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# Surface Waves in a Two Layered Fluid Model with an Inclined Magnetic Field

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**Abstract.** We consider a simple two layer fluid model to investigate surface waves arising due to the interaction between two fluids of different densities. The interface separates the convection zone and the atmosphere of the Sun. The upper fluid is under the influence of a magnetic field which is inclined at an angle to the interface while the lower fluid is field free. The wave vector is in a direction different from the magnetic field. Making the use of the continuity of the vertical velocity and the total pressure at the interface leads to a dispersion relation which is a polynomial of sixth degree. However, all the modes do not represent surface waves. The condition for the existence of surface waves and the variation of the normalized phase speed as a function of the different angles of the propagation vector is studied.

*Key words:* Surface waves, inclined magnetic field

## 1. Introduction

Studies in the recent past have enriched our knowledge on the dynamical behaviour of the Sun. The Sun exhibits oscillations at different frequencies with periods ranging from a few seconds to a few minutes. Oscillations such as the p modes (pressure) and g modes (gravity) enable us to probe into the interior of the Sun, namely the base of the convection zone, the radiative zone and the core. However, the atmosphere of the Sun interacts with the outermost boundary and this can lead to interfacial (Surface) waves.

Surface Waves are those that propagate on a sharp (discontinuous) interface. The energy of these waves is confined to within roughly a wavelength of the surface. They may propagate on magnetic structures that are more complicated than the single interface of which we are concerned in the present study.

Wave propagation in a magnetically structured configuration has been studied by several authors (see Roberts(1991) for a review). Roberts(1981), Somasundaram and

Uberoi(1982), Miles and Roberts(1989), Jain and Roberts(1991), Singh and Talwar(1993) have investigated the properties of waves arising on a single magnetic interface. In the present study, we have considered the properties of waves arising on a single interface wherein the magnetic field is aligned at an angle different from the propagation vector on the upper fluid while the lower fluid is field free.

In section 2, we derive the dispersion relation and discuss the condition for the existence of surface waves. Section 3 deals with discussion of the results while concluding remarks are made in Section 4.

## 2. Dispersion Relation

Consider $x = 0$ to be the interface between two compressible media denoting the region $x < 0$ with suffix "1" and the region $x > 0$ with suffix "2". Let the magnetic field be of the form

$$B_{01,2} = (0, B_{01,2} Cos\gamma_{1,2}, B_{01,2} Sin\gamma_{1,2}) \tag{1}$$

and the wave vector to be

$$k = (0, kSin\theta, kCos\theta) \tag{2}$$

Taking $\rho_{01}$ and $\rho_{02}$ to be the mass densities on either side of the interface and $c_{1,2}$ and $v_{A1,2}$ as the sound and Alfven speeds, the dispersion relation by solving the linearized magnetohydro dynamic equations and applying the boundary conditions can be written as (Uberoi and Satya Narayanan(1986))

$$\tau_1 \epsilon_2(\omega, k) + \tau_2 \epsilon_1(\omega, k) = 0 \tag{3}$$

where

$$\epsilon_1(\omega, k) = \rho_{01}(-\omega^2 + k^2 v_{A1}^2 Sin^2(\theta + \gamma_1)) \tag{4}$$

$$\epsilon_2(\omega, k) = \rho_{02}(-\omega^2 + k^2 v_{A2}^2 Sin^2(\theta + \gamma_2)) \tag{5}$$

and

$$\tau_{1,2}^2 = \frac{\omega^4 - k^2\omega^2(v_{A1,2}^2 + c_{1,2}^2) + k^4 c_{1,2}^2 v_{A1,2}^2 Sin^2(\theta + \gamma_{1,2})}{k^2 c_{1,2}^2 v_{A1,2}^2 Sin^2(\theta + \gamma_{1,2}) - \omega^2(c_{1,2}^2 + v_{A1,2}^2)} \tag{6}$$

In the incompressible limit, $c_{1,2} \rightarrow \infty$, $\tau_1^2 = \tau_2^2 = k^2$ and equation (3) becomes

$$\rho_{01}(-\omega^2 + k^2 v_{A1}^2 Sin^2(\theta + \gamma_1)) + \rho_{02}(-\omega^2 + k^2 v_{A2}^2 Sin^2(\theta + \gamma_2)) = 0 \tag{7}$$

which gives

$$\frac{\omega^2}{k^2} = \frac{B_{01}^2 Sin^2(\theta + \gamma_1) + B_{02}^2 Sin^2(\theta + \gamma_2)}{(\rho_{01} + \rho_{02})} \tag{8}$$

In the present study, we assume that $c_1/v_{A1} << 1$ and $B_{02} = 0$ so that equations (4), (5) and (6) can be written as

$$\epsilon_1 = \rho_{01}(-\omega^2 + k^2 v_{A1}^2 Sin^2(\theta + \gamma)) \tag{9}$$

$$\epsilon_2 = \rho_{02}(-\omega^2) \tag{10}$$

$$\tau_1^2 = k^2(1 - \frac{\omega^2}{k^2 v_{A1}^2}), \tau_2^2 = k^2(1 - \frac{\omega^2}{k^2 c_2^2}) \tag{11}$$

Introducing the nondimensional quantities $\alpha = \rho_{02}/\rho_{01}$, $\delta = c_2/v_{A1}$, $x = \omega/kv_{A1}$ and simplifying yield the dispersion relation

$$y^3 + Ay^2 + By + c = 0 \tag{12}$$

where $y = x^2$,

$$A = \frac{\delta^2(\alpha^2 - 1) - 2Sin^2(\theta + \gamma)}{(1 - \alpha^2\delta^2)}$$

$$B = \frac{(1 + 2\delta^2)Sin^2(\theta + \gamma)}{(1 - \alpha^2\delta^2)}$$

$$C = \frac{-\delta^2 Sin^4(\theta + \gamma)}{(1 - \alpha^2\delta^2)} \tag{13}$$

In order to discuss the existence of surface waves we look at the relation (3) . It is interesting to note that equation (3) will have real roots only when $\epsilon_1$ and $\epsilon_2$ are of opposite signs and decaying constants $\tau_{1,2}$ should both be positive for the roots to represent surface wave propagation. The root $\omega/k$ should lie in the region

$$min(v_{A1,2}Sin(\theta + \gamma_{1,2})) < \omega/k < max(v_{A1,2}Sin(\theta + \gamma_{1,2})) \tag{14}$$

In the present study, the above relation reduces to

$$min(v_{A1}Sin(\theta + \gamma)) < \omega/k < max(v_{A1}Sin(\theta + \gamma)) \tag{15}$$

## 3.Discussion

It is evident from the dispersion relation (12) that the phase speed of the surface wave depends on the parameters $\alpha$, $\delta$ and the angles $\theta$ and $\gamma$. We have restricted our calculation to specific parametric values. Figure 1 presents the normalized phase speed of the surface waves as a function of $\theta$ for $\alpha = 0.75$ and $\delta = 0.9$. It is interesting to note that the phase speed varies significantly for different $\theta$ for a given $\gamma$. The phase speed of the surface wave increases monotonically when $\gamma = 0°$ while it decreases monotonically for $\gamma = 90°$. However, for $\gamma = 40°, 60°$ the trend is monotonic increase upto a certain value of $\theta$ and then a monotonic decrease. We have carried out the calculation for other specific values of $\alpha$ and $\delta$. The results are similar and we skip the details for brevity.

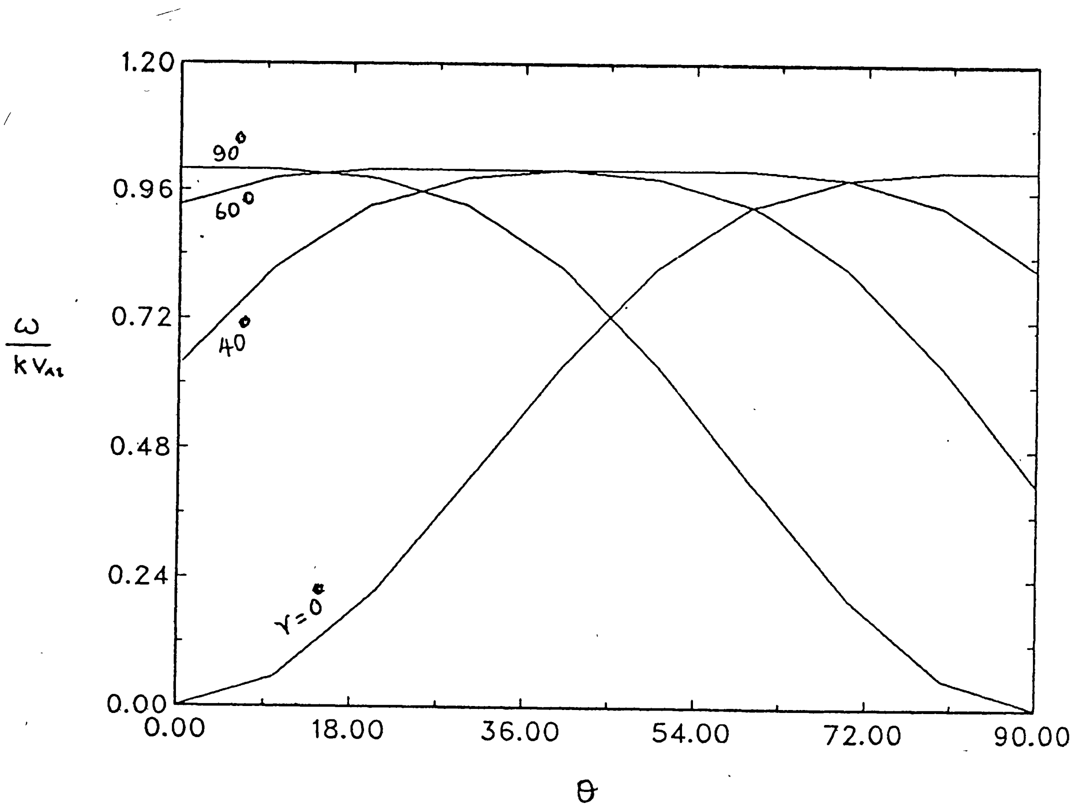


**Figure 1.** Normalized phase speed of the Surface Wave as a function of $\theta$.

## 4. Conclusions

The characteristics of Surface Waves can be different when we have an interface wherein the magnetic field is not parallel to the interface on the upper fluid while the lower fluid is field free. In particular, the phase speed of these waves depend significantly on the inclination of the magnetic field to the interface. More realistic magnetic field configurations in slab and cylindrical geometries will have to be considered to understand the interaction of the convection zone with the outer atmosphere of the Sun.

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