| Average | | Photometric Bands: I | | B | | V | | I | | $I_{23.5}$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Up | 0.65 | | 0.69 | | 0.74 | | 0.85 | | 0.85 | |
| $V_{tot}$ | r | 0.5 | | 0.58 | | 0.62 | | 0.71 | | 0.72 | |
| | Low | 0.31 | | 0.43 | | 0.47 | | 0.40 | | 0.41 | |
| | P | $10^{-4}$ | ★★★ | $10^{-8}$ | ★★★ | $10^{-7}$ | ★★★ | 0.001 | ★★ | 0.001 | ★★ |
| | Up | 0.64 | | 0.59 | | 0.61 | | 0.54 | | 0.57 | |
| $V_{turb}$ | r | 0.49 | | 0.45 | | 0.46 | | 0.21 | | 0.25 | |
| | Low | 0.29 | | 0.27 | | 0.27 | | 0.20 | | 0.17 | |
| | P | $10^{-4}$ | ★★★ | $10^{-5}$ | ★★★ | $10^{-4}$ | ★★★ | 0.40 | ns | 0.39 | ns |
| | Up | 0.20 | | 0.28 | | 0.33 | | 0.80 | | 0.80 | |
| $V_{grav}$ | r | –0.02 | | 0.09 | | 0.13 | | 0.63 | | 0.64 | |
| | Low | –0.24 | | –0.10 | | –0.08 | | 0.27 | | 0.29 | |
| | P | 0.83 | ns | 0.44 | ns | 0.30 | ns | 0.006 | ★★ | 0.005 | ★★ |
| df | | 52 | | 68 | | 56 | | 15 | | 15 | |

| | |
|---|---|
| r | Correlation coefficient |
| Up | upper limit on r (90% confidence interval) |
| Low | lower limit on r (90% confidence interval) |
| P | probability from t statistics |
| ★★★ | P < 0.001 (very significant correlation) |
| ★★ | 0.001 < P < 0.0 1 (quite significant correlation) |
| ns | not significant |
| df | degrees of freedom |

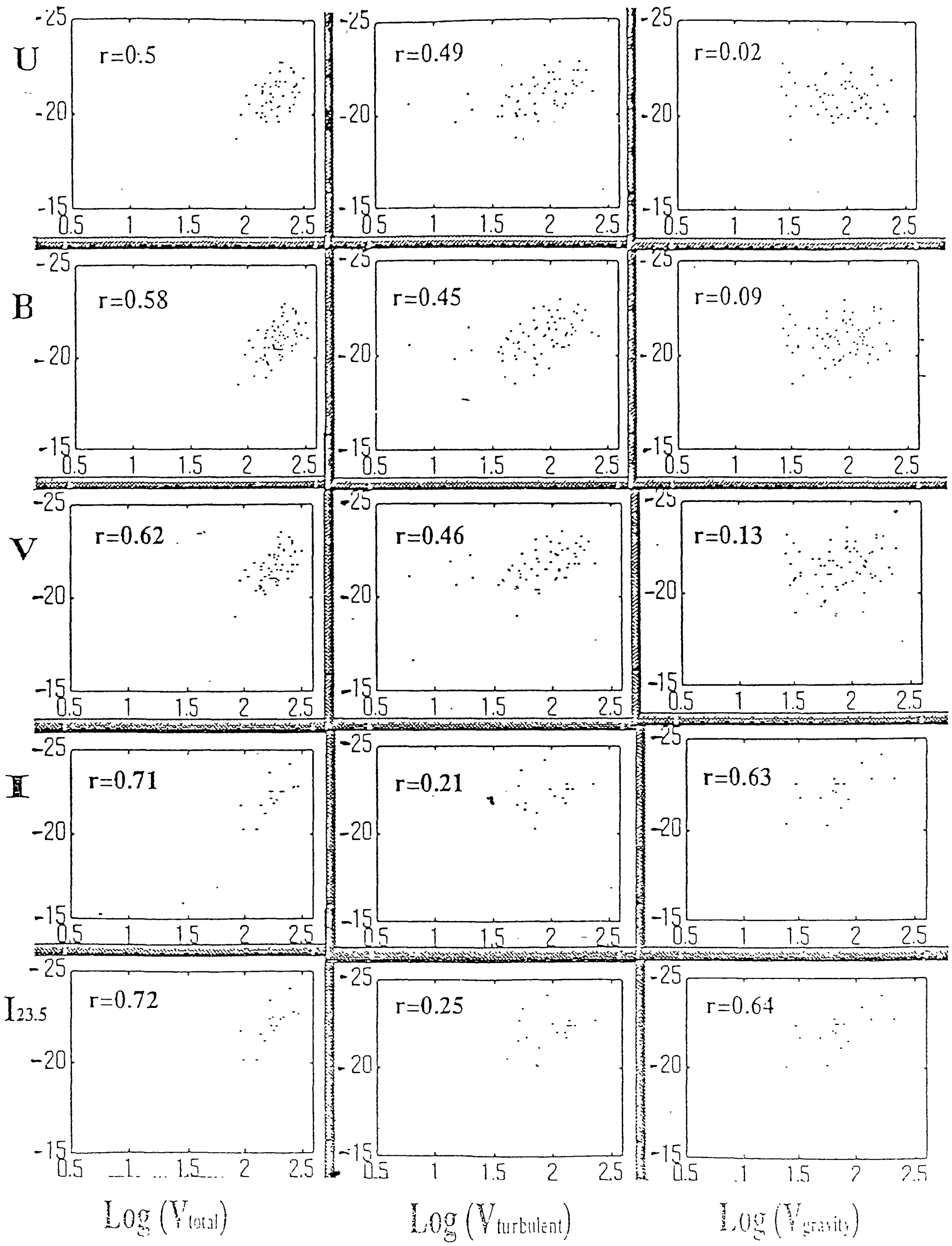


**Figure 5.** The plots of correlation data for each pair of velocity – luminosity variables.

First the total velocity $V_{tot}$ determined from the model fit correlates in the same conventional way as it does for a standard Tully - Fisher relation. We could confirm the normal trend wherein the correlations improve as we go to the longer wavelengths (from U to the I bands...). So, we confirm the Tully - Fisher relation in the first step. Next, we find that the turbulent component of velocity $V_{turb}$ correlates better than the gravity component $V_{grav}$ in the U, B and V bands. It is to be noted here that our model gives comparable values for both components $V_{turb}$ and $V_{grav}$ although the correlation of each with luminosity differs. Thus it is interesting to note that something other than the gravity - induced velocity is correlating better!. This trend is reversed as we approach the I bands, viz. the gravity component $V_{grav}$ correlates better. This is to be expected since the longer wavelength bands are more sensitive to the mass component (i.e., the gravity - induced velocity..)[3]. We believe that the turbulent component which our model for rotation curves resolves shows more than just scatter, for the correlations are statistically very significant, as shown by the values of P (ref. Table 1). We conclude that the inverse cascaded velocity field alongwith gravity component accounts well for the rotation curves of galaxies.

Thus it would be erroneous to interpret the observed spectral line widths as those due to motions induced by gravity alone. The contribution from turbulence is not only in the form of random motions, but turbulence gives rise to ordered motions too. This emphasizes the need to study the self-organizing aspects of turbulent media, which could contribute in the understanding of structure - formation in various astrophysical situations.

## Acknowledgements

Figure 1 has been reproduced from the article by Sir James Lighthill titled 'Fluid Mechanics' published in Twentieth Century Physics Vol. II edited by Laurie M. Brown, Abraham Pais and Sir Brian Pippard. The publishers are Institute of Physics Publishing, Bristol and Philadelphia and American Institute of Physics Press, New York.

This work has made use of the NASA Extragalactic Database.

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