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Steady parts of rotation and magnetic field in the Sun's convective envelope

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Abstract. We use the full set of Chandrasekhar's (1956) MHD equations for determining the steady parts of rotation and poloidal as well as toroidal magnetic field, in the Sun's convective envelope assuming incompressibility. The boundary conditions at the surface are taken from observations. A differential rotation is assumed to be present at the base of the convection zone. The resulting solution yields the isorotation contours similar to helioseismologically inferred rotation. However, the rotation given by the present study is much closer to the helioseismologically inferred rotation than that given by the previous study (Hiremath and Gokhale 1995b). This suggests that at the base of the convection zone, *differential rotation* is more likely than a rigid body rotation.
The toroidal part of magnetic field structure given by the present study is similar to that given by the previous study, and has values ~ 1 G just below the surface and $\sim 10^4$ G near the base of the convection zone.

Key words: sun: interior - sun : rotation - sun: magnetic field

1. Introduction

Helioseismological studies show that the Sun rotates differentially with a weak dependence on radius throughout its convection zone and has a nearly uniform rotation in the radiative core (Christensen-Dalsgaard and Schou 1988; Dziembowski *et.al.* 1989; Tomczyk *et.al.* 1995).

Based on the earlier works of Wasiutinski (1946); Biermann (1951) and, Kippenhahn (1963), many models were developed for the explanation of differential rotation of the Sun (*e.g* references given in Hiremath and Gokhale , 1995b). Only some of these models (Chan and Mayr 1994; Kitchatinov and Rudiger 1995) simulate satisfactorily the rotation contours given by the helioseismology.

Recently, we (Gokhale and Hiremath 1993; Hiremath 1994; Hiremath and Gokhale 1995a) have modeled the 'steady' (slowly varying on diffusion time scales) part of the Sun's poloidal magnetic field which isorotates as closely as possible with the helioseismologically observed internal rotation. From the steady part of the poloidal field, we modeled the steady part of the rotation. Though, the profiles of the derived rotational isocontours are similar to the observed contours, the values of rotational velocity are different from the 'observed ' values. Thus, the modeled profile may not represent the Sun's real rotational profile.

In our earlier studies (Hiremath 1994; Hiremath and Gokhale 1995a), we have shown that the 'steady' part of the poloidal magnetic field varies on time scales of age of the Sun. We expect similar order of time scales of variations for 'steady' part of rotation and toroidal componant of the magnetic field.

Hence, in later study (Hiremath and Gokhale 1995b), we solved the full sét of Chandrasekhar's (1956) MHD equations for 'steady' part of rotation and, toroidal and poloidal components of the magnetic field . In this, we used the boundary conditions for rotation and toroidal magnetic field same as those used by Nakagawa (1969) and obtained the rotation pattern in the convective envelope qualitatively similar to that given by helioseismology (Dziembowski *et.al* 1989).

In the present study, we make similar assumptions and approximations but with the following difference. ***Instead of assuming rigid body rotation at the base of the convection zone, we assume a differential rotation.*** This solution also yields isorotation contours similar to those obtained in the previous study. However, the internal rotation given by the previous study gave a r.m.s difference $\sim$ 50 nHz from helioseismologically determined internal rotation, whereas present study yields a r.m.s difference $\sim$ 9 nHz.

2. Equations

Following Chandrasekhar (1956), the magnetic field **B** and the velocity **V** in an axisymmetric system can be expressed as

$$\mathbf{h} = -\varpi\frac{\partial P}{\partial z}\hat{\mathbf{I}}_\varpi + (\varpi T)\hat{\mathbf{I}}_\varphi + \frac{1}{\varpi}\frac{\partial}{\partial\varpi}(\varpi^2 P)\hat{\mathbf{I}}_z\,, \tag{1}$$

$$\mathbf{V} = -\varpi\frac{\partial U}{\partial z}\hat{\mathbf{I}}_\varpi + (\varpi\Omega)\hat{\mathbf{I}}_\varphi + \frac{1}{\varpi}\frac{\partial}{\partial\varpi}(\varpi^2 U)\hat{\mathbf{I}}_z\,, \tag{2}$$

where $\mathbf{h} = \mathbf{B}/(4\pi\rho)^{1/2}$, ρ is the density, ϖ, φ, z are the cylindrical polar coordinates, with their axis along the axis of solar rotation; $\hat{\mathbf{I}}_\varpi$, $\hat{\mathbf{I}}_\varphi$, and $\hat{\mathbf{I}}_z$ are the corresponding unit vectors and; P, T, Ω, and U are the scalar functions which are functions of ϖ and z only. As in our previous study, Chandrasekhar's (1956) equations can be reduced to the following equations making reasonable assumptions

$$\Delta_5 T = 0\,, \tag{3}$$

$$\Delta_5\Omega = 0\,, \tag{4}$$

$$\frac{\partial}{\partial z}[\Omega^2 - T^2] = 0\,, \tag{5}$$

where Δ_5 is the five dimensional Laplacian operator.
The above equations yield

$$T(x,\mu) = \sum_{n=0,1,2,\ldots} \left[b_n x^n + c_n x^{-(n+3)}\right] C_n^{3/2}(\mu)\,, \tag{6}$$

$$\Omega(x,\mu) = \sum_{n=0,1,2,\ldots} \left[V_n x^n + W_n x^{-(n+3)}\right] C_n^{3/2}(\mu)\,, \tag{7}$$

subject to the constraint $\quad \Omega^2 = T^2 + f(\varpi),$ (8)

where $f(\varpi)$ is an arbitrary function of $\varpi = x sin\vartheta$; b_n, c_n, V_n, W_n are constants; $x = r/R_\odot$, $R_\odot$ is the radius of the Sun, $\mu = cos\vartheta$, ϑ is the co-latitude and $C_n^{3/2}(\mu)$ are the Gegenbauer polynomials of order 3/2. We assume $f(\varpi)$ to be in the form

$$f(\varpi) = \sum_{n=0,1,2,..} a_n \varpi^n \tag{9}$$

Here the function $f(\varpi)$ and the constants b_n, c_n, V_n, W_n have to be determined from the boundary conditions.

Nakagawa (1969) had assumed each of the functions T and Ω to be a series in Legendre polynomials and applied appropriate boundary conditions. In our model T and Ω are taken in the forms (6) and (7) which are solutions of equations (3) and (4).

3. Boundary conditions

3.1 Boundary conditions at the surface

At the surface we adopt the following boundary conditions for rotation and magnetic field, *viz.*,

$$\Omega = \Omega_{obs} \qquad \text{and} \qquad T = 0\,, \tag{10}$$

where Ω_{obs} is the observed surface rotation expressed in the form

$$\Omega_{obs} = A + B\cos^2\vartheta + C\cos^4\vartheta\,, \tag{11}$$

wherein A, B, C are the coefficients of rotation determined from the observations.

In the previous study, we assumed $T = 0$ on the surface. However, this condition can be derived from the continuity of the current at the surface, assuming that the field outside is current free (Nakagawa and Swartztrauber 1969).

3.2 Boundary conditions at the base

At the base of the convection zone, we assume that Ω, P and T are finite and continuous. There we assume a differential rotation of the form

$$\Omega(0.7, \vartheta) = A' + B' \cos^2 \vartheta\,. \tag{12}$$

4. Results and conclusions

We neglect the negative powers of x in equation (7), which are singular at the center. Assuming rotational symmetry about the equator, we take only even values of n in equations (6), (7) and (9). We solve equations (6)-(9) along with the afore mentioned boundary conditions. Taking the values A,B,C, $A^{'}$ and $B^{'}$ as given by the helioseismology (Goode and Dziembowski 1991), we have computed the rotation $\Omega(x, \vartheta)$ in the convective envelope. The resulting isocontours are presented in figure 1. Note that these rotational isocontours are similar to the helioseismologically inferred isocontours. Except for small differences in $\left|\Omega(x, \vartheta)\right|$, the isocontours obtained here are also similar to those obtained in the previous study.

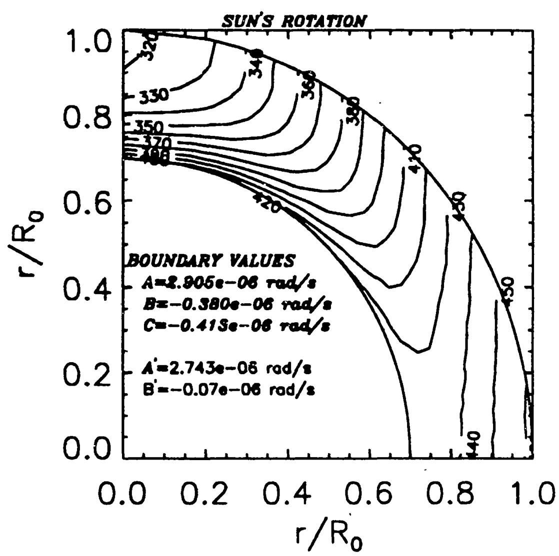

Figure 1. Steady part of Sun's rotation in the convective envelope. Rotational isocontours are in nano-Hertz.

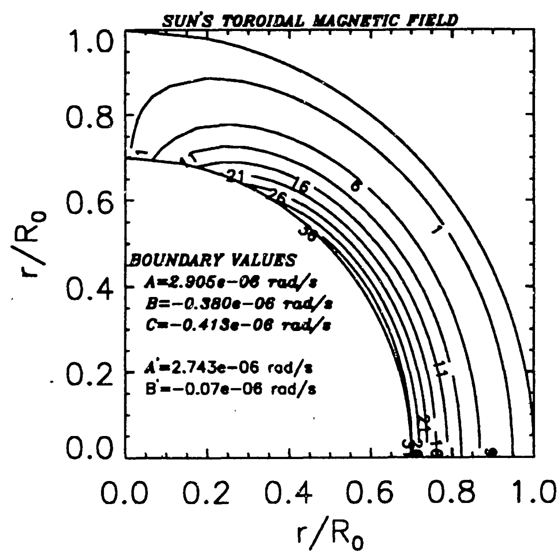

Figure 2. Steady part of Sun's toroidal magnetic field in the convective envelope. Isocontours of ωT are in 10^2 gauss.

We have computed the root mean square (r.m.s) deviations between $\Omega(x, \vartheta)$ and helioseismologically inferred rotation (Dziembowski *et.al.* 1989) in the domain $0.7 \leq x \leq 1.0$ and $10^\circ \leq \vartheta \leq 90^\circ$.

We find that the r.m.s deviation from the present study is ~ 9 nHz and that from the previous study is $\sim$ 50 nHz.

Thus, the present solution $\Omega(x, \vartheta)$ gives much better fit with helioseismological rotation than the previous solution. Assuming that the dominant part of the helioseismologically 'observed' rotation is 'steady', this study suggests that the steady rotation at the base of convection zone is more likely to be a ***differential rotation*** than a rigid body rotation.

Using density values of Spruit's model (1977) in the convection zone, we have computed the steady part of the toroidal magnetic field whose iso-gauss ($i.e.,\ \varpi T$) contours are shown in figure 2. This part of the field structure is weak ($\sim 1G$) near the surface and strong ($\sim 10^4 G$) near the base of the convection zone.

The structure of the iso-gauss contours is ***dipole like*** ($i.e$ one change of sign) field near the surface and ***quadrupole like*** ($i.e$ three changes of sign) field near the base of the convection zone.

It is interesting to note that, near the base of the convection zone, the strong toroidal fields are concentrated near the Sun's middle latitudes. Probabily, ***perturbations of these strong fields may be the sites of sunspots formations***, which are brought to the surface by buoyant forces.

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