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Surface Effects on Solar Oscillation Frequencies

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Abstract. The solar p–mode frequencies and eigenfunctions are modified by four processes operative in the atmosphere and the subphotospheric layers which can be, to a good approximation, treated as surface phenomena; they are the imperfect reflection at the outer boundary, scattering by convective eddies, effects caused by abrupt hydrogen ionization and non–adiabatic processes which are important mainly in the region where the product of the pressure and sound velocity is comparable to or less than the radiative flux. Propagation of acoustic modes of short period or large horizontal wave number $\ell \gtrsim 800$ critically depend on the atmospheric structure as well as the treatment of non–adiabatic effects. But the surface effects on low frequency modes can be, to sufficient accuracy, represented as a phase difference between the two solutions of the second order differential equation describing the adiabatic modes in the Cowling approximation. Most of these effects are frequency–dependent only, since the radial wave number of the modes is large compared to their horizontal wave number. However, it is possible to isolate the ℓ-dependent contribution, which becomes important when the magnitude of the (imaginary) Brunt-Vaisala frequency is comparable to the Lamb frequency.

Key words: solar oscillation, surface effects.

1. Introduction

The solar p-modes of frequency up to approximately 5 mHz are trapped in the subphotospheric layers. The rapidly rising Brunt-Vaisala frequency in the photosphere as well as the large acoustic cut-off frequency of the nearly isothermal atmosphere act as efficient reflectors of these modes. It is generally believed that since both these characteristic frequencies are independent of the horizontal wave number of the mode, the local radial wave number of the modes will also be independent of the horizontal wave number, ℓ. Consequently, it appears to be popular to remove a *best fit* polynomial component, which depends only on the frequency and not on the value of ℓ, from the oscillation frequency spectra prior to helioseismic inversion. This procedure is justified as the embodiment of our shortcomings in understanding, for instance, the non-adiabatic effects, the coupling between oscillation and convection, structure of the layer where hydrogen is partially ionized etc. However, Antia(1995) found that though this approximation is valid to first order, it is not correct to the accuracy of the measured frequencies.

We had earlier shown that short period oscillations having ℓ-values in the range of 1000 to 4000 can be trapped between the temperature minimum and the lower chromosphere, if there exists a sufficiently thick layer of the chromosphere which is nearly isothermal followed by a region of steep temperature gradient (Narasimha, 1992). These modes will be far from being adiabatic and better described as isothermal waves, though the radiative cooling time is not very small compared to the oscillation frequency (cf. Ando and Osaki, 1975). It is natural to expect that, for lower frequencies and ℓs some of the waves excited in the solar interior will propagate beyond the photosphere and their reflection from the atmosphere will be governed by the degree of penetration into the chromosphere.

The f-modes, in general, should be treated with caution in stars where processes operating at dynamical time scale are important and are coupled with the oscillation. Let us recall that an f-mode would have exhibited spatial oscillations similar to a p-mode in the envelope of a solar type star, if the radial wave number of the mode (k_r) is independent of the radius. (Note that we define k_r as the derivative of the phase Φ with respect to radial coordinate, r) However, the variation in k_r with radius in the outer layers of the star and the fluctuation in the phase due to the rapidly varying Brunt-Vaisala frequency will qualitatively change the wave propagation. For instance the f-mode could become a surface wave or it could be a g-mode in the interior which changes to a p-mode as it enters the convection zone.

Here we shall use the Cowling approximation to analyse the surface effects. The essential assumption in this work is that the effects of interest can be represented as a phase difference between the two linearly independent solutions of the second order differential equation describing the oscillations. This is valid if the damping of the wave is small in the narrow subphotospheric layer where the Brunt-Vaisala frequency and acoustic cut off frequency vary rapidly. We shall demonstrate the ℓ and ω dependence of the phase difference due to the major contribution to the surface effect, namely, modification of the Brunt-Vaisala frequency at the layer where the superadiabatic temperature gradient is the maximum. The relevance of the results to f-modes of various ranges of ℓs will be discussed.

2. Essential Mathematics

Following Unno *et al* (1989) the oscillations are described, in the Cowling approximation, by the following equation:

$$\frac{d^2\xi}{dr^2} - \frac{d\ell n|P|}{dr}\frac{d\xi}{dr} - PQ\xi = 0 \tag{1}$$

where

$$P(r) = \frac{r^2}{c^2}\left(\frac{\ell(\ell+1)c^2}{r^2\omega^2} - 1\right)h(r) \tag{2}$$

and

$$Q(r) = \frac{\omega^2 - N^2}{r^2} h(r)^{-1} \tag{3}$$

In the above expressions, c is the adiabatic sound velocity, N is the Brunt-Vaisala frequency,

$$h = \exp\left[\int^r \left(\frac{N^2}{g} - \frac{g}{c^2}\right) dr\right] \tag{4}$$

and the radial displacement is proportional to ξ.

It can be shown that the regions where the wave is propagatory satisfy the condition

$$c^2 k_r^2 = -PQ - f(P) > 0 \tag{5}$$

where

$$f(x) = |x|^{\frac{1}{2}} \frac{d^2|x|^{-\frac{1}{2}}}{dr^2} \tag{6}$$

This defines the turning points where k_r vanishes.

If k_r does not vary with radius rapidly, then it can be treated as the approximate local radial wave number of the mode. For an isothermal stratification under constant acceleration due to gravity,

$$c^2 k_r^2 = -PQ - \omega_{ac}^2 \tag{7}$$

where the acoustic cut off frequency for the isothermal atmosphere is $\omega_{ac} = c/(2H_P)$.

The phase of the wave satisfies the quantization condition,

$$\int_{tur.pts} k_r dr = \Phi_n = n\pi + \epsilon \tag{8}$$

where ϵ depends on the boundary conditions.

3. The Surface Effect

We shall consider the solar cavity to be a smooth medium except for a narrow subphotospheric layer. There is no scattering by irregularities except at the top layer. Consequently, the lower boundary conditions can be considered to be fixed and hence the equations for adiabatic oscillations will have one solution satisfying the lower boundary condition for each ω and ℓ. We shall treat the boundary condition as the specification of the phase of the wave at any time at one fixed point of the cavity. We then fix the upper boundary, for specification of the other boundary condition, to be just below the turbulent region. If we specify the phase of the mode at this point with respect to the lower fixed point, the frequency of the mode is determined from the quantization condition.

The entire uncertainties of the model as well as oscillation frequencies are confined to a narrow layer. The imperfect reflection or propagation into the atmosphere introduces a modification of the phase $\delta\Phi$ and corresponding change in the frequency $\sim \omega\delta\Phi/n\pi$. However, the nature of the wave in the atmosphere will depend on the structure of the layer. The non adiabatic effects can be similarly introduced because, at deeper layers, the measure of nonadiabaticity F^{rad}/Pc is small, where P is the pressure. However, we should point out that, non adiabatic effects which are responsible for the excitation or decay of the modes and the consequent line width cannot be incorporated using our approach. Our aim is to analyse the modification of the frequency only.

The main contribution to possible phase modification at the turbulent layer is due to the uncertainties in Brunt-Vaisala frequency. The effect can be approximated as follows:

$$\Phi = \int \sqrt{\left(1 - \frac{\ell(\ell+1)c^2}{r^2\omega^2}\right)(\omega^2 - N^2) - f_s}\frac{dr}{c} \tag{9}$$

where f_s represents the change in radial wave number due to stratification effects, including rapid variation of the wave number.

$$\delta\Phi \sim \int \frac{-\delta N^2 dr}{2c\sqrt{\frac{\omega^2 - N^2}{1 - \frac{\ell(\ell+1)c^2}{r^2\omega^2}} - F_s}} \tag{10}$$

Since the contribution is confined to a narrow layer of worse uncertainties, we can approximate the above expression as follows:

$$\delta\Phi \sim -\frac{\delta N^2 \Delta r/c}{\sqrt{\frac{\omega^2 - N^2}{1 - \frac{\ell(\ell+1)c^2}{r^2\omega^2}} - F_s}} \tag{11}$$

where the quantities are evaluated at the worst offending layer, and $\Delta r$is the thickness of the region of uncertainties.

For most of the modes, the variation of wave number can be taken into account through the acoustic cut off frequency, and hence

$$\delta\Phi = -\frac{\delta N^2 \Delta r/c}{\sqrt{\omega^2 - N^2 - \omega_{ac}^2}}\left[1 - \frac{\ell(\ell+1)c^2}{2r^2\omega^2}\right] \tag{12}$$

To get an idea of the numbers, various models of solar convection zone show difference of $\sim 10km$ in the position and thickness of the highly superadiabatic layer which contributes substantially to $\delta\Phi$. The variation in N^2 can be estimated from the possible range of maximum value for $\nabla - \nabla_{ad}$, which is of the order of unity. Consequently, the expected value of $\delta\Phi$ is a few times 10^{-2} and hence

$$\frac{\delta\omega}{\omega} \sim \frac{10^{-2}}{n} \tag{13}$$

where n is the radial order of the mode. For high values of ω and low values of ℓ, this component is nearly independent of ℓ. However, it is clear that for modes of lower radial order, the ℓ-dependency will be more important. If the main contributions to the surface effects were any phenomena associated with the hydrogen ionization zone, then the ℓ dependent term can be comparable to the term which depends only on ω.

4. The Fundamental Mode

The solar f-modes having degree $\ell \lesssim 1000$ are believed to be evanescent in the atmosphere and they propagate in the envelope (and core) to depths which decrease with increasing ℓ. In the solar envelope, the quantities we defined earlier, $P < 0$ and $Q > 0$ for these modes; consequently, they should generally propagate like *p-modes.* However, whether an f-mode will have oscillatory character along the radial coordinate will depend mainly on the quantity, k_r which is the derivative of the phase along the radial direction. It turns out that if the phase undergoes small–scale variation, it can change the sign of the quantity

$$(k_r^2 + k_H^2)c^2 - \omega^2 + \omega_{ac}^2 + N^2 - k_H^2 c^2 N^2/\omega^2 \tag{13}$$

The result will be that, even if the phase difference of the oscillation at a specified time, between the turning points is near zero, the wave will be damped (or growing) along the radial coordinate. We believe that the life time of f-modes should be a good indicator of their possible interaction with turbulent convection in the region where Brunt-Vaisala frequency changes quickly. A possible correlation between their life time, amplitude and departure of their frequency from the value expected for a smooth profile of N^2 could be a valuable probe of the models of convection in the hydrogen ionization zone.

5. Conclusions

We have treated the surface effects on the solar oscillation frequencies as a phase difference and consequent frequency shift in the mode. Understandably, the full effects of dissipation cannot be represented purely as a phase shift. But we expect that the frequency and wave number dependency of the surface effects should probably be correctly analysed from this approach.

We find that for low-order modes, the horizontal wave number dependent contribution to the frequency shift could be comparable to the frequency-alone dependent component. For f-modes, the phase shift and possible random phase introduced in the subphotospheric layers of the turbulent convection zone where hydrogen gets ionized, can be sufficient to change the spatial propagation characteristics along the radial direction.

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