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Periodicities in the Sun's "Torsional MHD Oscillations" and Planetary configurations

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Abstract. Recently, we studied the rotation of sunspot groups during 1879-1976 and detected ~ 22-yr and a few more other periods 'even' and 'odd' parity torsional oscillations of the Sun (Javaraiah & Gokhale 1995a, b). In the present paper we show that the amplitudes of these torsional oscillations of the Sun depend upon the temporal variation in the latitude (celestial) distribution of angular momentum of the solar system.

Key words: Solar rotation-magetic field- activity cycle-solar system dynamics

1. Introduction

Earlier, we have analyzed (Javaraiah & Gokhale 1995a, hereafter Paper I) the Greenwich data on sunspot groups of the period 1879-1976 and detected '22-yr' periodicity and a few more other periodicities in the ratio B/A (and also in B), where A and B are the coefficients in the standard law of differential rotation (see, Section 2). The temporal variation in B or in B/A represents an 'even-parity' (symmetric about the solar equator) torsional oscillations of the Sun. The dominant periodicities in B/A were found to agree, within their uncertainties, with the known periodicities in the photospheric magnetic field. On longer times scales, the variations in the amount of magnetic flux emergence per unit time were found to be related to the amplitudes of the torsional oscillations of period '22-yr'. Thus it was concluded that the solar cycle may be a manifestation of the Sun's torsional magnetohydrodynamic (MHD) oscillations (see also Layzer et al. 1979 and references therein). The periodicities of these oscillations were found to match with synodic periods of two or more consecutive planets. Hence it was suggested that planetary effects could provide perturbations needed to excite these oscillations.

Recently, (Gokhale et al. 1995), we have shown on theoretical grounds that the time-dependent inertial torque on the Sun, which is determined by the relative motions of the solar system bodies, is strong enough to force the torsional MHD oscillations, thereby maintaining these oscillations against the energy loss in the form of solar activity.

Recently, (Javaraiah & Gokhale 1995b, here after Paper II), we have also detected the '45-yr', '22-yr' and a few other periodicities in the temporal variations of the north(N)-south(S) asymmetries of A and B. Similar periodicities were also found in the N-S asymmetry of sunspot activity. The '22-yr' mode of oscillation in the asymmetries of A and B might be related to '22-yr' mode of 'even-parity' found in the magnetic field inferred from the sunspot data (Gokhale & Javaraiah, 1990). The dominant '45-yr' periodicity was found to match with the period of so-called 'Double-Hale' solar magnetic cycle (Fairbridge & Marcel, 1977). Hence, in Paper-II, we concluded that the periodicities in the N-S asymmetries of A and B may represent torsional MHD oscillations of 'odd-parity' in the rotation (i.e., antisymmetric about the solar equator) which correspond to even parity in the magnetic field.

The periodicities in the N-S asymmetries of A and B were found to match with the '*combination periods*' (reciprocals of *combination frequencies*) of major planets. *Combination frequencies* generally originate in non-linear interactions of oscillations (e.g. Main 1984). Hence, in Paper II, we speculated that the Sun may be behaving as a non-linear oscillator excited by the torque exerted by the solar system dynamics.

In the present paper we show that the amplitudes of the symmetric and antisymmetric torsional oscillations of the Sun depend upon the contributions of the major planets to the symmetric and asymmetric parts of the angular momentum of the solar system.

2. Determination of A, B and their N-S asymmetries

Since the present work is based on the results of Paper I and II, here we explain briefly the method of determination of A, B and their N-S asymmetries described in Paper I and II.

The Sun's differential rotation is well approximated by the formula

$$\omega(\lambda) = A + B sin^2\lambda, \tag{1}$$

where $\omega(\lambda)$ is the solar rotation rate at latitude λ, the parameter A represents the equatorial rotation rate and B is a measure of the latitude dependence of the rotation rate.

Fitting the whole sphere data to the above formula we computed A and B, during the 5-yr MTI (Moving Time Intervals).

The N-S asymmetry of the A (denoted by Aa) and that of the B (denoted by Ba) are defined in the following way:

$$Aa = (An - As)/(An + As), \quad Ba = (Bn - Bs)/(Bn + Bs), \tag{2}$$

where An (Bn) and As (Bs) are the values of A (B) derived from the N- and S- hemispheres data respectively.

Fitting separately the N- and S-hemispheres data to the equation (1), we obtained An, Bn, As and Bs during the 5-yr MTI. Using the equation (2), we determined the Aa and Ba during the 5-yr MTI. We have determined the periodicities in B, B/A, Aa and Ba using FFT and MEM analysis. From the analysis of the 'simulated' time seriese of B,

B/A, Aa and Ba, it was confirmed that all the periodicities found in B/A (and B), Aa and Ba were not artifacts of the uncertainties in the coefficients A, B, etc., (for details see Paper I and II).

3. 'Measure' of N-S symmetry and asymmetry in the distribution of angular momentum in the solar system

At a given instant the torque exerted on the Sun by the motions of a pair of planets will depend upon the variation in the contribution of the two planets to the angular momentum in the solar system. The orbits of the planets are slightly inclined to the ecliptic. This causes variations in the latitude (celestial) distribution of angular momentum in the solar system. The maximum N-S asymmetry in the torque exerted by a pair of planets will depend upon the contribution of the two planets tc the N-S asymmetry in the angular momentum distribution.

Let m_1 , m_2 ,.. be the masses of the planets $i = 1, 2, ..$ whose distances from the Sun are R_1 , R_2 ,... and I_1 , I_2 ,.. be the inclinations of thier orbits to the ecliptic. Since B is the coefficient of $sin^2\lambda$ in the equation (1), we define ($\chi_{[i]}$), a measure of the N-S symmetry in the maximum contribution of a set $[i] = [1, 2, ...]$ of major planets to the angular momentum in the solar system as:

$$\chi_{[i]} = \sum_i m_i R_i^2 \Omega_i sin^2(I_i), \tag{3}$$

where Ω_1, Ω_2,.. are respectively the orbital angular velocities of the planets i = 1, 2,...

We define a measure of N-S asymmetry of the contribution (ξ_{jk}) of any two planets 'j' and 'k' in the distribution of the angular momentum in the solar system as follows:

$$\xi_{jk} = |m_j R_j^2 \Omega_j S_j^3 - m_k R_k^2 \Omega_k S_k^3| \ / \ |m_j R_j^2 \Omega_j S_j^3 + m_k R_k^2 \Omega_k S_k^3|, \tag{4}$$

where $S_j = sin(I_j)$ and $S_k = sin(I_k)$.

We find that $\xi_{JS} = 0.473$, $\xi_{JU} = 0.964$, $\xi_{JN} = 0.507$, $\xi_{SU} = 0.987$, $\xi_{SN} = 0.791$ and $\xi_{UN} = 0.894$.

4. Results

In table I, we compare the estimated values of the relative powers (areas under the curves) in the peaks at the '*summation*' and the '*difference*' frequencies in the FFT spectra of B and Ba obtained from 5-yr MTI with the relative values of $\chi_{[i]}$ and of '$\xi_{[j,k]}$' respectively. The $\chi_{[i]}$ corresponding to a given frequency band of B is determined using equation (3) by summing over all the major planets whose *combination frequencies* fall within this frequency band. The '$\xi_{[j,k]}$' corresponding to a given frequency band of Ba is the sum of the values of ξ_{jk} calculated using equation (4) for all pairs of major planets j, k whose *combination frequencies* fall within this frequency band.

In table I, the relative values of powers P_B (and P_{Ba}) at the peaks in the FFT spectrum of B (and Ba) agree fairly well with the corresponding values of the $\chi_{[i]}$ (and $\xi_{[j,k]}$). This strongly indicates that there exists a relation between the parity of the Sun's torsional oscillations and the parity of the angular momentum distribution in the solar system.

Table I. In this table the symbols J, S, U and N denote orbital frequencies of Jupiter, Saturn, Uranus and Neptune respectively. ν_C represents the '*combination frequencies*' and T_C the '*combination periods*' (*in years*) of major planets, T_B (and T_{Ba}) are the periodicities (*in years*) detected in the symmetric (and asymmetric) part of B. Below each value of T_B and T_{Ba}, the level of signficance (in the unit of 1σ, standard deviation) is given. The estimate values of the relative powers at the peaks in the FFT spectrum of B (Ba) obtained from 5-yr MTI are denoted by P_B (P_{Ba}). The relative values of $\chi_{[i]}$ and $\xi_{[j,k]}$ are denoted by χ and ξ respectively.

ν_C	T_C	T_B	P_B	χ	T_{Ba}	P_{Ba}	ξ
$S-U$	45.37	45.0±15.0	0.71± 0.05	0.70	45.0±15.0	1.00±0.05	1.00
$S-N$	35.90	(2.5)			(3.5)		
$S-U$	21.81	20.0±4.0	1.00±0.05	1.00	20.0± 4.0	0.31±0.05	0.27
$J-S$	19.85	(4.0)			(2.7)		
$J-U$	13.81				13.3±1.5		
$J-N$	12.78	11.8±3.0	0.46±0.05	0.51	(2.3)	0.81±0.05	0.83
$J+N$	11.06	(~ 1.5)			10.5±0.5		
$J+U$	10.39				(3.4)		

5. Conclusions and Discussion

From the results in section 4 above, we have drawn the following conclusions:
(1) Amplitudes of the Sun's 'symmetric torsional oscillations' seem to be related to the symmetric part of the latitude (celestial) distribution of angular momentum in the solar system.
(2) Amplitudes of the Sun's 'antisymmetric torsional oscillations' seem to be related to the 'asymmetric' part of the latitude (celestial) distribution of angular momentum in the solar system.

However, the Sun's equator is inclined $7^o.17$ to the ecliptic. Taking this into account, the mechanism of interaction between the Sun's torsional MHD oscillations and the latitude (celestial) distribution of orbital angular momentum in the solar system should be investigated.

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