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Loss or gain in surface temperature and brightness of a binary system due to appearance of a third body from infinity in hyperbolic / parabolic orbit

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Abstract. Based on Roche equipotential surface, a theoretical model of a binary system has been constructed where the system is disturbed by a third body which comes from infinity in hyperbolic / parabolic orbit towards the binary, closes to it and finally goes away to infinity. Using this model, system's loss or gain in surface temperature and brightness have been calculated over the entire cycle of the system as a function of parameters like : (i) mass - ratio q between masses of the third body and the primary, (ii) eccentricity of the wide orbit, (iii) length of the semi - transverse axis a_2 of the wide orbit which is hyperbolic and (iv) distance d between the common centre of gravity and the focus of wide orbit for different angle α (see Fig. 1a). These calculations show an anomalous behaviour of loss or gain in surface temperature and brightness where phase of the cycle plays a very important role. At high mass - ratio q' anomaly is higher and at higher eccentricity e it is still higher.

1. Introduction

Some ten percent of all known close binary system such as Algol (β-persei) seem to possess a third companion, which can exert some perturbations on the motion of a close pair in course of time. The third companion, though too feeble to be seen, can however be detected spectroscopically. McLaughlin (1934), Eggen (1948), Kamp, Smith & Thomas (1951), Struve & Ebbighausen (1959), Kopal (1959) and others have discussed on such triplicate system. Recently Barman and Sagar (1996, in press) have described influence of a third on the luminosity of a contact binary system where the orbits of binary (close orbit) and third body (wide orbit) are closed. In the present paper the orbit of third body is open and the binary need not be contact. A third body in parabolic/hyperbolic orbit comes from infinity towards a binary system, comes closer and then finally goes away to infinity. In such case, we wish to calculate any loss or gain in surface temperature and brightness of the binary system. Calculation has been done over an entire cycle of the binary system for the parameters (i) q', ratio of masses between the third body and the primary component of the binary system,

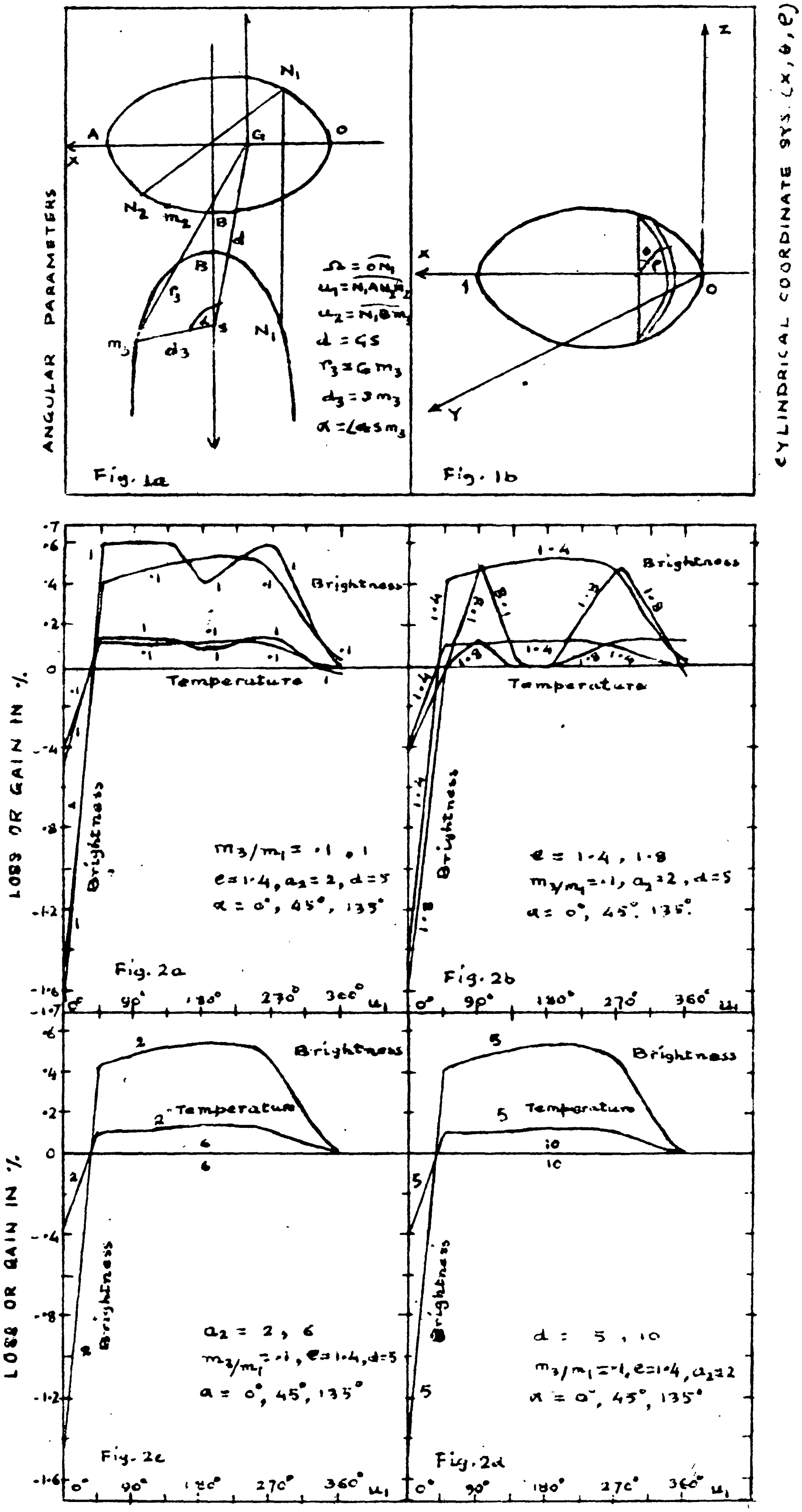
ANGULAR PARAMETERS
CYLINDRICAL COORDINATE SYS. (x, θ, ρ)
Fig. 1a
Fig. 1b
Brightness
Temperature
LOSS OR GAIN IN %
m3/m1 = ·1, ·1
e = 1·4, a2 = 2, d = 5
α = 0°, 45°, 135°
Fig. 2a
e = 1·4, 1·8
m3/m1 = ·1, a2 = 2, d = 5
α = 0°, 45°, 135°
Fig. 2b
a2 = 2, 6
m3/m1 = ·1, e = 1·4, d = 5
α = 0°, 45°, 135°
Fig. 2c
d = 5, 10
m3/m1 = ·1, e = 1·4, a2 = 2
α = 0°, 45°, 135°
Fig. 2d
0° 90° 180° 270° 360° u1

Figure 1.

(ii) eccentricity e of the wide orbit, (iii) Length of the semi-transverse axis a_2 of wide orbit, (iv) distance d between the common centre of gravity and the focus of wide orbit for different angle α subtended at focus of wide orbit by the common centre of gravity and the third body. The next section describes the model while the results have been discussed in the last section.

2. Theoretical model

Let the components m_1 (primary) and m_2 (secondary) of a binary system move round their common centre of gravity in a circular orbit. Another star of mass m_3 is moving in a hyperbolic (or parabolic) orbit (see Fig. 1a). We assume for simplicity that the common centre of gravity of the binary system is negligibly disturbed by the third body. The origin of the co-ordinate system is the centre of gravity of the primary component m_1; the line joining the centres of gravities of the components m_1 and m_2 is the x-axis; the z-axis is the axis of rotation of the primary component. The z-axis is taken perpendicular to the orbital plane of the binary system which is also the equatorial plane of the primary component.

Then following Barman and Sagar (1996), Mochnacki and Loughty (1972), we can express the normalized total potential C at a point e on the surface of the boundary system in cylindrical co-ordinates (see Fig 1) as :

$$C = \frac{2B_1}{\sqrt{x^2 + e^2}} + \frac{2B_2}{\sqrt{(x-1)^2 + r^2}} + \{(x + B_2)^2 + e^2 \sin^2\theta\} + \frac{2B_3}{r_3^3} \{P_2(\sigma) + \frac{(1-q)\,B_1}{r^3} P_3(\sigma)\} \quad (1)$$

where

$q = m_2 / m_1$, $q' = m_3 / m_1$, $B_1 = 1 / (1 + q)$, $B_2 = qB_1$, $B_3 = q' B_1$

$c = 2B_1 + / Cxm_1$

P_2 and P_3 are the second and third order polynomials of σ,

$$\sigma = \cos(u_1 - \Omega) \cos(u_2 - \Omega) + \sin(u_1 - \Omega) \sin(u_2 - \Omega) \operatorname{Cos} I \quad (1a)$$

where Ω denotes the longitudes of the node, I is the angle of inclination and u_1, u_2 are the longitudes of masses m_2, m_3 reckoned from Ω in the planes of respective orbits (see Fig 1a).

The last term of equation 1 is the disturbance created by the third body of mass m_3. The term, which is an approximation for an elliptical orbit of mass m_3 (Kopal 1959), has been extended to hyperbolic or parabolic orbit, then r_3 can be written as :

$$r_3^2 = d^2 + d_3^2 - 2\, d\, d_3 \cos\alpha$$
$$l / d_3 = 1 + e \cos\alpha \; (l = a_2 (e^2 - 1) \text{ for hyperbola, } l = Za_2 \text{ for pambola}) \quad (1b)$$

where d is the distance between it's focus and the common centre of gravity of the binary system, l is the semi-latus rectum of the hyperbolic (or parabolic) orbit, d_3 is radius vector or m_3 with respect to it's focus, α is the angle between d and d_3, and e is the eccentricity.

If C_1 be the initial potential corresponding to Lagrangian point L_1 for detached component, then photospheric potential Cp is conveniently defined in terms of fill-out ratio F as :

$$Cp = C_1/F, \; (C_p \geq C_1, \; O < F \leq 1) \tag{2}$$

The local surface gravity g for an element at the surface is given by

$$g = \sqrt{(f_x^2 + f_y^2 + f_z^2)}$$

where the derivates f_x, f_y, f_z of a function f (x, θ, e) = C - C_p are given in Appendix 1 (Barman and Sagar 1996) and the local effective temperature T is given by the gravity darkening law :

$$T = T_p \cdot (g / g_p)^{\beta}$$

where T_p is the polar temperature for the black body radiation. The exponent β has the value 0.25 for radiative atmosphere (von Zeipel 1924) while for convective atmosphere β is approximately 0.08 (lucy 1967).

Therefore loss or gain in effective temperature T can be written as :

$$[T - (T)_{\text{at } d_3 = \infty}] \, / \, (T)_{\text{at } d_3 = \infty} \tag{5}$$

Similarly loss or gain in surface brightness H at any wavelength can be written as :

$$[H - (H)_{\text{at } d_3 = \infty}] \, / \, (H)_{\text{at } d_3 = \infty} \tag{6}$$

where $$H = Hp \cdot \frac{e^{hc/\lambda k T_P} - 1}{e^{hc/\lambda k T} - 1} \tag{6a}$$

H_p being the brightness, h the Planck's constant, c the velocity of light, k the Boltzman constant, and the value of T being given by equation 4. For our calculation in equation 4 we have taken β = 0.25 and T_p = 6000°C which is the polar temperature or the primary component. Employing equations 4 & 5 we have calculated percentage loss or gain in temperature and brightness for an entire cycle of the system for the parameters m_3/m_1, e, a_2, d, α and the results have been presented in four figures 2a to 2d.

3. Results and discussions

Here the Roche surface arises simply from a combination of mass-point components, centrifugal forces and the disturbing forces. Since the common centre of gravity of the binary system is negligibly affected by the third body, we can therefore assume that the orbit of the system remains the same. During our calculation of the loss or gain in surface temperature and brightness, we have considered the mass-ratio m_2/m_1 = .54, the fill-out ratio F = 1.13 which are for W-Uma binary system (see Barman and Sagar 1996). For this system the value of the Lagrangian point L_1 is .563007. The angle α, which indicates distance of the third body in it's path, are taken for 0°C, 45°, 135°. We have always considered $\Omega = 0$, I = 0. The values of other parameters used in the calculations are marked in the Figures 2a - 2d. The curves have been drawn for the surface at $\alpha = .35$ and $\theta = 0$ which is near the neck of the binary.

Inspection of Figures 2a to 2d indicates that :

1. The percentage loss or gain in temperature and brightness are more or less same for the angles $\alpha = 0°$, 45°, 135°. For $u_1 < 36°$ there are losses but after that there are gains.

2. In Fig. 2a we see that the nature of the percentage loss or gain curves for temperature are not much different for the mass - ratio m_3/m_1 = .1 and 1. For $u_1 < 35°$ there is a loss, thereafter gain begins.

   Considering differences we see that at $u_1 = 0°$ the loss (– .42) for mass - ratio 1 is higher than the loss (–.37) for mass - ratio .1 and when gain for higher mass-ratio is always higher except places near $u_1 = 180°$. The nature of the curve for the brightness is almost same as that of the temperature, though the differences are much more prominent. For example we see at $u_1 = 0°$ the losses for the mass - ratios .1 and 1 are –1.46 and -1.64 respectively.

3. In Fig. 2b comparison between the curves of eccentricities e = 1.4 and 1.8 shows that behaviour of the curve e = 1.8 is much more anomalous when $u_1 > 45°$. For example in temperature curve, there is a gain (+ .13) at $u_1 = 90°$, there is neither loss nor gain during u_1 = 135° to 180° and thereafter gain begins. For brightness curve, there is a gain (+ .50) at $u_1 = 90°$, there is neither gain nor loss at $u_1 = 180°$, again it rises to .50 at $u_1 = 270°$ and thereafter it decreases to –.05 at 360°.

4. In Fig. 2c and 2d we see that if the semi-transverse axis a_2 increases to 6 or the distance d between the common centre of gravity and the focus increases to 10, there is neither loss nor gain.

4. Conclusions

From the above discussion it is clear that during the entire cycle of the binary system the percentage loss or gain of temperature or brightness at any point of surface is independent of the angle α and shows many anomalous behaviour. For some combinations of u_1 and u_2, this anomaly is large indicating that they play a very important role in generating anomalous behaviour of loss or gain curve. For only $u_1 < 36°$ there are losses. The anomalous gain (or loss) is generally higher for higher mass-ratios and for higher eccentricity the anamoly is still higher.

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